THE BUCKINGHAM-OSBURN SEARCHLIGHT ARITHMETICS

INTRODUCTORY BOOK

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The BUCKINGHAM-OSBURN SEARCHLIGHT ARITHMETICS

INTRODUCTORY BOOK

BY

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PREFACE

This is a teachers' book for use by teachers of the first and second years of school. It is preliminary to a series of pupils' books for the first six years of school and the junior high school or for the eight-year elementary school. It is intended to afford both the material and the method for pre-third-grade teaching according to a course of study which is fairly exacting but which is by no means extreme. Nevertheless, teachers whose course of study calls for less than the usual amount of work in the first grade may use the book without difficulty. Indeed, those who do no number work at all in the first grade will find the book suited to their needs in the second grade. They may not cover all the suggested work before the pupils are promoted to the third grade, but the ample review of pre-third-grade work presented at the beginning of the book for the third grade will afford material quite sufficient to permit these third-grade pupils to go on from the place where they stopped in the second grade.

We regard this point as important. Permit us to repeat it. Suggested first-grade and second-grade material is given in this book — the Introductory Book. Similar material is given in the first part of Book One, but the latter is a *pupils'* book, whereas this is a *teachers'* book. In the first place, therefore, pupils who have had little number work in the first and second grades may be taught the rest of the work here suggested after they have reached the third grade. This may be done by placing Book One in their hands and teaching them as much of the first part of it as they need. In the second place, pupils who cover the work of the Introductory Book before they reach the

third grade will, after they have been advanced to that grade, require only a portion of the early part of Book One, and this will be for purposes of review. In the third place, pupils who cover more ground than is laid down in this book for the first and second grades may have Book One placed in their hands sometime during their second school year, say at the middle of it. Such pupils, owing to the recency and intensity of their instruction in what we are calling the pre-third-grade work, will doubtless need to spend little time on the first part of their new book.

Although the amount of material covered by this Introductory Book is less in some respects than is often required in the first and second grades, in other respects it is greater. Experience in using the book, however, has shown that the work may easily be covered with reasonable effort. The progress of the pupils is facilitated in a number of ways:

1. Certain precautions are taken to insure success in teaching the fundamental addition and subtraction facts — precautions designed to reduce error and to prevent counting. The chances of error are reduced by posting the combinations for reference, by setting up an ideal of accuracy, and by encouraging "I don't know" as a proper response when the pupil is in doubt. Counting is prevented by the arrangement of the combinations for teaching purposes, by a whole-to-part rather than a part-to-whole method of presenting each combination in the first instance, and by the posting device already referred to.

2. The related addition and subtraction facts are taught together, thus giving each other meaning and support. In the controlled experiments which we have conducted under classroom conditions the "together" method proved to be superior to the "separate" method in five out of the six centers where these two methods were used.

3. A great deal of attention is given to practicing the combinations after they have been learned. All the combinations

occur a definite number of times and the more difficult combinations appear with additional frequency. Numerous games and devices are suggested, some primarily to provide drill and others chiefly to lend interest to the work.

4. Application, which is a form of practice, is a major purpose. As soon as a combination is learned it is used in problems. These are numerous and interesting. Moreover, we have made many suggestions for teaching the problems in such a way as to make them a vital, integral factor in the teaching rather than something apart. Application is likewise secured by the early use of the facts learned. As soon as the pupils have had the addition combinations whose sums are less than ten, they apply them in column addition and in adding two-place and three-place numbers without carrying. They also apply them in what we call higher-decade addition (addition by endings); and this in turn is a preparation for further column addition and for the process of multiplication, the latter being reserved, however, for the third grade. There is no need of tediously teaching all the addition facts before any of them are put to use. Similarly, the subtraction combinations with minuends less than ten are at once applied in higher-decade subtraction. This is partly for the sake of the drill; for otherwise the subtraction combinations would receive far less practice than the addition combinations, despite the fact that every investigation has shown their greater difficulty. Higher-decade subtraction is also a preparation for division, and it has high utility in its own right. Very many situations in practical life call for it. Finally, after the hard addition and subtraction facts have been taught — that is, those in which the sums and minuends are ten or more — they are likewise applied in column addition and in higher-decade work.

To a large extent this book is a product of research. We have drawn upon experimental data in deciding what we may expect

a child to know upon entering school. Conclusions as to the seriousness of error have led us to take measures to forestall it. The studies of children's vocabularies have aided us in writing our problems. Numerous results of investigation have been pooled in determining the difficulty of the number facts. The demonstrated need of a better distributed drill on the number facts has induced us to provide for every fact and to do so with an eye to its difficulty.

Our own investigations — some of them carried on for many years — have dealt with courses of study, with the situations, units, cues, and number relations exhibited in verbal problems, and with the difficulty of such problems. The data thus secured have constantly guided us in selecting material and in writing and grading it.

We have likewise invoked the aid of controlled experimentation in reaching several decisions. We have had the additive and the take-away methods of teaching the subtraction facts taught to pupils of equal ability in four different centers. In three of these four centers the take-away method proved to be superior. Accordingly, we have adopted the take-away procedure. As has been stated, the "together" and "separate" methods of teaching the addition and subtraction facts were tried with children of equal ability in six centers. In five of them the together method yielded the better results. We have therefore proposed that addition facts be taught along with their related subtraction facts. Again, our decision to present column addition as a downward process was based on its demonstrated superiority to upward addition in five out of six centers where the rival methods were tried.

Finally, the whole book in its original form has been subjected to the test of classroom use. In a very real sense, therefore, the entire method of teaching which the book exemplifies and the particular teaching material contained in it, both as to amount and as to organization, have been evolved on the basis

of experiment. On this basis we have selected many of the procedures and devices which appear in the following pages. Thus we have been led to the adoption of the domino patterns in teaching number ideas, to the utilization of a whole-to-part method of teaching a number fact, to the particular selection of games, to the separation of number facts having common elements, to the use of progress tests, to the extension of the fundamental facts in their higher-decade derivatives, and to a rich and varied offering of concrete verbal problems.

B. R. BUCKINGHAM
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THE BUCKINGHAM-OSBURN ARITHMETICS

INTRODUCTORY BOOK

CHAPTER I

IN THE BEGINNING

What we May Expect

There is good evidence for the belief that at least three fourths of the pupils who enter the first grade can at that time count to ten. On this point we need go no further than the work of Binet. In his intelligence scale he summed up the results of his own investigations and those of others. The Stanford Revision of this scale makes the counting of thirteen pennies — once in two trials — a typical performance of six-year-old children. Since the assignment of an ability to a certain age meant, with Binet and his successors, that about three fourths of the children of that age had the ability in question, we may not unreasonably infer that when children enter the first grade quite three fourths of them can count to ten. During the first two weeks of school all the first-grade pupils should be tested to determine their ability to count with and without objects from 1 to 10.

There is also reason to believe[1] that pupils at this stage of their development can tell by inspection or by counting how many objects there are in groups of *two, three,* and *four.* The preliminary testing should therefore include the examination of the pupils as to this ability. In conducting this examination

[1] E. L. Thorndike, The Psychology of Arithmetic, pp. 200–201. The Macmillan Company, 1922.

(which, of course, should be oral and individual), groups of two, three, and four pupils, or piles of two, three, and four books, or similar collections of any common objects may be used. Pictures are likewise useful for this purpose. Such questions should be asked as "How many boys are there?" "How many girls are there?" "How many books are there in this pile?" "How many horses do you see in the picture?"

The pupils who lack these abilities have very likely had no opportunity to acquire them. It is both unwise and unsafe to assume that they are stupid. They should be instructed individually with the intention of bringing them up to the accepted standard. In this work the services of some of the most capable pupils may be enlisted; for example, Alice, who can "count to a hundred," may help Sadie by hearing her count desks, panes of glass, or even dots and marks.

Children may also see how many things of different kinds they can name; for example, the things seen from the window, the things in front of the room, and the things in the room that are heavy, long, or red. Some rivalry may be developed to ascertain who can name the most things. This sort of activity — helping in counting — has excellent value as vocabulary drill. Indeed, when at a later time this idea is extended to the naming of things not present to the senses — games, toys, things to wear, kinds of animals, and the like — the vocabulary value is exceptional. This matter will be referred to again in Chapter XIII.

Counting to Twenty

Most of the pupils, then, either will come to school with the ability to count to ten or will acquire it through the method suggested above shortly after they enter school. Even in schools where no formal number work is done in the first grade, instruction in counting is generally recognized as valuable. The fact is that children of this age have a real need, and

therefore a desire, for counting. Without much delay, therefore, they should extend their counting to cover the numbers for which they have frequent use.

The first objective may very properly be to "count to twenty." This will be new work for a great many pupils, and it will not be easy. Imitation will be their main reliance and rhythm their greatest help. They may count as they step in walking. They may count as they clap hands or swing their arms. Closing their eyes, they may count while the teacher or a pupil raps on the floor, or they may count taps of a bell, clock-ticks, the bouncing of a ball, or the like. They may name, and count as they name, various classes of things as suggested in a foregoing paragraph. Objects in the room should be counted above ten — desks, panes of glass in windows, books on the teacher's desk, erasers in the blackboard tray, and, of course, children themselves in various groups, postures, and places.

The pupils should be encouraged to count things outside of school, such as the corners they turn, the streets they cross, the people they meet, the automobiles they pass, and objects of various kinds which they see in their homes. They should be asked to report the results of some of these observations, and, as they do so, interesting comparisons among their findings may be made.

There will be use immediately for such words as *more*, *less*, *same*, *many*, *most*, *least*, *few*, and *fewer*. These words should be freely employed and their meanings established. When looking about for something to count, Tom may have seen eight men working in Jones's garage and five working in Tuttle's print-shop. Were there *more* men working in the garage or in the print-shop? Which place had *less* (or *fewer*) men? Ethel may have found that twelve men were at work on building the new post office. Which place — the garage, the print-shop, or the post office — had the *most* men? Which the *least*? Robert

and Helen, let us suppose, each counted the automobiles parked between the schoolhouse and the corner. Robert reported five. On hearing this Helen said she counted the *same* number. How *many* did Helen count?

Learning Numbers as Applied to Collections of Things

Recognition of number groups is important. As has already been indicated, most children — quite three fourths according to the investigations of Hall — will be able to tell, either by inspection or by counting, the number of objects in groups as large as four. This work should be extended until under proper conditions they can recognize the number of things in a collection of six without counting. Pupils may be expected to *sense* 1, 2, and 3 in any arrangement and to recognize 4, 5, and 6 without counting when the items are in a standard arrangement. We shall find it useful to adopt, for the purpose of a standard, the arrangement found on dominoes. Frequent use should be made of the domino arrangements from 1 to 6, especially 4, 5, and 6, and drill should be provided in which children first answer the question "How many are there?" and then make the standard number patterns from dictation; for example, "Make three" and "Make six."

Pupils should likewise make collections of objects upon the request of the teacher or of a pupil selected either by the teacher or by the class. Thus collections of boys, pennies, balls, etc., may be called for.

When groups are larger than six, pupils will be expected to tell how many there are *by counting*. In this manner they will find out how many things there are in collections of 7, 8, 9, and 10. On the other hand, they will make up such collections when told to do so. The more capable pupils may even recognize without counting 7, 8, 9, and 10 things if they are in a standard pattern. Again the domino arrangements are

suggested. Their clearness is apparent and the groupings are familiar. Arrangements such as those of Fig. 1 will be found helpful.

Be careful, however, not to limit the notion of a given number to the particular domino arrangement of that number. The children should not suppose that "5" is "5" only when it is in the domino pattern. Objects and pictures in haphazard arrangement should be counted frequently. After a child has counted the number of objects in a group and has told how many there are, his answer may be checked by having him put the objects in domino form and then having him

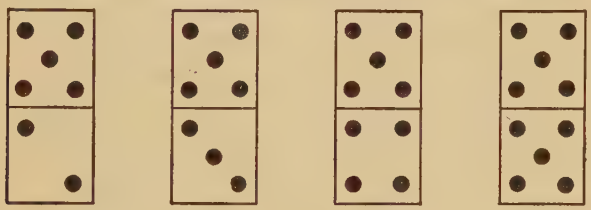

FIG. 1

compare his answer with the familiar pattern. It should be made clear, however, that the domino form is only a convenience to help us in telling how many there are and that it is but one of many possible arrangements.

When the plan just described is followed, a pupil's idea of a number should be twofold: first, that the number has a fixed place in a series and, second, that it tells the number of things in a group. For example, 5 is that number which is more than 4 and less than 6, and it is also the number which tells you how many there are in a certain definite collection of things which may be arranged like this ⁘. There are of course other meanings of 5, but these meanings do not appear to be necessary in this early stage of the pupil's learning. Indeed, it would be folly to attempt to teach 5 as "5 times whatever 1 is" until units of measurement have been taught

and pupils have been given some idea of measuring. Again, it would be manifestly impossible to teach 5 at this point as $2 + 3$ or $4 + 1$. This meaning of 5, however, will come later when the necessary facts are taught.

Reading and Writing the Numbers to Ten

Reading numbers. The next step is for the teacher to associate written Arabic figures with groups of objects whose numbers are already known. For example, the pupil already knows that :: means *four*. Let the teacher now show him "4" in connection with :: and let her bring out the fact that this is another way of expressing *four*. Pupils may have the standard patterns shown them with the Arabic numerals under each pattern as in Fig. 2. They should then be drilled in reading these numbers and in pointing to them. At first they should have both the dots and the figures before them, and they should point to 4, 6, 2, 9, etc., as these are called for by the teacher or by a selected pupil. Next the patterns, or dots, should be omitted and the same kind of drill in reading should be provided with the figures alone.

The teacher may sometimes give the pupils directions through the use of numbers written on the blackboard. For example, the teacher may say, "Mary, bring me (writing "6" on the board) books from the table," or "John may touch (writing "5" on the board) children in his row," and so on. Later the teacher may refer pupils to the pages of their reading books by putting numbers on the board.

Writing numbers. At first the pupils should *copy* numbers beginning with the easiest forms, such as 1, 4, and 7. The forms 0, 9, 6, and 5 are of intermediate difficulty. Special care must be exercised with 3, 2, and 8. Do not limit the copying of figures to the serial order. The presentation for reproduction should involve a random as well as a regular order.

Let it be understood at the outset that copying figures accurately is a difficult matter. Numerous investigations have brought out this point. For this reason considerable care should be devoted to the correct formation of the figures and to their rapid and accurate reproduction. In subsequent grades many mistakes are likely to be made through writing one figure for another when pupils are copying either from the

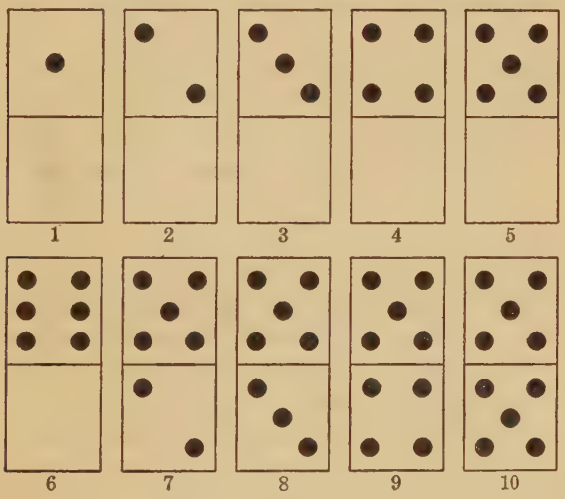

FIG. 2

board or from printed material. As far as possible this difficulty should be forestalled. Now is the time to do it.

The next step is the writing of figures from *dictation*. This, of course, is even more difficult than copying, but if there has been enough practice in copying, the added difficulty will hardly be apparent. Pupils should be expected to write the figures in order and at random, and they may also be expected to write "the number that comes after 6" or to write "the number that comes before 9." This writing and copying of numbers will be carried forward with the maximum of practice

and with greater interest after some of the addition and subtraction combinations have been taught.

The final step in the writing of numbers is what may be called *original writing*; that is, the forming of the characters in response to the child's own number ideas. These ideas, however, may be controlled for teaching purposes. The domino patterns may again be used. The pupils may be asked to write the number of noses or eyes they have, the number of fingers on one hand, the number on both hands, the number of windows in the room, or the number that tells how old they are.

Individual reporting may be connected with writing numbers. The children may be asked how many pets they have at home. Each one may write his answer and then read it aloud. Similarly he may write and report as to the number in his family, the number of streets he crosses in coming to school, the number of automobiles he counts on his way to school, the number of grocery stores he knows about, the number of games he can think of. As in the case of copying and writing from dictation, original writing will find its greatest application when the pupils begin to learn the combinations.

Subsequently the writing of the *words* "one," "two," "three," etc., and the correspondence of these words to the Arabic numerals should be taught. Whether and to what extent this is now done depends upon the ability of the pupils. In this as in other matters we are interested in setting forth the pre-third-grade work — the work which will qualify pupils for the study of arithmetic with the help of a textbook. In many matters it is by no means necessary that a particular order should be observed.

Games and Devices

The spirit of play should be evident in the primary grades. This spirit is far more than mere devices or games. It may exist without them, but most teachers find it easier to motivate

work by a more or less organized appeal to the play instinct. Certain very simple number games may be employed in connection with learning numbers and reading and writing them.

A matching game. In order to make number ideas more meaningful, certain games or devices for seat work may be used. They involve some effort on the teacher's part in

3	2	5	10	6
1	3	8	7	10
9	5	4	2	6

FIG. 3

preparing material, but most of the material, when once made, may be used indefinitely. A game like the one described below may be played.

On an ordinary sheet of drawing paper, mark off fifteen two-inch squares in three rows of five squares each. In the upper left-hand corner of each square write small Arabic numbers from 1 to 10 in random order. A sheet ruled and numbered may look like Fig. 3. You will need one of these ruled sheets for each pupil.

Now cut out sets of fifteen cardboard oblongs about $1\frac{3}{4}$ inches long and $\frac{7}{8}$ of an inch wide. Divide the oblongs into two equal squares. Then make domino patterns corresponding to the

numbers on the diagram. Let 7, 8, 9, and 10 be represented by 5–2, 5–3, 5–4, and double 5, as in Fig. 2.

It will be well to provide each child with a diagram and the corresponding "dominoes." Let him keep them in his desk and when he has finished assigned tasks and has time to spare let him play this matching game — a game in which he will place a "domino" in the square having the corresponding Arabic number. Fig. 4 shows the top row of Fig. 3 after the matching has been correctly done.

When the entire diagram is filled, the child lets the teacher know that he has finished. He may write his name in an assigned place on the blackboard, if he has learned to do so.

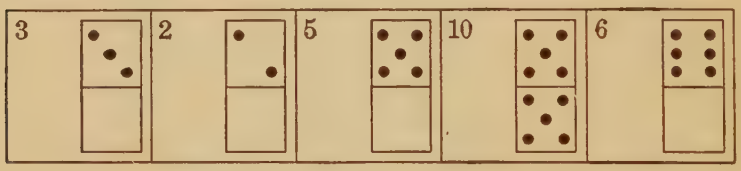

FIG. 4

The teacher at her first opportunity inspects the work and opposite the child's name writes the number of his correct matchings.

In making up this material for a class it is suggested that the diagrams, together with their accompanying sets of "dominoes," be varied as much as possible. If this is done, the charts and sets may be redistributed without the likelihood that pupils will memorize them.

A variation of the matching game just described may be used to require, not the activity of selecting the appropriate number of spots, but the opposite activity of selecting the proper Arabic number. In this case the material would consist of the 15 squares on the chart with the domino spots exhibited on them. Each pupil would then be furnished with 15 small squares, and these squares would have on them

the Arabic numbers corresponding to the "dominoes" on the chart. The game would then be to select the right number squares and place them on the chart, matching the proper "domino." The top row of a chart might then look like Fig. 5.

This matching game may also be played at the blackboard. The teacher may draw the diagram where the children can reach it, putting in the Arabic numbers. Pupils may be called on successively or two at a time to put in the spots. Or, after the children have learned to write the numbers, they may insert them, matching the domino patterns which the teacher has previously drawn on the blackboard.

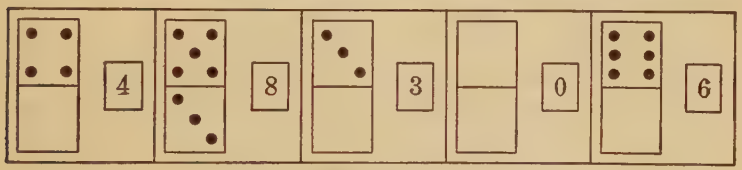

FIG. 5

Naming numbers. The teacher writes several numbers on the blackboard. A child runs to the board and touches any number he chooses, at the same time naming aloud the number, and runs back to his seat. Another child runs to the board and, after touching and naming that number, does the same to an additional one. A third child now runs to the board and touches and names the last-named number and one additional number. This is continued, each child touching and naming the last number and one other. At the present juncture, when the reading of numbers has not gone beyond 10, the same number may well appear on the board more than once. The game may continue to be played after the children have learned to read numbers beyond 10.

Identifying numbers. The teacher stands cards in the chalk tray with their numbers to the wall. She selects one pupil

from each of two rows. At the word "go" each of these pupils runs forward and takes a card. He looks at the number, names it, shows the card to the class, puts it back on the tray face to the wall, and runs back to his seat. He scores for his row if he names the number correctly and reaches his seat ahead of his opponent. Two other children, from the same rows, continue the procedure until every child in both rows has had a chance. The teacher keeps score and the row with the largest number of counts wins. In this game, when played by children who cannot read numbers beyond 10, more than one card may bear the same number. The game may continue to be used after the children have learned to read numbers beyond 10.

Marching game. The teacher prepares two types of cards: those bearing numbers, one on each card; and those bearing pictures illustrating numbers. Even arbitrary signs such as circles, dots, crosses, or marks will do. Cards of the first type are given to part of the class and cards of the second type to another part. These cards are either pinned to the clothing of the children or carried by them so that they may easily be seen. The teacher calls on a pupil who has a card of Type I. This pupil begins to march around the room. All the pupils who hold cards of Type II which express the same number may march with this pupil. Other pupils are called and are in turn followed by those who have the corresponding cards of Type II.

Other games. Many other simple devices characterized by the play spirit will be used by the resourceful teacher. Numbers may be read from the rounds of a ladder drawn on the blackboard. Children may be given packs of number cards in disarranged order, and they may be asked to arrange them in counting order. They may read numbers in connection with some of their seat work, such as stringing beads, with a number card to tell how many beads of each color to use, and

constructing articles with sticks. The play element may be introduced in the writing of numbers in most of the activities which have already been mentioned. The children may make flash cards. They may draw a ladder and insert the numbers. In counting, all sorts of interesting procedures may be adopted. One may be blindfolded and count to 20 while the chalk is hidden, or while the children hide in the game of hide and seek. One may count jumps while jumping a rope, blocks to be used in making a block house, the things needed in setting a table, and the like.[1]

Summary

Up to this point the object has been to have the pupils acquire the following abilities:

1. Counting to 20.
2. Recognition of groups of 1, 2, and 3 objects in any arrangement without counting.
3. Recognition of groups of 4, 5, and 6 objects in a standard arrangement (dots as in dominoes) without counting.
4. Recognition by counting of groups of 4 to 10 objects in any arrangement.
5. Reading the Arabic numerals from 1 to 10, both in sequence and at random.
6. Writing from copy or dictation or from number ideas the Arabic numerals from 1 to 10, both in sequence and at random.

[1] We are indebted to Professor Walter S. Guiler for many of these suggestions. See his "Teaching Arithmetic through Games and Other Pupil Activities." Edwards Brothers, Ann Arbor, 1924.

CHAPTER II

THE ORDER OF TEACHING THE BASIC COMBINATIONS

In no case should a combination be taught until the pupils have learned the meaning of the numbers of which it is composed. They should also be able to read and write these numbers. This includes the sum as well as the addends. In other words, do not teach the combination $\begin{array}{r} 5 \\ \underline{4} \end{array}$ until 5, 4, and 9 are known as number ideas, and not until each of these numbers can be read and written.

Some Principles to Bear in Mind

Preventing counting. You will soon see that we have adopted a special order in which to teach these all-important facts of arithmetic. Three principles have guided us in making this arrangement. First and foremost, our object has been to *prevent counting*. Writers on the teaching of arithmetic are agreed that counting causes slow and inaccurate work. We do not refer merely to finger counting. Even if you tie a pupil's hands he may still be counting just as truly as though his fingers were flying. A child may count with his tongue or his toes. He may tap with his pencil. The counting may be quite without outward manifestation. Indeed, the essence of counting is inside a pupil rather than outside him. The surest treatment is a preventive one.

Accordingly, we have arranged the combinations for teaching purposes in such a way as to facilitate the learning of the combinations purely as facts. For example, we believe that

to begin with $1 + 1$, and to follow this by $2 + 1$, $3 + 1$, etc., is to encourage counting from the outset. We therefore begin with $2 + 2$, not only because it fails to suggest counting but also because, as will appear later, it is almost the easiest of the combinations. Moreover, we have so distributed all the combinations involving 1 that no two of them are taught in succession.[1] This is one of several ways in which we have sought to prevent the habit of counting. The other ways, however, have to do with the teacher's method of instruction rather than with the order in which the combinations are taken up.

Reducing interference. Second, we have arranged the combinations for teaching purposes so as to reduce interferences. It has been suggested by some writers that if two combinations are learned together, the presence of the same addend in each will set up an interference.[2] For example, if "4 and 3 are 7" is learned together with "4 and 5 are 9," it is likely that the presence of the 4 in each case will cause some children to interchange the 9 and the 7. They should be spared this difficulty. Whether or not two combinations whose *sums* are the same interfere with each other is not so clear, for instance, $3 + 4 = 7$ and $2 + 5 = 7$. The likelihood of interference is still less when an addend of one combination is the sum in another, as $4 + 5 = 9$ and $2 + 3 = 5$. We do not know enough about the existence or the strength of such interferences, but in the absence of definite knowledge it is safer to keep similar combinations apart until each of them has been fairly well learned. The argument for this is much the same as the argument for teaching separately the spelling of *there* and *their*.

Taking account of difficulty. Third, the order of presenting the basic combinations which we are suggesting follows closely

[1] 1 and 1, a combination which may generally be omitted so far as presentation is concerned, is an exception (see page 20).

[2] Garry C. Myers, The Prevention and Correction of Errors in Arithmetic, p. 29. The Plymouth Press, Chicago, 1925.

the learning difficulty of each of them. A number of investigators have brought out the fact that these combinations are by no means of equal difficulty. They agree that the size of the sums has much to do with the difficulty of the addition combinations and that in like manner the size of the minuends affects the difficulty of the subtraction combinations. It is true that there are important exceptions to this rule, exceptions too great to be neglected in teaching. It is entirely practicable, however, to divide the combinations into an easy and a hard group according as the sums or minuends are less than 10 or are between 10 and 18. If this is done, there will be, in both addition and subtraction, 55 easy combinations, including those which involve zero, and 45 hard combinations. Thus 100 combinations are required in each operation, or 200 in both. We often hear of "the 45 combinations." The limitation of the 100 addition combinations to 45 assumes (1) that a combination is the same whether it occurs in direct or reverse order and (2) that the zero combinations may be neglected. Neither of these assumptions is true. This is abundantly attested by evidence now in hand: $2 + 5$, for example, is considerably different from $5 + 2$; and such combinations as $8 + 0$ and $0 + 4$ are too hard to be neglected.

Grouping the Easy Combinations

The order of procedure may be still further adapted to the principles we have set up by dividing the so-called easy combinations, those whose sums or minuends are less than 10, into subgroups for teaching purposes. As has been said, there are 55 easy addition combinations and 55 easy subtraction combinations, or 110 easy combinations in all. We have separated these easy combinations into five teaching groups, as will be seen in Table I. Group I has five so-called teaching units. A teaching unit is a certain amount of material which

is to be presented together. In this instance, it consists of a series of related combinations. The first teaching unit in Group I is the $2 + 2$ unit.[1] It is intended, however, that this unit shall contain not only $2 + 2$ but also the related subtraction combination $4 - 2$. As is implied in the fact that these two combinations are put in the same teaching unit, they are to be taught in close connection with each other. Since $2 + 2$ is a double there are of course no reverse combinations and only two facts are therefore included in the teaching unit. The second teaching unit is the $1 + 4$ unit.[2] It consists not only of $1 + 4$ but also of the three related combinations $4 + 1$, $5 - 1$, and $5 - 4$; and these are to be taught together. Since all of them taken as a related group are designated in Table I by the addition combination $1 + 4$, the *name* of this teaching unit may be taken as "one and four." The combination by which a teaching unit is named may be called its key combination. It is always an addition fact and, when not a double, is the one in which the smaller number is written and read first. For example, the name, or key combination, for the second unit in Group I is $\underline{\overset{1}{4}}$, read "one and four," not $\underline{\overset{4}{1}}$. The key combination is sometimes called the direct addition combination or fact.

Groups II, III, and V likewise have five teaching units, and each of these units has two or four combinations according as it is or is not a double. Group IV consists of the zero combinations, of which there are 19 in addition and 19 in subtraction.[3]

[1] This is printed as $\underline{\overset{2}{2}}$ in the third column of Table I.

[2] Printed as $\underline{\overset{1}{4}}$ in the third column of Table I. It is to be understood that these combinations are *read downward*. Therefore $\underline{\overset{1}{4}}$ is the same as $1 + 4$, or 1 and 4.

[3] See Chapter XI.

TABLE I. PLAN FOR TEACHING THE EASY ADDITION AND SUBTRACTION COMBINATIONS

(Combinations are to be read downward, that is, $\frac{1}{4}$ is to be read "one and four.")

TEACHING GROUP	TEACHING UNITS IN ORDER OF PRESENTATION	KEY COMBINATION (DIRECT ADDITION FACT)	DIFFICULTY INDEX [1]
I	1	$\frac{2}{2}$	26.7
	2	$\frac{1}{4}$	39.2
	3	$\frac{3}{3}$	38.5
	4	$\frac{1}{7}$	38.4
	5	$\frac{1}{1}$	39.3
	Group average		37.0
II	1	$\frac{1}{2}$	36.2
	2	$\frac{4}{4}$	42.0
	3	$\frac{1}{6}$	40.1
	4	$\frac{4}{5}$	47.9
	5	$\frac{1}{3}$	38.2
	Group average		40.2

[1] A combination of the results of eight studies by five investigators.

TABLE I (*Continued*)

TEACHING GROUP	TEACHING UNITS IN ORDER OF PRESENTATION	KEY COMBINATION (DIRECT ADDITION FACT)	DIFFICULTY INDEX
III	1	2 3	40.9
	2	1 5	39.9
	3	2 4	43.1
	4	1 8	44.5
	5	2 5	46.9
	Group average . . 20 . .		43.4
IV	Zero combinations 38		
V	1	3 4	45.2
	2	2 6	47.8
	3	3 5	50.0
	4	2 7	50.4
	5	3 6	51.3
	Group average . . 21 . . .		49.2

In Table I the last column shows for each teaching unit the difficulty of all the associated facts in that unit taken together. For example, in the case of $\frac{1}{4}$ the figure 39.2[1] represents the combined difficulty of the four combinations $1 + 4$, $4 + 1$, $5 - 1$, and $5 - 4$.

The order of presentation suggested in Table I does not conform strictly to the second and third principles stated above. Any arrangement of the combinations which would in every case keep apart those having the same numbers would depart so widely from the order of difficulty as to be undesirable from that point of view. On the other hand, an arrangement strictly according to difficulty would often bring together combinations involving the same numbers. A compromise is the best procedure, and Table I is the result of many attempts to secure an order which would do as little violence as possible to the principles we have laid down. In no case is the irregularity serious. For example, the teaching units $1 + 7$ and $1 + 1$, taught consecutively in Group I, do indeed have an addend in common. But the two combinations are so radically unlike, one being a double and having a very small sum, that no interference in learning is likely to occur. In Group II, $4 + 5$ and $1 + 3$ come together and the same is true of $2 + 3$ and $1 + 5$ in Group III, but the interference due to a *sum* and an addend being the same is almost certainly slight. In Group V two instances occur in which consecutive sums are the same. In one case $2 + 6$ is followed by $3 + 5$ and in another $2 + 7$ is followed by $3 + 6$. As has already been pointed out, however, identical sums in successive combinations probably constitute very weak cases of interference; they may even facilitate learning instead of hindering it.

[1] This figure is not a per cent. It represents a merger of all the available studies concerning the difficulty of these number facts for elementary-school children.

The Difficulty of Each of the Easy Combinations

It has been suggested above that the order of teaching the combinations should take account of their relative difficulty, but since it is economical of time to teach related combinations together, the order of presentation cannot be finely adjusted to difficulty. An example will illustrate. In Table I the difficulty index of the four related combinations designated by the key combination $\underline{7}^{1}$ is given as 38.4. This, as has been explained, is the average difficulty of $\underline{7}^{1}$, $\underline{1}^{7}$, $\underline{-1}^{8}$, and $\underline{-7}^{8}$. The difficulty index of *each* of these four combinations is as follows:

$\underline{7}^{1}$	35.0
$\underline{1}^{7}$	38.5
$\underline{-1}^{8}$	44.0
$\underline{-7}^{8}$	36.0
Average	38.4

This varying difficulty of particular combinations within a teaching unit must be taken care of in the drill on these combinations after they have first been taught. With respect to the teaching unit $1+7$, the indications are that more practice must be given to $\underline{-1}^{8}$ than to $\underline{7}^{1}$. Moreover, although these figures do not show it, the subtraction combinations need

more practice than the addition combinations, and this is true in general. Subtraction is harder than addition.[1]

One of the investigators to whom we have referred reported the per cent of errors made by pupils on all the addition and subtraction facts.[2] Table II is arranged from his data. It shows the difficulty of all the so-called easy combinations, that is, the 55 addition combinations whose sums do not exceed nine and the 55 corresponding subtraction facts whose minuends do not exceed nine. In the table the "per cent of error" entered after each combination is the per cent of 7,414 pupils in Grades III to VIII inclusive who missed the combination in question. The 110 combinations are arranged in the order of their difficulty, beginning with the easiest. Observe that the range of difficulty is very wide. Only 0.2 per cent of these pupils got $0 + 0$ wrong, that is, only two per thousand. On the other hand, 17.1 per cent, or 171 pupils per thousand, missed the combination $8 - 5$. Observe too that the subtraction facts are considerably harder than the addition facts. This means that we must give more attention to the subtraction facts. We must teach them carefully and drill on them often.

Two other points in the table are worthy of notice. (1) There is an appreciable tendency for the "large" combinations to be hard, large combinations being those whose sums or minuends are large. In general a teacher must put more effort on the large combinations. (2) The zero combinations in subtraction stand high in the table; that is, they are hard. It is plain that we cannot neglect these zero combinations either in teaching or in drill.

1

[1] The reason the figures given for the $\frac{7}{-}$ teaching unit fail to show that the subtraction facts are harder than the addition facts is that, in computing the difficulty index, the combinations were ranked from each operation separately. Evidently a subtraction combination might rank as easy among subtraction combinations and yet be hard in comparison with addition combinations.

[2] Frank L. Clapp, "The Number Combinations, their Relative Difficulty and the Frequency of their Appearance in Textbooks." *Bureau of Educational Research Bulletin No. 2*, University of Wisconsin, July, 1924.

TABLE II. THE DIFFICULTY OF EACH COMBINATION UP TO SUMS AND MINUENDS OF 9 (After Clapp)

(This is not the order in which the combinations are to be taught.)

COMBINATION	PER CENT OF ERROR	COMBINATION	PER CENT OF ERROR	COMBINATION	PER CENT OF ERROR
0 + 0	0.2	8 + 1	4.5	4 − 3	6.8
0 − 0	0.4	7 + 2	4.6	5 − 4	6.9
2 + 2	1.7	0 + 5	4.6	2 − 1	6.9
1 + 7	1.8	6 − 6	4.6	7 − 5	7.0
2 + 1	1.8	0 + 1	4.7	6 − 4	7.0
1 + 6	2.1	0 + 7	4.7	6 − 1	7.0
1 + 3	2.1	4 − 2	4.7	2 − 2	7.2
5 + 4	2.3	8 − 1	4.7	4 − 1	7.2
1 + 1	2.4	7 + 0	4.8	8 − 7	7.3
3 + 3	2.5	5 + 1	4.9	2 + 6	7.3
6 + 1	2.5	0 + 9	4.9	9 + 0	7.5
2 + 0	2.6	0 + 3	5.0	9 − 5	7.5
1 + 8	2.7	1 − 1	5.2	7 − 6	7.6
4 + 0	2.8	5 − 2	5.2	6 − 2	7.9
3 + 1	2.9	5 + 3	5.2	8 − 6	8.0
4 + 1	3.0	1 + 2	5.2	7 − 2	8.4
4 + 4	3.0	4 + 2	5.2	9 − 0	8.6
2 + 5	3.1	1 + 0	5.3	6 − 0	8.8
7 + 1	3.3	5 + 2	5.3	9 − 3	8.9
4 + 3	3.4	8 + 0	5.4	7 − 3	8.9
5 − 1	3.4	5 − 3	5.4	8 − 8	9.1
1 + 5	3.4	9 − 1	5.6	8 − 2	9.2
3 + 2	3.5	9 − 9	5.6	9 − 6	9.2
5 + 0	3.6	4 − 4	5.8	5 − 0	9.3
7 − 7	3.6	2 + 7	5.8	3 − 0	9.6
1 + 4	3.7	6 + 3	6.0	4 − 0	9.6
3 − 2	3.8	9 − 8	6.0	9 − 4	9.9
6 + 0	3.9	3 + 5	6.2	9 − 7	10.3
0 + 8	4.0	2 + 3	6.3	2 − 0	10.4
4 + 5	4.2	3 + 4	6.4	8 − 0	10.4
3 + 0	4.3	0 + 6	6.5	7 − 0	12.2
3 − 3	4.4	8 − 4	6.5	8 − 3	13.0
6 + 2	4.4	6 − 5	6.5	1 − 0	14.1
2 + 4	4.4	5 − 5	6.6	7 − 4	14.6
0 + 2	4.4	3 − 1	6.8	9 − 2	16.2
0 + 4	4.4	7 − 1	6.8	8 − 5	17.1
3 + 6	4.4	6 − 3	6.8		

Summary

This chapter has been devoted to a statement of theory and principles; and this summary is intended to show the considerations which have guided us in arranging for teaching purposes the fundamental combinations in addition and subtraction.

We have said that these combinations should not be taught until the meaning of the numbers involved in them is known and not until these numbers can be read and written. In deciding what the teaching order shall be we have attempted (1) to prevent counting, (2) to reduce interference in learning, and (3) to take account of the difficulty of the learning material.

We have first divided the 100 addition facts into easy and hard combinations, the easy combinations being the 55 whose sums are less than 10. Similarly we have divided the 100 subtraction facts into easy and hard combinations. The easy subtraction combinations are likewise 55 in number and their minuends are always less than 10. Next, we have divided the 110 easy combinations in addition and subtraction into five teaching groups, each group being more difficult than the one before it. The hard combinations we have left for future treatment (Chapter XXI). Finally, each of the teaching groups except the fourth, which is devoted to the zero combinations, has been further subdivided into teaching units, each composed of the four or, in the case of doubles, two combinations involving the same three numbers. Each teaching unit is designated by a so-called key combination which gives the unit its name, that is, the $\underline{3}^{2}$, or two and three, unit, which comprises $2+3$, $3+2$, $5-2$, and $5-3$.

Just as we have sought to reduce interference in learning, so we have sought to increase facilitation. We therefore

advise the teaching of the subtraction combinations along with the related addition combinations. With $2 + 3$ teach the related $5 - 2$. With $2 + 5$ teach the related $7 - 2$. Both these types of facts must be learned, and they can never be learned as easily as they can when they lend each other mutual support.

Finally, we have given what we regard as the best determination which our present knowledge permits of the difficulty of each of the easy addition and subtraction combinations. Earnest teachers of pre-third-grade arithmetic will be able to make many uses of this table in adjusting their emphasis both in teaching and in drill. In particular they will find that they cannot neglect combinations involving zero and that they must teach skillfully and drill extensively on the subtraction facts.

CHAPTER III

THE FIRST GROUP OF COMBINATIONS

The First Teaching Unit

The first teaching unit is "two and two." It involves the combinations $2 + 2$ and $4 - 2$, or, as we shall write them for some time in our pre-third-grade work, $\underline{2}$ and $\underline{-2}$. Before you begin, be sure the pupils know the numbers which enter into the combinations. This is a general precaution applying to all teaching units. Your teaching procedure will be of little avail if the pupils do not know the numbers about which you and they are talking. In the actual teaching process, the usual pedagogical principles should be in evidence, together with this added principle peculiar to teaching the basic combinations: "The children must not arrive at the answer by counting." The method which follows may be used in teaching any combination.

Teaching "two and two are four." Let the teacher call to the front of the room four boys or four girls. The teacher first asks and secures an answer to the question "How many boys [or girls, as the case may be] are there?" At this point, that is, at the presentation of the *total* group, the pupils may count if they cannot give the correct answer without doing so. In this case, however, the pupils will almost certainly be able to reply correctly without counting. After securing several responses to the question "How many boys are there?" the teacher may divide the four boys into groups of two, keeping the groups close together.

Again let the teacher ask: "How many boys did you say there were?" or "How many boys are there altogether?" The answer should be obtained and repeated if there appears to be any doubt about it.

TEACHER. How many boys are there here? [*Indicating one of the groups*]

PUPIL. Two.

TEACHER. How many boys are there here? [*Indicating the other group*]

PUPIL. Two.

TEACHER. How many boys are there altogether?

PUPIL. Four.

TEACHER. Then two boys and two boys are how many boys?

PUPIL. Four.

The teacher may then apply the same method to other concrete material, such as books, pencils, splints, and beads. But in all such cases the counting, if there is any, should be *before* the whole group is separated into two parts.

After the number fact has been presented in concrete form the teacher should write on the board the form $\begin{matrix}2\\4\end{matrix}$, and as she writes she should say, "Two and two are four." Then the same statement should be obtained from the pupils and they should write it.

The corresponding subtraction fact. After having taught $\begin{matrix}2\\4\end{matrix}$ the teacher should at once present the corresponding subtraction fact $\begin{matrix}4\\-2\\2\end{matrix}$. It is an unnecessary waste of time to teach the subtraction facts apart from the related addition facts. In other words, just as we should avoid interferences in learning, so we should take advantage of circumstances that

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facilitate learning. This fact may, as in the case of the addition fact, be represented with several kinds of material. For example:

TEACHER. [*Holding four books*] How many books have I?

PUPIL. Four.

TEACHER. If I now take away *two* books [*taking two from the pile and putting them out of sight*] how many books have I left? [*Showing the remaining two*]

Or the teacher may say: "I have four books in my hands. I take away two of them. How many have I left?"

At this point a question arises as to the form in which the subtraction facts shall be stated.[1] We may have the children say "four less two are two" or "four minus two are two" or "two from four are two." There are other variations, but these are the most common. For reasons that come out clearly in the subtraction of large numbers, it is much to be preferred that the minuend be named first, followed by the subtrahend and the remainder in that order. It will therefore be best not to use at present the form "two from four are two." The word "minus" is a new term, and in order to concentrate upon as few difficulties as possible, let us reserve this term for use later on. The form "four less two" remains to be considered. It is true that this form is often used when pupils are learning their first subtraction combinations, and no serious error will be made in adopting it. We believe, however, that an easier statement is possible. Accordingly we recommend that a less elegant but more readily understood form be used at this period of the pupils' training. If we say (enduring what to our adult ears is a somewhat clumsy expression) "5 take away 3 are 2," we shall have a form of expression which the child will easily grasp and one to which we can suit the taking away of objective material showing a visible

[1] For additional treatment of this question, especially as to the advantages of the take-away method of subtraction over the addition method, see Chapter IV.

remainder. A fuller form of question may often be used in this early teaching of subtraction, such as "If from 5 I take away 3, how many are left?" The pupil may make the statement in the form "If from 5 I take away 3, I have 2 left."

The essential points that we wish to make are: (1) Let the minuend be named first, then the subtrahend, and the remainder last. (2) Use the words "take away" and, in the actual teaching, suit actions to the words. (3) Use frequently the words "have left" or their equivalent.

After having taught $4 - 2$ by means of concrete materials, the teacher should write on the board the form $\begin{array}{r} 4 \\ -\,2 \\ \hline 2 \end{array}$. As she writes she should say "4 take away 2 are 2," and explain that the mark in front of the 2 means "take away." Later it may be called "less" and still later "minus." The children should repeat the statement. Then they should copy the form and write it from dictation. "Don't forget the mark that says 'take away.'"

Do not add to the difficulty of learning the combinations by presenting them in horizontal form. Reserve expressions such as $2 + 2 = 4$ and $4 - 2 = 2$ until much later.

Posting the combinations. A suitable place should be provided in the front of the room for posting the complete form of each combination as soon as it is taught. By "complete form" is meant the combination *with the answer*. A bulletin board of ample size is ideal. The purpose will also be well served if the space along the top of the blackboard is used. The combination cards may be hung upon a wire stretched over this space.

The combinations for posting should be written on separate cards and in figures large enough to be read from any part of the room. After a combination has been posted, it should

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remain posted until all the pupils know it. The cards, however, should be faced toward the wall on the occasion of a progress test. (See pages 51, 86, etc.)

Make it clear to the pupils that they are to look at these cards whenever they are not sure of the answer to a combination. *Never reprimand a child for consulting them.* Encourage him to do so. That is what they are for. Do not worry because the pupils continue for some time to rely on the posted cards. They will cease to do so when the cards are no longer needed, for the sufficient reason that it is easier and more satisfying to say or write the correct answer without having to look for it.

Two main purposes are served by this posting device. One of these is our old friend the *prevention of counting.* We have adopted a certain order of teaching the combinations with this object in mind. We have suggested a method of presentation with the same end in view; and now the permanent display of the combinations is dictated by the same purpose. Why should a child, if he is asked to give the answer to a combination, take the trouble to count when the answer is before him? Is it not easier and more satisfactory to read it and be sure of getting it right than it is to labor over it by counting and then to run the risk of getting it wrong? In fact, the doctrine of satisfiers and annoyers (the law of effect) in psychology teaches us that counting is so generally resorted to by children, and adults too, not because it is satisfying in itself but because it is less annoying than getting the wrong answer. If now we introduce something which is more expeditious and more satisfying than counting, there will be no counting. Such a procedure will be far more effective than admonition or the setting up of a system of penalties and rewards.

The second purpose served by this posting device is the prevention of error. Modern pedagogy emphasizes the importance of forestalling mistakes rather than of correcting them after they have occurred. Psychologists have asserted

the seriousness of error. The evidence presented by Myers[1] is to the effect that "a mistake is immeasurably serious. Once it is made, we are never sure it will not be made again in just that same way." Accordingly in all your work on these combinations strive earnestly to banish guessing. Encourage the pupils to say "I don't know" when they are not sure. In this work they should never "take a chance"; and we as teachers should be careful not to put them in a position, at least in respect to the basic facts of arithmetic, where they are tempted to take a chance. The conditions should be such that they either know the combinations or can readily find them.

It may not be necessary, however, that *all* the combinations be posted. Perhaps the key combinations will suffice. Perhaps the posting of $\begin{matrix}3\\5\\8\end{matrix}$ will enable the child to get the additional facts $5+3=8$, $8-3=5$, and $8-5=3$. If the posting could be reduced to this minimum, it would be a great advantage. First, not nearly as much room would be required for the cards, and a pupil could find any given card more easily. Secondly, the relationship between combinations involving the same three numbers, for instance, 3, 5, and 8 in the set just mentioned, would be more clearly perceived, with undoubted benefit in the learning of these related facts.

We suggest, however, that all the *addition* facts be posted, for example, in the set just mentioned, that both $\begin{matrix}3\\5\\8\end{matrix}$ and $\begin{matrix}5\\3\\8\end{matrix}$ be displayed. This will mean that the two subtraction facts may be read more easily. The addends, or components, appear *in the same order* in the addition fact and in the corresponding

[1] Garry C. Myers, The Prevention and Correction of Errors in Arithmetic, 75 pp. The Plymouth Press, Chicago, 1925.

subtraction fact. Thus $\begin{array}{r} 8 \\ -\,3 \\ \hline 5 \end{array}$ is the subtraction fact corresponding to the addition fact $\begin{array}{r} 3 \\ 5 \\ \hline 8 \end{array}$; and $\begin{array}{r} 8 \\ -\,5 \\ \hline 3 \end{array}$ is the subtraction fact corresponding to the addition fact $\begin{array}{r} 5 \\ 3 \\ \hline 8 \end{array}$. Of course, there is no fundamental objection to the posting of all the combinations, both those in addition and those in subtraction. As has been explained, however, this will require a great deal of space. It may also prove confusing to the pupils in locating a desired combination, and it lacks the advantage of bringing the combinations of a given set to the support of each other through an apprehension of their relationship.

After having taught $2 + 2$ and the corresponding subtraction fact $4 - 2$, the teacher will post the form $\begin{array}{r} 2 \\ 2 \\ \hline 4 \end{array}$. She should teach the children to get from this not only the fact "two and two are four" but also the fact "four take away two are two."

Application in problems. The term "problems" as we shall use it means a description in words of a situation involving numerical relationships and requiring a solution through the use of number facts or operations. With us, a problem is thus distinguished from a requirement that abstract solutions be made — such as a task in column addition, compound multiplication, short division, or even a simple number combination. All such abstract requirements we shall call not problems but "examples."

The desirability of using objective material as a basis for the *teaching* of number facts is not generally questioned. The purpose is to make abstract numerical statements meaningful

by giving them concrete *origin*. They should also have concrete *application*. There is no assurance that the child who knows "five and two are seven" will use it in finding out how many rabbits a boy would have if he owned five to begin with and bought two more. Yet it is precisely in the form of problems that the number facts are of use. It would obviously be difficult to make out a case for the intrinsic value of "five and two are seven."

There is, therefore, every reason to introduce verbal problems early. In fact, as soon as a combination has been taught and while it is merely on the lips of the pupils ready to take its flight when something else comes to mind, concrete applications should be made.

For example, in connection with the teaching unit "two and two are four," problems like these may be proposed: (1) "Mr. Brown had two horses in the barn and two in the field. How many horses did Mr. Brown have altogether?" (2) "Sadie made four paper dolls. She lost two of them. How many paper dolls did she have left?" (3) "One summer morning I saw two boats on the lake and there were two people in each boat. How many people were boating on the lake?" This last is, of course, a multiplication problem.

Such problems should be more than "proposed," however. They should be made vivid. Often the problems may be dramatized. The first of the problems just mentioned may be made highly objective by making believe that two children behind the teacher's desk are the horses in the barn while two other children in front of the class are those at large in the field. The second problem will be more easily understood if the paper dolls are at hand, or at least something to represent them. At any rate, we urge that verbal problems, made concrete, interesting, and vivid, be used as an inseparable part of the drill on number facts from the moment the number facts are first taught.

The Second Teaching Unit

Teaching the addition facts. In the suggested order of teaching the combinations, $\begin{array}{r}1\\4\\\hline 5\end{array}$ comes next. Show the pupils five objects. Be sure that they recognize them as five things. Then separate one of them from the other four, but not so far that the five idea is lost. Be just as sure that the pupils recognize the *one* and the *four* as you were that they recognized *five*. Use large objects so that everyone can see them. Children are always at hand to serve as objects. If they are used, it is preferable that they all be of one sex. If they are boys, get from the pupils the statement that "one boy and four boys are five boys" and also the reverse statement that "four boys and one boy are five boys." Obtain similar statements from the pupils relative to other concrete material. Then write on the board the form $\begin{array}{r}1\\4\\\hline 5\end{array}$, saying "one and four are five" as you write. Call upon pupils to say the combination and to write it. Then treat $\begin{array}{r}4\\1\\\hline 5\end{array}$ in the same way. Remember that combinations in addition and subtraction are always to be read *downward*.

The related subtraction facts. After these two statements have been developed, namely, $\begin{array}{r}1\\4\\\hline 5\end{array}$ and $\begin{array}{r}4\\1\\\hline 5\end{array}$, the teacher should next develop the corresponding subtraction statements. The subtraction statement corresponding to "one and four are five" is "five take away one are four." This may be visualized, if one cares to do so, by putting the two facts in the usual vertical form: $\begin{array}{r}1\\4\\\hline 5\end{array}$ and $\begin{array}{r}5\\-\,1\\\hline 4\end{array}$. In the latter combination it

will be noted that the order of mentioning the numbers is the same so far as the 1 and 4 are concerned. In other words, the 1 is named before the 4 in each case. Similarly the subtraction statement corresponding to "four and one are five" is "five take away four are one."

In order to teach the pupils the idea "five take away one are four," it is again desirable to use objects. As before, the five objects should first be recognized as five. Again one of these objects should be separated from the other four. The teacher, being sure that the children have the idea "five" in mind, should now conceal one object and ask the question "How many are left?" It should not be difficult to obtain the answer "four," that is, four of whatever the objects are. If necessary, this may be supplemented by similar manipulations of other objects. Response to the following full form of the question should be secured: "If from five books I take away one [book], how many have I left?" After obtaining such answers with reference to several types of material the teacher is ready to obtain the generalized statement "five take away one are four." The teacher then writes on the board the form

$$\begin{array}{r} 5 \\ -\ 1 \\ \hline 4 \end{array}$$

, reading it as she writes. The pupils then read the form, copy it, and write it from dictation. The subtraction statement "five take away four are one" should be taught in a similar manner.

If conditions permit, the corresponding subtraction facts should be taught at the same time the addition facts are taught and with the same objective material. For example, while the five children are being used to teach $1 + 4$ and $4 + 1$, one of the children can be "taken away" and hidden behind the door, with the evident result that four of them "are left." Similarly four of the children may be taken away into the hall or hidden behind the teacher's desk, the class observing

that one child is left. Some will feel that the presentation of four facts is too exacting for pupils of this age. This may be true when the earlier facts are being taught — say, the facts of the first teaching group — without being true a little later. The success of this grouping of four facts at a time will depend largely upon the extent to which the teacher can present them as *one fact*, that is, as a single idea expressed in four different ways.

All the answers required in connection with the key combination, $\begin{matrix}1\\4,\\5\end{matrix}$, should be secured repeatedly. The pupils should read, copy, and write from dictation these four related combinations:

$$\begin{matrix}1\\4,\\5\end{matrix}\quad\begin{matrix}4\\1,\\5\end{matrix}\quad\begin{matrix}5\\-1,\\4\end{matrix}\quad\text{and}\quad\begin{matrix}5\\-4\\1\end{matrix}$$

Two cards, one containing the combination $\begin{matrix}1\\4,\\5\end{matrix}$, and the other the combination $\begin{matrix}4\\1,\\5\end{matrix}$, should be placed beside the combination $\begin{matrix}2\\2\\4\end{matrix}$ in the place already chosen for the permanent posting of the combinations. If at any time the children need to look up one of the subtraction facts, they should get it from its corresponding addition fact. From $\begin{matrix}1\\4\\5\end{matrix}$ they should be shown how to read "five take away one are four"; and from $\begin{matrix}4\\1\\5\end{matrix}$ they should likewise learn to read "five take away four are one."

The four facts in a teaching unit. As has already been pointed out, except in the case of doubles like $2 + 2$, there are four facts connected with each teaching unit. They are: (1) the direct addition combination (sometimes called the key

combination); (2) the reverse addition combination; (3) the direct subtraction combination; and (4) the reverse subtraction combination. It will be well for the teacher to have the meaning of each of these terms clearly in mind.

The direct addition fact, as we use the term, is an arrangement in which the larger number is below the smaller number. Since all combinations are to be read from the top down, this means that in the statement of the *direct* combination the smaller number will be named first, followed by the larger number, and then by the sum of the two numbers. In the set of combinations about which we have been speaking, $\begin{matrix}1\\4\\5\end{matrix}$ is therefore the direct addition combination. The reverse addition combination presents a larger number above a smaller one, for example, $\begin{matrix}4\\1.\\5\end{matrix}$ This is read "four and one are five."

In teaching a given set of four combinations, the direct addition fact should be taught first. The reason is that the pupils will be less tempted to secure the answer by counting if to a small number they have to add a large one than they will be if to a large number they have to add a small one. The reverse addition fact, since it may be secured from the direct addition fact, does not offer the same temptation to count. The subtraction combinations scarcely invite counting. If, therefore, the teacher will always remember that the direct addition fact is the one in which the smaller number is thought of first, followed by the larger one, and if she will begin each teaching unit with the direct addition fact, the undesirable tendency to obtain answers by counting will be reduced.

Of course, all this is merely a matter of organizing the material for teaching and learning purposes. The teacher will not make the mistake of supposing that it means any priority

in importance or with respect to teaching effort for the direct addition combinations. The direct addition combination, $1 + 4 = 5$, is no more important than $4 + 1 = 5$, the corresponding reverse addition combination. Moreover, the subtraction facts, independent of the question of importance, require *considerably more* practice than the addition facts, because they are harder. It would be a most unfortunate mistake if a teacher permitted the addition facts to take precedence over the subtraction facts in point of emphasis.

The direct subtraction combination corresponds to the direct addition combination. It names the addends or components (the numbers which in the addition fact are above the line) *in the same order*. In the set of combinations which we are using for purposes of illustration, the direct subtraction combination is $\begin{array}{r} 5 \\ -\,1 \\ \hline 4 \end{array}$. Here the addends of $\begin{array}{r} 1 \\ 4 \\ \hline 5 \end{array}$, the direct addition combination, appear in the same order as in that combination.

The reverse subtraction combination similarly corresponds to the reverse addition combination. In this set it is $\begin{array}{r} 5 \\ -\,4 \\ \hline 1 \end{array}$, corresponding (according to the order of the addends or components) to $\begin{array}{r} 4 \\ 1 \\ \hline 5 \end{array}$.

To summarize, then, the four related facts in this teaching unit are as follows:

Direct addition	$\begin{array}{r} 1 \\ 4 \\ \hline 5 \end{array}$	Direct subtraction .	$\begin{array}{r} 5 \\ -\,1 \\ \hline 4 \end{array}$
Reverse addition . . .	$\begin{array}{r} 4 \\ 1 \\ \hline 5 \end{array}$	Reverse subtraction .	$\begin{array}{r} 5 \\ -\,4 \\ \hline 1 \end{array}$

Telling the Whole Story

Not only are all the related facts to be taught together but they should frequently be drilled together. Suppose, for example, the teacher asks for $\frac{1}{4}$, and suppose it is understood that this time the pupils are to "tell the whole story." Under these circumstances the response should be: "One and four are five; four and one are five; five take away one are four; five take away four are one." The order of these statements is immaterial.

This telling the whole story is important. Of course, the teacher should not always require it, perhaps not even generally, but it should be one of the recognized ways of securing repetition of the facts to be learned. Moreover, this kind of repetition is secured in such a way that error is very unlikely to creep in. The child who has said "three and four are seven," and he may look at the posted combinations if he is in any doubt, will hardly go wrong in saying the other three related facts.

Children should not be expected to "tell the whole story" when a subtraction fact is presented. There are two reasons for this. First, the formulation of the related facts from a given subtraction fact is rather difficult. Secondly, to confine this device to the presentation of addition facts will secure a drill that is better distributed. As we have already shown, children make many more mistakes in subtraction combinations than they do in addition combinations. Accordingly we must seek every chance to emphasize the subtraction facts. In particular these should get more drill. If they are repeated when the addition combinations are presented — when children tell the whole story — they will secure extra repetition. Adopt, therefore, as one of your rules of drill the requirement that pupils frequently tell the series of related combinations, especially when an addition combination is the one first called for.

Varying the Response

It is good procedure to adopt different points of view in drill. We recognize this in every subject. In history we reorganize material which has been taught chronologically so as to get a view of it topically. In teaching French we have the pupil practice not only translating French into English but also translating English into French. In arithmetic we should likewise avail ourselves of the advantages of different points of view. In particular in the treatment of these addition and subtraction combinations we should call for the items from which the sum or difference arises. In every combination there are three elements, and if two of them are given the third is obtainable.

Therefore in connection with the teaching and drill on "one and four are five" the following questions should be frequently asked and answered: "One and four are how many?" "One and how many are five?" "How many and four are five?" The same varied attack may be made upon the subtraction facts. In connection with "five take away one are four" the following questions may be asked: "Five take away one are how many?" "Five take away how many are four?" "How many take away one are four?" It is admitted that these questions will be much neater when the time arrives to use the word "less" or the word "minus" instead of "take away."

The same variety may be secured visually. In connection with the written form $\frac{1}{4}$, we shall have not only the arrangement $\frac{1}{?}$ but also the arrangements $\frac{?}{5}$ and $\frac{1}{5}$. The task for the pupil is to give the correct figure to replace the question mark. The same or perhaps even better results may be secured by merely leaving blanks for the child to fill in either orally or in writing.

The Accuracy Ideal

From what we have said about the seriousness of error and from the precautions we have suggested to prevent error, you will be prepared to find us emphasizing accuracy as a primary objective. Number work is nothing if it is not accurate. This is true not only in the primary grades but in every other grade. It is true in life. Of late, speed has entered into competition, so to speak, with accuracy. For example, many of the recently published tests in arithmetic have tended strongly to suggest that speed may in some way stand in the place of accuracy and compensate for the lack of it. Nothing could be further from the truth. No one is interested in how quickly a child can get a thing wrong. It appears that there are two kinds of speed. One is the rate at which one does a good job; the other is the rate at which one does a job without regard to its quality. In arithmetic, since in the last analysis only the good job — the accurate result — is worth anything, speed is useless unless it is based on accuracy.

Not that readiness of response in our drill work is not to be sought; but rather that since this readiness must be secured in connection with accuracy, it will be obtained in the only way it is worth obtaining when accuracy is made the first objective. It is therefore urged that an ideal of correct response be set up. Aim to have no wrong answers. Let the pupils, if they are in the least doubt, refer to the combinations posted before them. Let the pupil say "I don't know" or "I don't remember" without being reproached.

The Remaining Teaching Units

The third teaching unit in Group I is represented by the key combination $\frac{3}{6}$. The fourth unit is $\frac{1}{7}$ and the fifth $\frac{1}{1}$.
$\frac{}{8}$ $\frac{}{2}$.

Multiplication and Division

The children have now been taught three doubles, namely, $2+2$, $3+3$, and $1+1$. At this point the corresponding multiplication and division facts should be taught. It is not at all necessary, however, to use the words "multiplication" and "division." It is suggested that this work be begun in connection with $3+3$. Show that we have another way of saying "three and three," namely, "two three's." When figures are used this should be written "2 3's." Do not use the expression "three times three" nor the form 2×3. It is doubtful if many children ever really understand the meaning of "two times three."

The idea is merely to show that one 3 and another 3 are clearly two 3's. In other words, we have a second way of saying *the same thing*. If, therefore, 3 and 3 are 6, then two 3's are likewise 6. Teach the children to say and write the number fact, "2 3's are 6." If there is any difficulty in reaching this result, go through the objective procedure as already suggested. For example, bring forward six boys; divide them into two groups of three boys each; ask the pupils the number in each group; and ask how many groups of three, or how many three's there are. Having obtained the response "two," the full statement may be obtained: "Two groups of three boys are six boys." Other objective material may be used in the same way and the full statement shortened to its abstract form "2 3's are 6."

The same objective procedure may be carried to the point of asking "How many 3's does it take to make 6?" or "There are how many 3's in 6?" This is the division form of the question. The full answer may be "The number of 3's in 6 is 2." It is difficult to make a brief statement of a division fact in such a way as to carry the divisor first, the dividend second, and the quotient last. Such a statement, however, is much

needed. We recommend the form "3's in 6 are 2." As in the case of the multiplication fact, show the children that this is only another way of saying a thing which they already know.

Do *not* introduce the reverse multiplication fact or the reverse division fact. Do not teach 3 2's are 6, for the simple reason that the children have not yet had the summation of three 2's. In fact, they will not have this until they reach column addition and get the answer to a column like this:

$$\begin{array}{r} 2 \\ 2 \\ \underline{2} \end{array}$$

For the same reason do not teach "the number of 2's in 6 is 3," or "2's in 6 are 3." This fact likewise depends upon combining 2 and 2 and 2.

Much loose thinking on the part of both pupils and teachers results from the notion that 2 3's is the same as 3 2's or that 5×7 is the same as 7×5. These facts refer to quite different conditions.

Summarizing, therefore, we may point out that we have added to the three-and-three-are-six situation two important facts, namely:

2 3's are 6

3's in 6 are 2

Now teach the multiplication and division facts corresponding to the other doubles of the first teaching group. These doubles are

$$\begin{array}{r} 1 \\ \underline{1} \\ 2 \end{array} \quad \text{and} \quad \begin{array}{r} 2 \\ \underline{2} \\ 4 \end{array}$$

The new facts to be taught are therefore:

2 1's are 2	2 2's are 4
1's in 2 are 2	2's in 4 are 2

Illustrative Problems

Below are given a few problems in which the combinations of Group I are applied. Note that in the subtraction problems the conditions describe the *taking away* of one number from another and require as an answer the number that is *left*. The other types of subtraction problems — such as those in which one component of a total is given and the other required or those in which it is required to find how much more or less one number is than another — are reserved for introduction later. These take-away problems are definitely related, and intentionally so, to the take-away method of teaching the subtraction combinations.

Moreover, they are just as definitely related to the order in which the subtraction facts are stated. You will recall our suggestion that a subtraction combination be stated in the form "five take away one are four." Now these problems when they involve subtraction invariably present the minuend first, followed by the subtrahend, as does the combination in the form we have just given. In other words, the problems do not, as yet, require the pupil either to turn his figures around or to adopt the form of expression "one from five is four." We should be introducing this difficulty if we gave the following problem: "Fred went to school at one o'clock and came home at five o'clock. How many hours was he away from home?" Here the form of the problem favors the statement "one from five is four," a statement which we have been avoiding. It is true that in attempting this problem a pupil may turn the numbers around so that they may stand in the order to which he is accustomed. In any event, however, the problem would introduce a difficulty which we do not desire to have the child cope with at this time. How confusing it is to a beginner to encounter various ways of stating subtraction combinations we do not know, nor do we

know how much confusion arises from the promiscuous wording of subtraction problems; but in the opinion of competent teachers these difficulties are considerable. Later, we may have the pupil express the abstract combinations in the form "one from five is four." Moreover, whether or not this alternative form is used, we shall expect him to solve all types of problems to which subtraction applies.

The reader will observe that these problems are grouped under the key combinations which they exemplify. This organization has been adopted in order that teachers may find problems to illustrate the combinations they may have in mind. No order of treatment, however, is intended to be suggested by this arrangement. The teacher will use the problems as they are needed, not as they are printed. Moreover, she will add many problems of her own, for more will be required than are given here. Observe that problems 5 to 8 inclusive under "Two and two," "Three and three," and "One and one" are multiplication and division problems. Do not use these problems until after the teaching of multiplication and division as provided for on pages 44 and 45.

Of course the problems are to be presented orally; the teacher is to read them to the pupils. In formulating them we have made no attempt to keep within the supposed reading vocabulary of first-grade children. If we are reasonably within the oral vocabulary of the children we are content. Our effort has been to make the problems interesting rather than to exemplify an infantile word list.

Two and two

1. Anna had 2 pennies in her bank and put in 2 more. How many pennies did she have in her bank then?

2. There were 2 pictures hanging on the wall in our room. Then we hung 2 more. How many pictures did we have on the wall then?

3. There were 4 bags of candy on the Christmas tree. We took off 2 bags. How many bags were left?

4. There were 4 little birds in a nest; 2 hopped out. How many birds were left in the nest?

5. Lucy wanted to mail 2 letters. She had to buy a 2-cent stamp for each letter. How many cents did it cost her to mail the letters?

6. A shoemaker has to mend 2 pairs of shoes. How many shoes must he mend?

7. Apples were 2 cents each. How many apples could Ella buy with 4 cents?

8. It is time to go home from the party. Mr. Rogers says he will take 4 little sisters home, but he can carry only 2 at a time. How many times will he have to go?

One and four

1. One little boy was trying to shovel the snow from the back walk. By and by 4 little boys came to help him. How many little boys were there then?

2. Baby had only 1 toy. Santa Claus gave him 4 new ones. How many toys did Baby have altogether?

3. There were 4 birds sitting on a branch; 1 more came. How many birds were on the branch then?

4. There were 4 kittens sleeping in a basket; 1 more jumped in. How many kittens were in the basket then?

5. Harry's big brother had 5 dollars and spent 1 dollar. How many dollars did he have left?

6. There are 5 people in our family. If 1 goes away on a visit, how many people will be left at home?

7. There were 5 eggs in the hen's nest. Charles took away 4. How many eggs did he leave?

8. There were 5 oranges in a dish. We used 4 for breakfast. How many oranges were left in the dish?

Three and three

1. One evening 3 stars came out from behind the clouds and then 3 more came out. How many stars could we see then?

2. One night there were 3 roses in my garden. The next morning there were 3 more. Then how many roses had I?

3. Mother had 6 eggs in the ice box. She boiled 3 for lunch. How many eggs were left?

4. There were 6 bananas on the table. The children took 3 to school. How many bananas did they leave on the table?

5. Mrs. Blake earns 3 dollars a day. How many dollars does she earn in 2 days?

6. Miss Hall was driving a 2-seated automobile with 3 people in each seat. How many people were riding in the automobile?

7. Picture cards cost 3 cents each. How many can May buy for 6 cents?

8. Mildred wanted to carry 6 big books down to the living room. She could carry only 3 at a time. How many times did she have to go down to the living room?

One and seven

1. Harriet cut out 1 paper doll. Her little friends cut out 7. How many paper dolls did they all cut out?

2. Mother had 1 cake of soap and the grocer brought her 7 cakes. How many cakes of soap did she have then?

3. The children wrote 7 words on the blackboard and the teacher wrote 1. How many words did they all write?

4. William's suit cost 7 dollars. His cap cost 1 dollar. How many dollars did his suit and cap cost?

5. There were 8 words on the blackboard. The teacher erased 1. How many words were there still on the blackboard?

6. Fred had 8 rabbits in a pen and 1 got out. How many rabbits stayed in the pen?

7. There were 8 people in our room and 7 went out. How many people were left in our room?

8. There were 8 little chickens out in the garden. Mother Hen said, "Cluck! Cluck!" and 7 of them came back. How many little chickens were left in the garden?

One and one

1. Henry has 1 apple in his hand and 1 in his pocket. How many apples has he?

2. If you spend 1 cent for candy and 1 cent for a pencil, how many cents do you spend?

3. Carl had 2 marbles. He gave away 1. How many marbles had he left?

4. Julia had 2 sticks of candy. She gave her sister 1. How many sticks of candy had Julia left?

5. Each day Jack's pet hen laid 1 big white egg. How many eggs did she lay for Jack in 2 days?

6. Margaret bought 2 sticks of red candy at 1 cent each. How many cents did she pay for the candy?

7. How many 1-cent flags could you buy with 2 cents?

8. There were 2 bags of potatoes down in the field. John was to bring them up in his cart. He could carry only 1 bag at a time. How many times did he have to go?

The Progress Test

Twenty combinations have now been taught and applied in verbal problems. It is desirable that all the children be tested as to their ability to give the answers to these twenty combinations. The best way to give this test, if facilities are at hand and the ability of the class permits, is to mimeograph the combinations and give the children sheets on which they are to complete each combination by writing the correct answer. While the progress test is being given, the combinations posted for reference should be taken down or covered. If facilities do not permit mimeographing, or if the children have not yet advanced far enough in the writing of numbers,

the test may be given to each pupil orally. Oral testing may either be addressed to the class as a whole, thus involving the writing of the answers, or it may be addressed to each child separately. In the latter event the teacher may write down the answers which the pupil gives orally. The test which follows may be used to find out whether the children know the twenty combinations taught up to this point.

Test on Combinations of Group I

4	5	3	8	4	5	1
1	−4	3	−7	−2	−1	7

2 2's are
3's in 6 are
2 1's are

2 3's are
1's in 2 are
2's in 4 are

2	2	1	6	1	8	7
2	−1	4	−3	1	−1	1

Using a class record sheet. After giving the foregoing test it is highly desirable that the teacher collect the data systematically on a class record sheet. The things she wants to know are essentially: (1) the pupils who make mistakes; and (2) the combinations on which they make them. Thus there are two points of view. According to the first, the pupil is considered, with the result that individual attention is given to him in conformity with his needs. According to the second, the particular combination is considered, and drill is provided on the combination in question according to the extent of error revealed by the score sheet.

The following form is suggested in connection with the progress test on Group I. It may easily be set up on ordinary letter-size paper, $8\frac{1}{2}$ inches by 11 inches. Square-ruled, or coördinate, paper with squares of one fourth or one fifth of an inch is especially suitable.

Class Record Sheet

City_____ School_____

Grade or Class_____ Teacher_____ Date_____

Names of Pupils	Combinations																									Total Errors per Pupil
	4 1	5 −4	3 3	8 −7	4 −2	5 −1	1 7	2 2's	3's in 6	2 1's	2 3's	1's in 2	2's in 4	2 2	2 −1	1 4	6 −3	1 1	8 −1	7 1						
John Doe		/																								
Jane Roe			/		/																					
Total errors per combination																										

If a pupil misses a given combination, put a score mark opposite his name and under the combination he missed. In the form above John Doe is recorded as having missed $\frac{5}{-4}$ and Jane Roe as having missed $\frac{3}{\underline{\ }}$ and $\frac{4}{-2}$. After the entries have been made, the score marks should be counted horizontally and the totals entered in the last column. These will tell the number of combinations each pupil missed. Correspondingly the score marks should be counted vertically and the sums entered on the bottom line. These will tell the number of times each combination was missed. With this information in hand the teacher will be able to locate the individual difficulties of pupils and also the combinations on which more drill is needed by the whole class.

CHAPTER IV

ON TEACHING THE SUBTRACTION COMBINATIONS

As you are no doubt aware, there is a good deal of argument nowadays as to the way in which subtraction should be taught. Some favor the "addition" method and some the "take-away" method. If the controversy concerned nothing more than teaching the combinations in the primary grades, the decision, while it would be of some importance, would not be fundamental. The truth is, however, that one must at this early date in the teaching of arithmetic make a decision which is far-reaching in its consequences.

The Take-Away Method Preferred

In spite of the attractiveness of the new ideas about additive subtraction, we have decided, as you have already observed, to teach the subtraction combinations *as subtraction.* We believe that those who use this book are entitled to know why we have made this decision. Let us therefore get the problem clearly before us.

The additive procedure. If you teach subtraction by the addition method, you introduce in connection with the combination $1 + 4 = 5$ the question (together with its corresponding reply) "One plus what [or how many] equals five?" When the pupil answers "four," he is held to have subtracted by the addition method. We doubt whether he has thought anything about subtraction, and very likely he should not. To him the question and its answer are only another way of getting at the same addition fact that he has already learned. This will be

evident if we consider the full answer to each question. To the question "One and four are what?" the answer is "One and four are five." To the question "One and what are five?" the answer is also "One and four are five." Only by oral emphasis can a distinction be made.

We have already advised the teacher to vary her drill on the addition facts by using exactly the same form of question that the teacher of additive subtraction uses in teaching subtraction (see page 42). When, therefore, it is urged as a reason for teaching additive subtraction that we thus fortify the learning of the addition facts, it is proper to reply that we may and should secure this advantage by teaching and practicing the addition facts themselves, without any reference whatever to subtraction.

The take-away procedure. If you are teaching subtraction by the take-away method, besides teaching the fact $1 + 4 = 5$ you also teach the fact $5 - 1 = 4$. This is indeed a subtraction fact. It conforms to the idea of subtraction. It *is* subtraction. It represents, as subtraction should from its fundamental meaning, a withdrawing or taking away of a part from a whole, and the fact that the other part remains. This idea is too useful to be left out of account in school work.

The subtraction idea in life. In the first place, the world thinks of subtraction as a take-away process, as a withdrawal, a deduction from a larger to a smaller amount. Trade discount is identified with subtraction. It is an amount taken from the list price. One does not think of it, although it is admitted that one could, as the amount by which the list price exceeds the net price. Far less does one think of it as an amount added to the net price to produce the list price. The discount is an amount saved; and one who secures an overcoat marked down from \$40 to \$32 invariably figures how much *less* he paid by purchasing at \$32 rather than how much *more* he would have paid if he had purchased at \$40. The depreciation

of a building or of equipment is regarded as a decrease from original value, not an increase over present value. Losses in business are thought of as deductions from cost, not as amounts added to receipts. It is true that the additive idea is found in such a situation as "I need so much but I have only so much; how much must I get?" The take-away idea, however, seems to be fundamental to a greater diversity of conditions. As things are now, the thought of the people is, on many matters, subtractive rather than additive.

The take-away method in the school. In the second place, our pupils live in a school world which is adjusted to the subtractive rather than the additive idea of subtraction. Throughout the course of study there are many places where either the take-away or the addition idea must be adopted. In most of these instances, no matter what the early treatment of the subject may have been, the decision is in favor of the take-away idea. In other words, no matter whether in the first and second grades the subtraction combinations were taught by the addition or the take-away method, the take-away method for most part prevails in later work. No doubt all the arithmetic work, and algebra work too, could be adjusted to the addition idea. But such an adjustment has never been made, and in our opinion it is largely because of this fact that the take-away method scores its victory.

It sometimes happens, for example, that a pupil is taught the combinations by the addition process and is afterwards required to explain "long" subtraction by the take-away method. But the real objection to the addition method of subtraction lies still farther on in the course of study. It lies in the inconsistent teaching of division and of the subtraction of fractions and denominate numbers. The addition method of subtraction needs no justification in theory, but before it can be a real success in practice it must be carried out consistently. This no one is at present ready to do.

Applications in Problem Solving

Both the take-away and the addition methods of subtraction are readily applicable in verbal problems. Two very common situations in life require subtraction. One is represented by the problem in which we find how much is left, the other by the problem in which we find out "how much more." The take-away idea corresponds to the first of these and the addition idea to the second. Consider the following problem: "John had five marbles and lost two. How many had he left?" This problem clearly favors the take-away idea, and when subtraction is taught by the take-away method its application to this problem is easy. It is fairly easy to show that John has three marbles left because five less two are three; but to explain to a pupil that John has three marbles left because two and three are five is by no means as easy. Consider, however, this problem: "Philip is seven years old and George is five years old. How much older is Philip than George?" It is clear that the addition method of teaching subtraction is more easily applied to this problem than the take-away method. It is easier for a pupil to see the bearing of 5 and 2 are 7 than it is to see that he needs to find 7 less 5.

Making the application to contrary types. In teaching the application of the combinations to problems, those who teach either method of subtraction will have a rather delicate transition to make to the type of thinking which is not favored by the method they choose to teach. It does not appear, however, that the transition is any more difficult one way than it is the other way; and if the tasks are of equal difficulty, there is no argument for either method of teaching subtraction. The real need here is for teachers clearly to distinguish the two types of subtraction problems and to provide carefully for the necessary transition. We suggest the use of an intermediate type as facilitating this transition, a type in which the whole group

is naturally separated into two groups (decomposition). Consider, for example, this problem: "Five children are playing a game. Two of them are boys. How many are girls?" There is no withdrawal or subtraction idea here, at least not obviously; nor is there except by inference the idea "how much more?" Such problems can be made in great abundance, the essential point being that the total group must be composed of two readily distinguishable subgroups. Mary may have a string of red and white beads. Knowing the total number of beads and the number of white beads, the problem is to find how many red ones there are. Similarly a mother hen may have black chicks and white chicks. There may be green books and red books on the teacher's desk. Helen may have red ribbons and blue ribbons. Robert may have big marbles and little marbles.

There is no use in pretending that a difficulty does not exist. Either method of teaching subtraction must be carefully handled in its application to contrary types of reasoning problems. We have recognized this fact by introducing none but the take-away type of verbal problems at first, and then gradually introducing what we have called the transitional type and the "how much more?" type. In other words, we have in the early presentation of verbal problems deliberately favored the type we have adopted to teach; but we have not failed to introduce the other types. We believe this is good pedagogy.

Additional Reasons for Preferring the Take-Away Method

If we may be permitted to pursue the matter a little further, we should like to point out briefly some of the additional reasons which induce us to select the take-away method of subtraction for this series of arithmetics.

In the first place, contrary to the usual idea, the addition method is *not* generally the method of making change. If you give the grocer a dollar bill for a 35-cent purchase, he gives you change and as he does so he says "forty" (putting down a nickel), "fifty" (putting down a dime), and then very likely "seventy-five, one dollar" (as he puts down two quarters). This is not the addition method of subtracting 35 from 100. A *series* of additions is made instead of a single addition. Moreover, *the answer is not given.* Unless you stop to count your change after you have received it all, or unless you actually subtract 35 from 100 mentally, you do not know how much change you received. You are generally satisfied to start with the grocer at 35 and count up with him as he lays down the coins. Strictly speaking, only when the change consists of a single coin is the method that of additive subtraction.

In the second place, as usually taught, the addition method when applied to long subtraction leads to a proceeding which is far from straightforward. Consider the following example:

8645
1231.
7414

As the addition method is usually taught, the pupil is told to think "1 and 4 are 5 [write 4], 3 and 1 are 4 [write 1], 2 and 4 are 6 [write 4], 1 and 7 are 8 [write 7]." In each case he thinks first of the middle number, then apprehends the top number in connection with it in order that he may next think of the bottom number. He finally thinks of the top number and comes back to the bottom number in order to write it. The take-away method permits a very much more direct procedure. The child thinks from the top down, for example, "5 less 1 are 4 [write 4], 4 less 3 are 1 [write 1]," etc.[1]

[1] It is true that one way of handling the subtraction combinations according to the addition method permits a direct eye-and-hand movement. This is the method which teaches the combination used in units' place in the above example in the form "$5 = 1 + 4$." This, however, is not the usual form employed in teaching the subtraction combinations by the addition method.

In the third place, the take-away method, especially when extended to long subtraction, provides a means of checking the answer by applying another operation, namely, adding the remainder and the subtrahend. Additive subtraction provides no real check, since in combining the remainder and subtrahend one simply repeats the former process.

In the fourth place, the published experimental evidence concerning these rival methods, though meager, is quite as favorable to the take-away method as to the addition method. It is certain that more people use the take-away method,[1] and the most competent experiment hitherto reported shows that, at least so far as the teaching of the combinations is concerned, the take-away method is *somewhat* more satisfactory.[2]

[1] J. T. Johnson, "The Merits of Different Methods of Subtraction," *Journal of Educational Research* (November, 1924), Vol. X, pp. 279–290. A number of other investigators give testimony to the same effect; for example, McClellan, Ballard, and Taylor.

[2] Cyrus D. Mead and Isabel Sears, "Additive Subtraction and Multiplicative Division Tested," *Journal of Educational Psychology* (May, 1916), Vol. VII, p. 261.

CHAPTER V

EXTENDING NUMBER IDEAS

Although it is expected that while the children are learning the combinations they will continue to be given opportunity to count and to read and write numbers, it is suggested that after the combinations of Group I have been taught, a number of days be spent during which no new combinations are taken up. During those days let the previous gains be consolidated; let the old combinations be reviewed and applied in problems; and let the new work consist of an extension of the range of number ideas.

Assuming that the children are now able to count to 20, as has been previously provided, let the first step in this extension be the counting by tens to 100. Along with this will go the reading and writing of the numbers 20, 30, 40, · · ·, 100. At this time note the character "0" and give it its name, zero.

The Number Chart

A systematic arrangement of the first 100 numbers, called the number chart, will frequently be referred to in this and the subsequent books of this series. The complete chart is given on the following page.

It is well for the teacher to have this completed chart in mind, even at this early stage. Since, however, the pupils have only learned to count by 1's to 20 and by 10's to 100, the teacher will at first use no more than the two left-hand columns and the top row of the chart, as shown in the incomplete chart on the following page.

THE COMPLETE NUMBER CHART

0	10	20	30	40	50	60	70	80	90	100
1	11	21	31	41	51	61	71	81	91	
2	12	22	32	42	52	62	72	82	92	
3	13	23	33	43	53	63	73	83	93	
4	14	24	34	44	54	64	74	84	94	
5	15	25	35	45	55	65	75	85	95	
6	16	26	36	46	56	66	76	86	96	
7	17	27	37	47	57	67	77	87	97	
8	18	28	38	48	58	68	78	88	98	
9	19	29	39	49	59	69	79	89	99	

INCOMPLETE NUMBER CHART

0	10	20	30	40	50	60	70	80	90	100
1	11									
2	12									
3	13									
4	14									
5	15									
6	16									
7	17									
8	18									
9	19									

This fragment of the number chart should be conspicuously displayed and kept permanently in view. As time passes and the pupils increase their knowledge of numbers, the chart should grow. Moreover, the pupils should learn to make the chart, or as much of it as has been developed. Its uses are many, and some of them we shall point out from time to time. Many teachers have found it advisable to put the number chart, after it can be used in its completed form, on a curtain

which may be hung in front of the class and lowered and raised at will. There are some advantages, however, in having it always in view.

Even before the chart has been developed very far, it may be used to assist pupils in reading and writing numbers and in locating them. One may "find," "point to," "touch," or "write" 15, 6, 11, 18; the number between 12 and 14; the number next larger than 17; the number next smaller than 13; the number that 3 and 3 make; the number that tells how many children there are in a certain row in the classroom; the number that tells what 8 less 7 are; or the number that tells us how many this is (Fig. 6).

FIG. 6

The further extension of number ideas will aim to develop counting by ones to 100 and the reading and writing of numbers to 100. It has already been suggested that a period comprising an indefinite number of recitations be devoted to this enlargement of the pupils' grasp of numbers. Whether the teacher prolongs this period or extends the counting, reading, and writing of numbers while teaching the combinations of the second and later groups is relatively immaterial. Our own preference would be for getting the extension of number ideas well under way and then for developing it to the desired degree while the next number facts are being taught.

We have already suggested the first step in the extension of number ideas, namely, the counting to 100 by tens. This serves to set up the skeleton of the first 100 numbers considered as an organized body of knowledge. Thus the "decades" are established, the 'teens, twenties, thirties, and so on. When these are learned, the filling-in between the tens is not difficult. To the twenties the child will annex one, two, three, and so forth; the same with the thirties, and so on. The chief trouble comes in passing from a 9 in one of the decades to the next decade, as, for instance, from 59 to 60.

If, however, the pupils have thoroughly learned to count by tens to 100 this difficulty will be small.

Drill in counting should be provided on all reasonable occasions. Certain suggestions have already been given on pages 4–6. The difference here is, however, that the counting runs much higher and so involves a different set of objects. For example, it will no longer be of much consequence to count the number of rows of seats in the room or the number of seats in a given row. The counting, however, of all the seats in the room is appropriate. So also is the counting of the pupils present in each school session. Collections of more than 20 but less than 100 objects are desirable. These may be books in the classroom library, panes of glass in the schoolroom windows, or the teacher's supply of pencils or crayons. Some of these collections vary in size at different times, thus affording more or less continuous practice. Moreover, it is well to have several pupils count a given collection of objects in order that they may verify one another's work. Do not fail to have pupils do some independent counting. Each one is in daily contact at home, in the store, and on the street with situations involving numbers of objects, and these situations are to a large degree different for different children. Accordingly we may have them count the things which come within their experience and report on these matters to the class. Some of these efforts we may dignify by the name "investigations."

Along with all this should go the writing of numbers, not only in connection with the counting but also in connection with any other number work which the class may be doing. Moreover, in addition to this more or less incidental reading and writing of numbers, there should be definite drill having proficiency in these activities as its sole object. Such drill will require pupils to read numbers which have been written on the board expressly for the purpose of being read. It

will also require them to copy numbers and to write them from dictation. In both the copying and the writing from dictation, emphasis should be placed upon writing the numbers one under another, as is done in column addition. Since the children have not yet been taught about numbers of more than two digits, they may without any appeal to general principles, such as "units under units and tens under tens," get the idea of setting the one-digit number under the right-hand part of the two-digit number above it. In other words, the children may learn from imitation so that the column-addition arrangement will be made without difficulty.

The number chart (see page 62) will now be completed and given a permanent place in the equipment of the room. Its use in assisting the children to count, read, and write numbers will be very great. It will also be of inestimable value in helping them to an idea of the decimal structure of our notation. It is not difficult when the proper time comes to develop with the aid of the chart the idea of tens and units. The pupils have already counted by tens, and both from the picture of the numbers and from their sound they can readily apprehend that 20 means two tens, that 30 means three tens, and so on. The 'teens are of course irregular. They will have to be learned without much assistance from their sound.

Ones and Tens

It should be pointed out that in the number chart the first column is the ones' or units' column, and that the numbers found in this column are also found in every other column. Decide whether to use the term "ones" or "units." Perhaps the former is preferable, at least for the present. It may be shown that in order to make the big numbers we use one or more tens besides the ones' figures. A number with one ten is found in the 'teen column, the column of one ten

and a ones' figure. A number with two tens is found in the 20 column, and so on. All this may be illustrated by the use of small objects. One or more groups of ten may be counted and tied into bundles. To represent the number 16, it may be shown that one ten-bundle is used; and that this number is found in the 'teen column, the column of one ten and a ones' figure. To represent 23, it will take two ten-bundles, and this number is found in the twenty column or the column in which every number has two tens. Using the chart, pupils may then find, read, and copy numbers containing a specified number of tens and ones. For example: "Find the number that has two tens and four ones." "What is this number called?" "Copy it." "Copy the number that has one ten and six ones."

Or, instead of letting the chart come at the end of this kind of work, it may come at the beginning. For example, the number 23 may first be selected from the chart. The teacher may, with the assistance of the pupils, count out 23 objects to represent this number. She may then count ten of these and tie them into a bundle or put them in a pile. A second ten may then be counted and put together. There are three objects remaining. There were 23 in the beginning and there are now two ten-groups and three left over. The "23" of the chart shows this fact. The 2 shows the number of ten-groups, or the "tens," and the 3 the number of ones. After the teacher has demonstrated facts of this sort with the assistance of the pupils, the latter may be encouraged to "make" designated numbers with objects. In this work frequent use should be made of the terms "ones" and "tens."

Perhaps an even better way to develop the idea of ones and tens is to use either real or toy money. A number of "pennies" and dimes are all that one needs. The pupils may easily be taught that a dime is the same as ten pennies. Probably many of them know this already.

If the number 23 is under consideration, twenty-three pennies may be counted out. Then ten of them may be replaced by a dime; another ten by a dime; and the fact brought out that the 23 cents is now made up of 2 dimes and 3 pennies. The dimes are tens, because it takes ten pennies to make each of them. The pennies are just ones. Hence the "2" of 23 is the number of tens and the "3" is the number of ones.

It is not our intention to suggest that this kind of teaching be done at any particular time. Much will depend upon the aptitude of the class and the time which the course of study permits for arithmetic before the third grade. We wish, however, to emphasize that this kind of work when once begun should not be dropped. It should be continued from time to time in connection with the chart until it is thoroughly mastered.

Counting by 2's, 3's, and 5's. Finally in connection with "extending number ideas" have the children do some of the commoner kinds of counting by amounts greater than one. We often hear people count by 2's and sometimes by 3's. Eggs, for example, are quite generally counted by 3's. Children in some of their games count by 5's. It appears therefore that there is something like a "social need" for counting by 2's, 3's, and 5's.

Write on the blackboard the series of numbers from 1 to 20. Underline 2, 4, 6, and so on. The series will then look like this:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

You may have the children read all the numbers from 1 to 20. Then have them read only the underlined numbers. The odd numbers (that is, 1, 3, 5, 7, · · ·) may then be erased and the children may be drilled on reading the even numbers. Twenty is a minimum limit for this work, although the counting may well be carried further if the children enjoy it and appear to

profit by it. Show the pupils how to count small objects by taking two at a time. Let them count the same objects by ones and observe that they get the same result.

A similar procedure may be followed in teaching the pupils to count by 3's. Thirty is the natural limit; but this limit may be exceeded under favorable circumstances.

Counting by 5's will present no difficulties. Children generally "pick up" this ability outside of school. The counting should be carried to 60 and may go as far as 100. Sixty is suggested as a limit because there are 60 minutes in an hour and because the clockface is divided into 5-minute intervals. It will be a good plan to have children count by 5's, pointing as they do so to the numbers on the clockface. As they point to I they may say "5"; as they point to II they may say "10"; and so on.

CHAPTER VI

DRILLS AND GAMES. I

Some authorities would have no formal number work done in the first grade at all. Others call for a rather exacting program. We incline to the belief that definite achievement on the part of first-grade pupils should be expected. It is our experience that work which is left to be done incidentally is not done satisfactorily by more than fifteen or twenty per cent of the classes concerned. Whether systematic number work is begun in the first grade or later, it is generally agreed that the spirit of informal activity more or less on the play level is desirable in the early school years. The value of motivation in the necessary drill work can never be overestimated and least of all in these grades.

Certain games and devices for insuring practice in number work have been utilized by successful teachers for many years. It is our purpose to mention a few of these at this point, although, because of the small amount of teaching that has thus far been done, their applicability is limited. More games can be played after a larger number of combinations have been taught, and a great many more after *all* the number combinations, in addition, subtraction, multiplication, or division, have been covered. With the small number of combinations to be practiced at this juncture, only a few devices, and those of a very simple nature, can be used.

It is especially appropriate that these games should appear in a teachers' book. Indeed, the common practice of including them in pupils' books is of questionable value. The teacher should be in a position to retain the initiative in such matters.

Identifying Numbers

Game No. 1. The teacher gives each pupil a number. This may be done orally or by pointing to numbers in the number chart and calling each pupil's name in connection with his number. The pupil at once writes his number on paper and pins it to himself. The teacher then calls a group to the front of the room by writing their numbers on the blackboard. The pupils who have thus been selected turn their backs to the blackboard. The teacher then erases one of the numbers and tells the pupils to face about, that is, to face the blackboard. The pupil whose number is missing calls the missing number and takes his seat. This procedure continues until the group is seated. The exercise may be varied in a number of ways.

Game No. 2. A number is pinned to each pupil. The pupils then form a circle. (If at this particular juncture they do not know numbers above 20, the circle will probably include only part of the class; but there can be more than one circle.) One pupil passes around the outside of the circle calling a certain number. The pupil who has this number immediately tries to tag the child who calls the number. If the wearer of the number is successful, he names the number and is allowed in turn to call a different number, whereupon he is pursued by the wearer of that number.

Practicing the Combinations

Drill cards. Much of the element of play may be present when ordinary drill cards are used. Sides may be chosen and scores taken. Cards may be "captured." They may be "matched" with answers. They may be sent as "letters" to the person whose "address" is the answer. For suggestions of this sort the reader is referred to pages 105 and 166.

We have already said that the combinations, at least in their addition form, are to be posted for reference. These posted combinations are in *complete form*, that is, they show the answers. For use in games and for other drill purposes a *drill form* is also needed, one in which the answers are omitted. Moreover, unlike the complete cards used for posting, these drill cards must exhibit every possible combination. The subtraction facts must be included, and not left to be inferred from the addition facts. Indeed, for reasons already stated, the subtraction facts should be practiced more frequently than the addition facts.

The sign of subtraction, having been taught and interpreted first as "take away" and perhaps afterwards as "less," will be used in the drill forms of the combinations. For example, the drill forms in connection with the combination $\begin{array}{r} 1 \\ 4 \\ \hline 5 \end{array}$ are as follows:

When cards containing addition facts are used, adopt frequently the procedure which we have called "Telling the whole story" (see page 41).

A bean-bag game. Mark on the floor fifteen large squares in three rows of five each. In each of these squares put one of the twenty combinations which have already been learned, as shown on the following page.

If some combinations do not need drill, the number of squares may be reduced or some of the combinations may be repeated or a number of squares may be left empty. The selection and placing of the combinations will naturally be different at different times. The pupils may or may not choose sides, as desired.

2's in 4	2 1's	$\begin{array}{r} 8 \\ -\ 7 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ -\ 1 \\ \hline \end{array}$	3's in 6
$\begin{array}{r} 8 \\ -\ 1 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ -\ 3 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ -\ 4 \\ \hline \end{array}$	2 3's	$\begin{array}{r} 3 \\ 3 \\ \hline \end{array}$
$\begin{array}{r} 7 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ -\ 2 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ -\ 1 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ 4 \\ \hline \end{array}$

Give each pupil three throws of a bean bag. He must give the answer to the combination in the square upon which his bean bag falls; and if he does so, he scores one. If the bean bag falls on a line, the teacher acting as umpire should decide which combination is indicated, either by moving the bag or by calling out the combination. If the bean bag falls in an empty square or outside the entire drawing, or if the pupil misses the combination, he gets no score. Do not deny him a score if he finds his answer by consulting the posted combinations. He has shown that he does not think he knows when he does not know; and he has shown the unwillingness to guess which we desire him to feel. The highest score for a pupil on his three trials will be three. If sides are chosen, the teacher may keep score on the board by using marks. The counting of these marks afterwards will serve as a drill in counting. There should not be more than 6 pupils, however, on each side until the class can count beyond 20.

A variation of this game may be played by drawing the same figure on the blackboard and using a soft rubber ball

instead of a bean bag. The game may, of course, be used to advantage all through the primary grades while number facts are being taught and drilled.

Crossing the stream. Draw on the blackboard the "stones" in the bed of a stream. Mark each of these stones with a combination. Suppose there are seven of them and that each is marked with one of the addition combinations. In order to cross the stream the pupil must give each combination; for example, in order to pass the stone marked $\underline{4}^{1}$ the pupil must say "1 and 4 are 5." Again, the stones may be marked with the subtraction, multiplication, or division combinations, and the crossing of the stream may thus be invested with new hazards. Use the subtraction combinations more frequently than the addition combinations. Sometimes addition or subtraction stones may both be used at the same time. The mishaps of the children in crossing the stream, their beckoning friends on the other side, their hesitation in making the attempt — these are some of the things which may be played up. Like the bean-bag game described above, "Crossing the stream" may be played anywhere in the primary grades, so far as the number facts are concerned. Children, however, will outgrow it before long.

The ladder game. This familiar game lends itself well to use when only a few combinations have been taught. It will be no less useful later when many combinations have been presented. It will afford intensive drill on material selected because of its difficulty. Draw a ladder on the board. Its upper end may be represented as reaching the top of a wall or the boughs of an apple tree or the window of a burning house. The wall may conceal a magic garden, the tree may have ripe fruit on it, while the third situation may involve a rescue. Below each rung a combination may be written. The successful climbing of the ladder, of course, depends upon giving the

right answer to the combination. The game may be to see who can reach the top of the ladder and look into the magic garden or gather apples from the tree or save the little girl at the window. The drawing may be quite crude and yet be effective. A ladder for addition may look like Fig. 7. Before the pupils can climb this ladder they may be required to "tell the whole story" of each combination (see page 41). Similarly a ladder for the subtraction facts may be constructed. Use more subtraction than addition and do not omit the facts involving multiplication and division.

The ladder scheme lends itself to various interesting extensions. Children who have climbed the ladder to peep over the wall into the magic garden may be asked "What do you see?" They may reply in terms of some combination; for example, "I see two red birds and one blue bird." The combination may be completed by the same child, "I see three birds"; or another child may say, "He sees three birds." If this child answers correctly he may have his turn, first climbing the ladder to the top of the wall and then telling what he sees, for example, "I see three roses and three lilies." Response: "He sees [I see] six flowers." Or "I see two big trees and two little trees" or "I see one silver apple and one gold apple."

1 7
2 2
1 4
3 3
7 1
1 4
4 1

FIG. 7

Another variation may be made by putting under each rung *two* combinations, one in addition and the other in subtraction. The game may be to go up adding and come down subtracting.

CHAPTER VII

THE SECOND GROUP OF COMBINATIONS

After an interval, long or short according to the capability of the class, during which the teaching of new combinations will give way to the extension of counting and to the reading and writing of numbers to 100, the second group of combinations should be begun. According to Table I of Chapter II, these are represented by the key combinations

$$\begin{array}{ccccc} 1 & 4 & 1 & 4 & 1 \\ \underline{2} & \underline{4} & \underline{6} & \underline{5} & \underline{3} \end{array}$$

The average difficulty of the facts in Group II has been found to be 40.2, while the corresponding figure for the facts in Group I is 37.0. Again the teacher is reminded that the suggested order for teaching these combinations is by no means haphazard. Although a strict order of difficulty has not been achieved within the group, the fact is that these combinations are not much different from one another in point of difficulty, and that so far as this basis is concerned the teaching order is almost immaterial. The determining condition in this group is the avoidance of interfering influences. The arrangement is therefore such that the same number does not appear in two consecutive combinations.

The Method of Presentation

In presenting these combinations there should be the same avoidance as before of a resort to counting. It is quite permissible to count the entire group, for example, 9, in presenting the combination $4 + 5 = 9$. There the purpose is the true

purpose of counting, namely, to find out how many there are in a group of things taken as a whole. In teaching this combination, after the teacher has divided the 9 children or books or erasers into subgroups of 4 and 5, each of these may be separately apprehended, by counting if necessary. The point, however, is that nowhere in the procedure should any child begin at 4 and count 5 more, thus reaching 9. The essential treatment of each combination may be summarized as follows:

1. Present the total group and have the pupils recognize how many there are in it.

2. Divide this total group into two smaller groups corresponding to the addends of the combination in question. Be sure that these groups are not widely separated from each other. The totality should not be lost sight of.

3. Have the pupils recognize the number of things in each of the two groups.

4. Recall to their minds the size of the total group.

5. Get the statement of the direct addition combination in concrete form, such as "4 children and 5 children are 9 children."

6. Proceed in the same way with different concrete material if this procedure appears to be advisable.

7. Obtain from the pupils the statement of the direct addition combination in abstract form, such as "4 and 5 are 9."

8. Write this on the board in vertical form, saying the combination as you write.

9. Have the pupils copy it and write it from dictation.

10. Obtain the same statement repeatedly and apply it in concrete problems.

11. Obtain the corresponding direct subtraction fact — "9 take away 4 are 5." Write it; have it written; have it repeated; apply it in concrete problems — all as in the case of the direct addition fact.

12. Post the combination with those already taught, using the addition form for this purpose, that is, $\begin{array}{r} 4 \\ \underline{5} \\ 9 \end{array}$. Have this read both ways; that is, "four and five are nine" and "nine take away four are five."

13. By reference to concrete material if necessary, teach in a similar manner the reverse addition and subtraction combinations, "5 and 4 are 9" and "9 take away 5 are 4." If the ability of the pupils permits, the reverse addition fact can be taught at the same time the direct fact is taught. If, for example, 9 children are divided into two groups, one of 5 children and the other of 4 children, it is just as clear that 4 children and 5 children are 9 children as it is that 5 children and 4 children are 9 children. However, unless you are sure that the pupils are quite able to handle the two facts together, each fact should be treated separately.

14. Drill and apply these reverse combinations as indicated in the case of the direct combinations.

15. Post the reverse combination, $\begin{array}{r} 5 \\ \underline{4} \\ 9 \end{array}$, beside the direct combination in the place reserved for permanent display.

16. In connection with a double, teach the corresponding multiplication and division facts. For example, with $4 + 4$ teach 2 4's are 8 and 4's in 8 are 2.

An Alternative Method

No matter how good the foregoing method may be, it will become monotonous. For the sake of variety, if for no other reason, the following alternative method is suggested. The object in this method, as in the one previously described, is primarily to avoid counting.

4

Suppose the combination $\underline{5}$ is about to be taught. Let the teacher select 9 objects large enough to be visible to the entire class but not too large to be easily handled. Sticks of crayon will do very well. Some way of displaying these on the desk or a low table should be arranged, and the pupils should be permitted to gather close around the teacher.

Let the teacher first show 4 crayons by laying them on the desk or table.

TEACHER. How many pieces of chalk are there here?

PUPIL. Four pieces. [*Counting if necessary*]

The teacher should now conceal these 4 crayons, perhaps by putting a book in front of them or by turning a small box over them. Let her now show 5 more pieces of chalk on the table, and let her obtain from the children as before a statement that there are 5 pieces. Let her now recall the number of crayons there were the first time, namely, 4; and let her get a restatement of the number now displayed, namely, 5.

Let the teacher now place the 5 crayons out of sight along with the 4 crayons already concealed. Let her then show the entire group and ask the children to tell her how many there are. They will easily be able to count them and ascertain that there are 9.

It will be a good plan to repeat the process from the beginning, emphasizing the 4 crayons shown first, then the 5 crayons shown second, and finally the 9 crayons resulting from combining these. From this the statement "4 crayons and 5 crayons are 9 crayons" may be secured.

A similar procedure may be followed, using for concrete objects children, buttons, or even books, if they are not too large to be concealed. If children are to represent the groups, they may hide behind the teacher's desk or go to the rear of the room. Finally the general statement may be written on the board, read, copied, and written from dictation as before.

In this method the essential points are that each addend or component is presented separately and identified, that the one first presented is concealed, and that the other addend after identification is concealed with it. The concealment is intended to prevent the child's seeing the group in such a way as to suggest counting from the next unit after the first addend; that is, when 4 and 5 are presented, counting "5, 6, 7, 8, 9." Another important point is that when the entire group is brought to view it is to be identified as a whole. Counting is entirely permissible at this time.

A method identical in principle may be used in teaching the subtraction facts. Let us suppose that the fact "9 less 4 are 5" is to be taught. Let 9 crayons be exhibited, counted, and identified as to number. Let the question "How many pieces of chalk are there?" be answered. Then let all the crayons be hidden from view. They may be concealed behind a book or a pile of books. From the 9 things thus concealed let 4 be taken and brought to view. These should be counted, if necessary, and every pupil should realize that there are four of them. Then let the question be raised, "If from 9 pieces of chalk I take away 4 pieces, how many are left?" At this point let the remainder be brought to view. The books which conceal the remainder may be thrust aside. The dénouement will reveal the answer to the question, namely, five of whatever articles are being used.

This method of hidden operations lends itself very well to drill, especially where the objects added or subtracted are pupils. The device of having two successive groups of children go to the rear of the room where they cannot be seen, such as 5 pupils and 3 pupils, creates considerable interest. Similarly the device of having 8 pupils go to the rear of the room or out into the hall and of having 3 of them come forward gives a dramatic verification of the answer to the combination 8 less 3 are 5.

Verbal Problems

General and particular problems. It will be observed that as each combination has been taught we have suggested its immediate application in concrete problems. Now no book of the character of this one can hope to list the most appropriate problems, because these must be largely local and occasional in character. We can be reasonably sure that all children are interested in some things and that to a certain degree they possess a common heritage and common instinctive tendencies. Problems based on this assumption will indeed be valuable but doubtless not so much so as problems based upon a particular local condition; and, as far as individual pupils are concerned, the most valuable problems of all are probably those which are made up to fit each child's interest. It is true that group teaching does not permit us to adjust our materials with this degree of refinement, at least not as a matter of general policy, but these advantages should be used when special need and reasonable opportunity permit. The teacher should keep a record of the problems she uses, especially of those that seem to work well.

Supplementing the printed problems. On page 83 we give a list of problems illustrative of the combinations of Group II. Since these combinations will have to be reviewed for a long time, the problems we present will be insufficient material for the purpose. They may be used as far as they go, and the situations to which they refer may afford suggestions to teachers for devising additional problems. For example, among the problems which exemplify "one and six" are the following situations, which may be used as the basis of other problems: Bo-peep and her lost sheep, goldfish in a globe, buttons on a blouse, the purchase of a pencil box and pencils, roses on a bush, cherries on a branch, marbles given away, and eggs in Biddy's nest.

Taking time to teach problems. At the same time we should like to express our belief that teachers quite generally use up their problem material too rapidly, that they do not get from it all it has of value. This value, we believe, can be secured only when close and somewhat prolonged attention is given to each problem. Our advice therefore is: "Do not burn up the material; do not waste it by skimming over it. A few problems thoughtfully handled and well analyzed are more valuable than a great many problems superficially presented." Our point is that in the application of number work to concrete situations, if those situations are really meaningful, the object is not only to set up situations requiring reasoning but also by these means to *train* children in reasoning. Now, by its very nature, the reasoning process, especially in its early stages, is deliberate. Speed is about the last thing we want. Speed with accuracy in giving the fundamental combinations is indeed desirable, it is essential. But in the analysis of situations to the point where the proper choices are made, sure and reasoned judgment is greatly to be preferred to snap judgment. Nothing is more inimical to reasoning in problem work than the attitude of the child who blurts out an answer long before either he or anyone of his mental maturity could have arrived at it by justifiable means.

Making the problems vivid. It is not sufficient that problems be well made or that they present real or interesting situations clearly and effectively. As part of the *teaching* of the problems, perhaps the most necessary part, the pupils should be made keenly aware of these situations. If they are really going to think about the problems and think to some purpose, they must appreciate their meaning. Sufficient care and enough time should be taken in their presentation to bring this meaning before them vividly. This may be done in various ways, sometimes by merely talking about the problems, sometimes by a kind of make-believe, and sometimes

by dramatization. For example, in connection with the first problem under "One and six" given later, one girl may represent Bo-peep, another may represent the one lost sheep that came home first, and six others may represent those that came home later. Again, pictures may be sketched — they need not be artistic — to illustrate problems. The goldfish problem (2 under "One and six") or the problem about roses or the one about cherries (5 and 6 in the same set) may be so treated. The point we wish to make is that considerably more should be done than merely reading the problems to the children.

Having pupils make problems. As soon as pupils can begin to make their own problems they should be encouraged to do so. Teaching children to solve problems is not easy. It is generally neglected. Undoubtedly this difficult task will be made easier if the pupils so far join in the activity as to create problem situations. If the problems we give them are well within their ability to comprehend, then we may not unfairly expect them to reverse the process and to formulate problems of like character.

In this they will at first need considerable help. A problem may be used as a model. For example (4 under "One and three"): "There are 3 bears in a little house in the woods. If 1 bear comes to visit them, how many bears are there in the house?" Children may then be asked to tell another story about bears. The details supplied them may extend as far as appears necessary. If they need much help, they may be asked to tell another story about three bears, or about three bears in a little house in the woods. They may likewise be given the combination which is to be illustrated. For example: "Tell a story about bears; use 3 take away 1 in it."

Similarly such proposals may be made as "Make a problem [tell a story] about Tom and his rabbits; use 3 take away 2

in it." Or one like this: "Go to the store and buy a banana and an orange. Make a problem about it. Use 4 and 4."

Pupils who make problems should be permitted to call on someone to solve them. The play element can easily be introduced into this procedure. Moreover, the alert teacher may find that pupils will supply interesting problem settings which have not occurred to her. All such suggestions should be jotted down and recorded for future use.

Illustrative Problems

One and two

1. Betty spent 1 cent for a pen and 2 cents for a penholder. How many cents did she spend for both?

2. Robert had 1 cent and earned 2 cents. How many cents had he then?

3. Jane paid 2 cents for a card and 1 cent for a stamp. How much did she pay for the card and stamp?

4. I had 2 flowers and Kate brought me 1 more. How many flowers did I then have?

5. A little boy had 3 stars on the blackboard. One morning he was late and lost 1. How many stars did he then have?

6. There were 3 bowls of soup on the bears' table, and 1 was eaten up. How many bowls of soup were left?

7. There were 3 children playing on the porch; 2 went into the house. How many children were left on the porch?

8. Julia had 3 flowers and gave 2 to her teacher. How many flowers had she left?

Four and four

1. Harold spends 4 cents for an orange and 4 cents for a banana. How many cents does he spend?

2. Frank sold 4 quarts of cherries in the morning and 4 quarts in the afternoon. How many quarts did he sell that day?

3. Ralph picked 8 quarts of berries and sold 4 quarts. How many quarts would he keep?

4. There were 8 books on the desk. The children took 4 of them. How many books were left on the desk?

5. A picture post card and a stamp to send it cost 4 cents. How many cents would it cost to send 2 cards?

6. Joe fed his pet squirrel twice a day. Each time he fed it, he gave it 4 nuts. How many nuts did he give it each day?

7. Mrs. Jones has some large jugs. It takes 4 quarts to fill one. How many jugs will 8 quarts fill?

8. At Lulu's party she will have 8 children and she can have 4 children at each game table. How many game tables will she need?

One and six

1. Bo-peep lost her sheep. First 1 came home, then 6 more came. How many sheep came home in all?

2. We had 1 goldfish in a globe and we put in 6 more. How many goldfish did we then have in our globe?

3. There were 6 buttons on a little boy's blouse. His mother sewed on 1 more. How many buttons were then on the blouse?

4. Jane paid 6 cents for a pencil box and 1 cent for a pencil. How many cents did she pay for both?

5. There were 7 roses on a bush. Alice picked 1. How many roses were left?

6. There were 7 cherries on a branch. Robin Redbreast picked 1. How many cherries were left on the branch?

7. Edward had 7 marbles and gave 6 of them to his little brother. How many marbles had he left?

8. There were 7 eggs in Biddy's nest. Farmer Brown took away 6 of them. How many eggs were left in Biddy's nest?

Four and five

1. We had 4 gold stars and 5 white stars on our blackboard. How many stars were there on our blackboard?

2. Polly earned 4 cents and her father gave her 5 cents. How many cents did she have then?

3. Henry sold 5 papers in the morning and 4 at evening. How many papers did he sell in all?

4. Tom had 5 little rabbits and 4 big rabbits. How many rabbits did he have altogether?

5. A farmer has 9 milk cans in his wagon. He takes out 4. How many cans are left in the wagon?

6. There were 9 men working on the road, and 4 of them went away. How many men were left to work on the road?

7. There were 9 little pigs in a pen; 5 ran out to play. How many little pigs were left in the pen?

8. There were 9 apples on a tree; 5 dropped off. How many apples were left on the tree?

One and three

1. A teacher sent 1 boy to the blackboard, and then 3 more boys. How many boys did she send to the blackboard?

2. One little pig went out through the fence, and then 3 went out. How many little pigs went through the fence?

3. Every day I buy 3 quarts of milk in the morning and 1 quart at evening. How many quarts of milk do I buy each day?

4. There are 3 bears in a little house in the woods. If 1 bear comes to visit them, how many bears are there in the house?

5. There were 4 little kittens in a basket, and 1 jumped out. How many kittens stayed in the basket?

6. There were 4 lighted candles on a birthday cake; 1 candle burned out. How many candles were left burning?

7. There were 4 jack-o'-lanterns in the toy store. The storekeeper sold 3 of them. How many jack-o'-lanterns were left in the store?

8. Brother brought 4 big sticks of wood for the fireplace; 3 of them were burned up. How many sticks of wood were left?

The Progress Test

The following test is intended to permit the teacher to check the success of her work up to this point, at least so far as the combinations are concerned. The twenty facts of Group I and the twenty facts of Group II are included. If

practicable, the test should be given in mimeographed form, and the pupils should be asked to write the missing answers. The purpose will be served, however, if the test is put on the board or given orally. In any event a record of each pupil's response on each combination should be made. A class record sheet similar to that suggested for the test on Group I should be used. Additional drill should be given to individual pupils on certain combinations according to the results shown on this class record sheet.

Test on the Combinations of Groups I and II

7	8	7	9
−6	−4	−1	−4
4	1	1	6
−3	7	4	−3
3	3	5	5
1	−1	−4	4
5	7	2	2
−1	1	2	−1
2	3	9	8
1	−2	−5	−1
4	1	4	1
−1	1	1	6
1	6	4	4
3	1	5	4
3	4	8	1
3	−2	−7	2
2 4's are	1's in 2 are	4's in 8 are	3's in 6 are
2's in 4 are	2 3's are	2 1's are	2 2's are

The foregoing exercise does not test the number ideas of the pupils nor their ability to write numbers, except as these ideas and this ability are implied in the completing of the combinations. The teacher should therefore give a supplementary test, for the most part oral, in which pupils will have an opportunity to do such of the following things as they have been taught:

1. Counting to 100 by 1's and by 10's.
2. Reading numbers pointed to on the number chart.
3. Finding dictated numbers on the chart.
4. Describing them as to tens and ones.
5. Copying numbers within 100.
6. Writing numbers within 100 from dictation.
7. Writing numbers from dictation and putting them one under another.
8. Solving verbal problems.
9. Making verbal problems from given conditions.

CHAPTER VIII

MEASUREMENTS

It is entirely possible to make this important division of primary work tedious and repellent. The teacher's problem is to avail herself of the undoubted interest which children have in certain forms of measurement. If we seek to identify these forms, it is certain that we shall include among the most promising of them the measurement of length or distance. The idea of distance enters very early into the life of children. They have to go from one place to another, and some of their destinations are farther away than others. "Far" and "near," therefore, early come to have intimate personal meaning. Some things within the early and continued experience of children are longer or shorter than others; and they are intensely concerned among themselves as to which one is taller or "bigger" than another. Clearly, therefore, the child's interest dictates that measurements of length and units of length should be introduced early.

Units of Length

The foot. Perhaps the most natural unit is the foot. It is large enough to be obvious, yet not too large to be easily used. Moreover, many children, especially the boys, may have already used the term "foot" as a unit of measurement. The common practice among boys of measuring how far they have jumped by putting heel to toe and counting the number of their "feet" is often observed. It may afford an interesting approach to the subject to show that after a boy's jump is

measured by his own feet the result may be quite different from that of the same jump when measured by little Annie's feet. Children can easily be brought to see that you can't tell how many feet Bill can jump unless you know whose feet you are talking about.

It is appropriate in this connection to tell the children that many, many years ago people measured, as children now do, each man by his own feet. They saw, however, just as the children can be induced to see, that it made a great deal of difference whether the feet used in measuring were large or small. So the people decided to use one man's foot in order that everybody's measurement would be the same. Naturally the man whose foot they would use would be the king. This was all right as long as they had the same king, but when the Very Tall King with his big feet was followed by the Very Short King with his small feet there was trouble. Finally people hit upon the idea of having the same length for a foot no matter how many kings might come and go. "And," the teacher may conclude, showing a foot ruler, "this is the length of the foot they decided to have."

A supply of cheap rulers should be available, and the children should be given frequent opportunity to measure lengths such as the dimensions of the schoolroom, the blackboard, the doors, and windows, which lend themselves reasonably well to the application of the foot as a unit. Do not let the children suppose that the foot is the foot ruler, but rather that it is the *length* represented by the foot ruler.

Have them draw lines on the blackboard both with and without the assistance of the ruler, lines first one foot long and later more than one foot long. Have them estimate how long a foot is. Let them do this by marking off distances on their desks or at the board. Afterwards they should check their estimates by using the ruler. They should also estimate distances of two feet, three feet, and the like, frequently

checking their estimates by actual measurement. A great deal of interest may be aroused in this sort of thing. Tom may estimate that the broom handle is two feet long, Helen may think it is three. The views of Tom and Helen may be reconciled when Jack takes a foot ruler and measures the handle. Perhaps it turns out to be four feet long (to the nearest foot).

Half a foot. At this point almost inevitably, and certainly inevitably if the teacher does a little managing, the idea of half a foot will come up. It has been abundantly shown that the word "half" and the general idea of its meaning are in the possession of children of this age. Be sure to capitalize this knowledge or partial knowledge. Do not at this time develop formally the idea of one half. Use it and let the children use it. Bring out the idea that on the foot ruler half a foot runs from the left end to the middle, and that from the middle it is just as far to the right end, and so there are two halves in a foot, and each of these halves is the same length.

As soon as the children can freely use the idea of half a foot, their resourcefulness in measuring the length of a desk or table will be increased. They should learn that if something is four feet long and half a foot over we say it is four and a half feet long. They may have practice in estimating and drawing lines or setting distances on the schoolroom floor to meet assigned conditions; and in these conditions lengths like two and a half feet, three and a half feet, and so on, will be used.

The inch. Very soon, however, the need of a smaller unit will be felt. The broom handle which Jack measures is, let us say, considerably over three feet long and somewhat less than four feet long. Moreover, it is not very near to three and a half. The children are perplexed. This is a fine problem situation. There is a "felt need." It is time to teach the unit "inch." The broom handle may turn out to be 3 feet 9 inches

long. Show the children in this and similar instances how to measure in feet and inches and how to say and write the result.

Children should draw lines and estimate distances of one inch, checking their results by the inch markings on the foot ruler. They should likewise draw lines and estimate distances of two inches, three inches, and so forth, testing their estimates by measurement. The same procedure should be followed in drawing, with and without the aid of the ruler, distances of a certain number of feet and inches, for example, 2 feet 3 inches, and 1 foot 9 inches. The words "foot," "feet," "inch," and "inches" should be taught so that they may be read.

Of course, in teaching the inch we shall also teach that "twelve inches make one foot." This can be verified by counting the inch markings on the ruler.

The half inch. First-grade and second-grade children will easily see that many lines, distances, lengths, and so forth are not a whole number of inches long. About as far as it is advisable to go in accuracy at this time is to the half inch. Let the matter arise naturally from the measurement of some distance which falls about midway between two whole numbers of inches. Very likely some of the children will know what this means. Generally, however, they will not. When the matter arises, it will be well to state that this distance is, let us say, five and one half inches long. It will then be appropriate to raise the question "What is half an inch?" and to devote some time to answering it.

As in the case of the half foot the essential thing is to show that the half as applied to an inch means one of the two equal parts of an inch, that is, that there can be only two such parts and that each of them is just the same length as the other. Children should be shown the half-inch marking on the foot ruler and should be made familiar with the fact that it is just as far from the five-inch mark to the five-and-one-half-inch mark as it is from the latter to the six-inch mark.

Measuring heights. Of all the applications of the inch and foot, the one which will most surely interest the children is the application to their own heights. As we all know, there is considerable rivalry among children as to which is taller. If the teacher shows them how to measure one another, they will continue to make applications of the inch and foot, both in and out of school hours, long after the teacher has ceased to give attention to this matter.

Applying these Units

Home study and investigation. Have pupils measure things at home, such as rugs and rooms, the height of chairs and tables, windows, doors, and the like. A great deal can be made of this sort of work by putting it on the basis of investigation and report. Children will discover to their surprise that practically all chair seats are eighteen inches high and tables thirty inches high. Doors and windows have different heights and different widths, but many of them are alike. The use of certain quantitative adjectives should be encouraged, such as the positive, comparative, and superlative forms of "much," "little," "long," "short," "wide," "far," "near," "apart," "tall."

Verifying the combinations. It is a good plan to have pupils test, by means of measurement, some of the combinations they have learned. For example, if they draw a line one inch long and then, beginning at the right-hand end of that line, draw another line three inches long in the same direction, they can measure the resulting line and find that it is four inches long; in other words, that one inch and three inches are four inches. The same sort of thing can be done even more effectively by cutting paper of various lengths and putting the cuttings together. The subtraction combinations are also neatly illustrated objectively by these methods.

Units of Capacity

Another type of measurement which appeals to children with sufficient force to justify its inclusion here is measurement of capacity. The units of this sort with which they are most likely to be familiar are *teaspoonful*, *tablespoonful*, and *cupful*. They are certain to know spoon and cup, and the spoon they commonly use is, of course, the teaspoon. Have the children dip water with a teaspoon to fill a "big" spoon, or tablespoon, and let them see that it takes three teaspoonfuls to make one tablespoonful. It will be a good plan to have them learn this as a fact. They may count the tablespoonfuls of water as they pour them into a cup, and find that they must dip the spoon 16 times before the cup is full. Therefore, there are 16 tablespoonfuls in one cup. Dry material such as flour, sugar, and sand should also be used, but with such material the spoon or cup should be made level by scraping off the excess with the back of a knife. All these units, as actually found in the home, are more or less inexact. The teacher should have at hand standard measuring teaspoons, tablespoons, and cups. These may usually be purchased without difficulty. If measuring spoons and cups cannot be had, the teacher should be sure to test those that may be collected from the homes to see that they are of the right size.

The fact that two cupfuls are a *pint* may be used in teaching the meaning of the latter unit. The fact itself should be learned, and also the corresponding fact that a cupful is one half a pint. Again, as in the case of half a foot and half an inch, be careful to show that the cupful is half a pint because it is one of the two equal parts of a pint, that is, that there can be only *two* cupfuls in a pint and that, of course, each of them is the same. The facts regarding the cupful and pint should be abundantly illustrated by having the children handle materials, both dry and liquid, with the actual measures. They

should do the same with the pint and the *quart*, learning the facts, 2 pints are 1 quart, and 1 pint is half a quart. Have at hand not only quart and pint cups (liquid measure) but also quart and pint boxes such as are used to measure berries.

The idea of one half may now be applied for the fourth time. Show that the pint is one half a quart because it is one of the two parts of a quart that are just the same (equal). We may now generalize about one half. We have always found that it is one of the two equal parts of something. We may therefore say that "one half of anything is one of the two equal parts of it." The pupils should be shown how to write $\frac{1}{2}$ and should be given practice in writing such facts as "$\frac{1}{2}$ a pint is a cupful" and "$\frac{1}{2}$ a quart is a pint." They should answer such questions as "*What part of* a quart is a pint?"

Children should be trained in estimating the number of cupfuls, pints, or quarts of water in a pan, basin, or other container (not necessarily full). They should "guess" the number of quarts in a pile of nuts or pints in a heap of sand or cupfuls in a quantity of flour. They should verify their estimates by measuring the materials. The children should also be encouraged to estimate and measure things at home. Use, and expect the children to use, the appropriate quantitative adjectives. Problems should, from this point on, make free use of the terms and equivalences taught in this chapter as far as they are within the number facts which the children are supposed to know.

Half of a number. The pupils have already been taught the facts

2 1's are 2	2 3's are 6
2 2's are 4	2 4's are 8

Now, since 2 4's are 8, if 8 is separated into two parts each of the same size, one of these parts is 4. In other words, 4 is one of the two equal parts of 8. Recall the similar

facts as to cupfuls and pints, pints and quarts, showing that in each case the former is half the latter. The teacher may then ask, "Four is what part of 8?" If the idea of one half has been well presented in this lesson and in previous lessons, most of the children will see that 4 is $\frac{1}{2}$ of 8. The teacher should then write on the board "$\frac{1}{2}$ of 8 is 4," reading this fact as she writes. Finally, the fact should be posted unless the fact 2 4's are 8 or $\frac{4}{8}$ is considered sufficient.

Similarly, from the fact 2 3's are 6, teach the fact $\frac{1}{2}$ of 6 is 3 (one of the two equal parts of 6 is 3); from the fact 2 2's are 4, teach $\frac{1}{2}$ of 4 is 2; and from the fact 2 1's are 2, teach $\frac{1}{2}$ of 2 is 1. Thus four *partition* facts will have been taught. They involve finding $\frac{1}{2}$ of each of the even numbers from 2 to 8 inclusive. These facts should be posted and they should form part of the regular drill material.

Coins

We cannot neglect in our treatment of measurements at this early period the various coins about which children hear so much and in which they are sure to have an interest. Most children know something about the cent, nickel, and dime. Their ideas of these coins should be made specific. The teacher should be sure that they can identify them, and that they know the relation between them, namely, that five cents have the same value as one nickel, and that a dime may be made up of ten cents, or two nickels, or a nickel and five cents. Children should play store and give and receive change by counting real or toy money. Under usual circumstances the quarter, half dollar, and dollar may be taken up at this time or not much later. The ability to identify coins and to know their equivalents is the aim. In buying and selling, the units and half units already learned should be freely employed.

Continuing the Work in Measurement

All through the pre-third-grade number work measurement should be frequently employed. At no time should it be dropped. As has already been indicated, the basic number facts may be illustrated. Estimates may be made, as well as actual measurements with the appropriate units. The facts of equivalence, such as "two pints make a quart," may continue to be stated and illustrated. Such relationships occasionally may be put into the verbal problems which the pupils are to solve. For example: "Charles picked a quart of raspberries before dinner and a pint after dinner. He put them into pint boxes to sell. How many pint boxes did he have to sell?" This problem, of course, requires the use of $2 + 1$, with the added requirement that the equivalence of a quart and two pints be known. Although extensive use of this equivalence is not yet possible because of the pupils' limited knowledge of number facts, problems involving one quart and two pints or one pint and two cupfuls may now be introduced. Problems involving one quart subtracted from three pints or four pints are likewise within the number facts already taught. When the combinations of Group III have been taught, the relation between quart and pint may be further illustrated in problems where one quart is added to or subtracted from three, four, or five pints.

For the most part, problems requiring a knowledge of the relation between inch and foot should be delayed until some of the combinations involving twelve have been taught. Certain problems of this kind, however, may be solved by actual measurement — supplemented by counting — and without recourse to the number facts involved. For example, a pupil may, by drawing or by cutting paper, find out how many inches there are in 1 foot 4 inches, how many more inches there are in 1 foot than in 5 inches, or how many more inches there

are in 2 feet 3 inches than in 1 foot 8 inches. Problems of a more highly developed character may be solved by measurement, if the teacher gives proper guidance. Consider the following problem: "Mother and little Patsy were sweeping. Mother's broom was 4 feet 5 inches long. Patsy's was 3 feet 6 inches long. How much longer was mother's broom than Patsy's?" The teacher may have a line drawn 4 feet 5 inches long to represent mother's broom. A line 3 feet 6 inches long beginning directly under the left end of the first line may be drawn to represent Patsy's broom. If the lines are carefully drawn and are close together, pupils can easily measure how much longer mother's broom was than Patsy's.

This method of solving problems, by measurement and counting rather than by computation, may be considerably extended. Do so by all means, if conditions permit. Pupils will thus obtain better ideas of the meaning of the units and a certain skill in the use of the instruments of measurement. Moreover, the hand training involved will be of much value.

Illustrative Problems

The following problems are intended to suggest to the teacher the application of the facts taught in this chapter. Nearly every one of these problems is a type, and many others may be made like it. For example, only one paper-cutting problem in each operation is given. The teacher should devise others of a similar nature.

One half of a number

1. Polly and Pearl were twins. They always divided their good things into halves. One evening father brought them a box from the baker's. In the box were 2 fine cream puffs. Polly and Pearl peeped in, and then each took $\frac{1}{2}$. How many cream puffs did each one take?

2. Grandpa Brown was going to give away 2 bushels of his apples. He divided them equally between 2 families, so that each family got $\frac{1}{2}$ of them. How many bushels of apples did he give each family?

3. Jack and Tom were camping. "Oh," said Jack, "we have just 4 eggs left for our breakfast." Tom said, "That's enough; we'll each eat half of them." How many eggs did each boy have?

4. Mrs. Harper had 4 peaches. She divided them equally between her 2 little boys. What part of the peaches did each boy receive?

5. Mrs. Race bought 6 lemons. The children wanted them for lemonade; but Mrs. Race wanted them for pies. Finally she said they would share them. She would let them have $\frac{1}{2}$ of them for lemonade. How many lemons did the children have for lemonade?

6. Kate wants to ride on her pony to her aunt's house 6 miles away. Mrs. Brown's house was just halfway there. So Kate said she would stop at Mrs. Brown's to rest. How many miles did Kate ride before resting?

7. If sugar is 8 cents a pound, how many cents will $\frac{1}{2}$ pound cost?

8. There are 8 plums in a pound. How many plums are there in $\frac{1}{2}$ a pound?

Measures of length

1. Tom's foot is 8 inches long and Robert's is 6 inches. How many inches shorter is Robert's foot than Tom's? (To be done by measuring.)

2. The Very Tall King's foot was 1 foot 2 inches long. The Very Short King's foot was only 10 inches long. How many inches longer was the Very Tall King's foot? (To be done by measuring.)

3. How many inches make a foot?

4. How many inches are there in half a foot? Count on your ruler if you do not know.

5. Cut a strip of paper 8 inches long. Now cut off 3 inches of the strip. Measure the piece left. How many inches is it? Then 8 less 3 are how many?

6. Cut two strips of paper, one 4 inches long and the other 3 inches long. Put the strips together end to end, and measure the length all together. How many inches *should* it be? Do you find it so?

7. Cut two strips of paper each 4 inches long. Put them together end to end. Measure the length of both together. How many inches? Then 2 4's are how many?

8. Cut a strip of paper 6 inches long. Fold it so as to bring the ends together. Now unfold it. You have divided it into halves. Measure each of these halves. How many inches long are they? Then $\frac{1}{2}$ of 6 is how many?

9. With your ruler draw a line $6\frac{1}{2}$ inches long. Draw another line 5 inches long. How many inches longer is the first line than the second? Use your ruler to find out. How many inches shorter is the second line than the first?

To the Teacher. Have these lines drawn horizontally, parallel to each other, and with left-hand ends aligned, like this ——————.

10. Without using your ruler, tell how long you think the window sill is. Guess how tall your teacher is. Guess how wide the cupboard is. Now measure the window sill, the teacher's height, and the width of the cupboard. How near was your guess in each case?

11. Without using your ruler, draw a line which you think is 3 inches long. Draw a line which you think is $5\frac{1}{2}$ inches long. Draw still another line which you think is 8 inches long. Now measure each of the lines you have drawn and see how near your guess was. Were you nearest right in the first, second, or third line?

12. Without using your ruler, mark off on the floor a distance which you think is 2 feet. Now measure the distance with your ruler. Did you guess too much or too little? Mark off what you think is $3\frac{1}{2}$ feet on the floor without using your ruler and again see whether your guess was too much or too little.

13. Put your ruler away. Now draw a line which you think is 1 foot long. Take your ruler and measure the line. How many inches is your line more or less than 1 foot? Throw your paper away or turn it over and try again. See if you can draw a line more nearly a foot long this time.

14. Little Jerry's step is only half as long as his father's. His father steps 2 feet 6 inches. Cut a piece of string 2 feet 6 inches long. Double it and cut it in halves. The length of one of these pieces of string is the length of little Jerry's step. What is the length of his step?

Measures of capacity

1. What part of a quart is a pint?

2. How many pints are there in 3 quarts? (To be done by measuring.)

3. How much more are 2 quarts than 3 pints? (To be done by measuring.)

4. Pour into a basin what you think is a pint of water. Now measure what you have put into the basin and see how nearly right you were able to guess.

5. Pour what you think is a quart of water into a basin. Now measure the water and see how near you were able to guess.

To the Teacher. Provide about 8 quarts of dry material. Nuts, pop corn, or peanuts are suggested. For measures the ordinary pint and quart berry boxes are recommended. In the following problems "nuts" will be used. Substitute whatever you have provided.

6. Without measuring, make a pile of nuts which you think contains 4 quarts. Now measure the nuts. How nearly right were you?

7. Jim picked 5 quarts of raspberries. Then he wanted to sell them, but they are sold in pint boxes. How many pints did Jim have? Play that your nuts are Jim's raspberries. Measure out 5 quarts, then fill a pint box with them as many times as you can.

8. Measure 10 cupfuls of nuts. See how many quart boxes you can fill with these nuts. See how many pint boxes the rest of the nuts will fill. Then how many quarts and pints are 10 cupfuls?

Coins

1. Fred went to the store to buy a top. It cost 5 cents. Fred had a nickel. Did he have enough to pay for the top?

2. How many cents are a nickel and 4 pennies?

3. Chocolate bars are a nickel apiece. How many can Jerry buy for a dime?

4. Herbert bought a pound of sugar for 8 cents. If he gave the storekeeper a nickel, how many pennies should he also give him?

5. Susie bought a bun for 3 cents. She gave the storekeeper a nickel. How many cents did he give back to her?

6. Alice and Marie are playing store. Alice has two nickels and Marie has a dime. Marie says her money is as much as Alice's. Is this true?

7. Marie wants John to give her some pennies for her dime. How many pennies should John give her?

8. How many dimes will Joseph need to pay for a baseball that costs half a dollar?

9. A quarter is two dimes and how many nickels?

10. Philip has a quarter and 3 pennies. How many cents has he all together?

11. How many quarters will Henry need to buy a necktie that costs a dollar?

12. How many handkerchiefs can Sam buy for a dollar if they cost half a dollar each?

CHAPTER IX

DRILLS AND GAMES. II

The same devices which have been described in connection with the drill on the combinations of Group I may, of course, be used with all the combinations taught up to this point. For example, the drill cards with emphasis upon accuracy rather than speed may be utilized for a long time. The bean-bag game will continue to be useful, and the children may still play crossing the stream and the ladder game. Several other play activities which have proved successful are described here.

Fishing game. As the teacher sketches on the blackboard a representation of fish in the water, she may say: "Today we are going to take a little fishing trip. I am sure all the boys would like to go and perhaps some of the girls would like to go too." As the teacher continues to talk in this manner she completes her sketch and writes in the water the combinations which are to represent the fish. These, of course, are the combinations on which the children need drill. "Well," says the teacher, "here we are, ready to go fishing, and what a lot of fish there are! Edward, you may get into the boat and catch some of them." She then points to various combinations representing fish and Edward tries to catch the fish by telling the combinations. If he is successful he may be allowed to call on another pupil, or the teacher may keep the matter in her own hands, thus directing the drill toward those who particularly need it. The combinations may be changed as often as desired. This game is not intended to represent a contest; but by giving each child the same number of oppor-

tunities and by scoring the number of fish he catches, the element of contest between individuals or "sides" may be introduced. The picture below indicates the nature of the sketch for this game.

Flower game. The teacher says: "Let us play that today we are going on a long walk over the hills and through the fields. The sun is shining and bright flowers are waving in

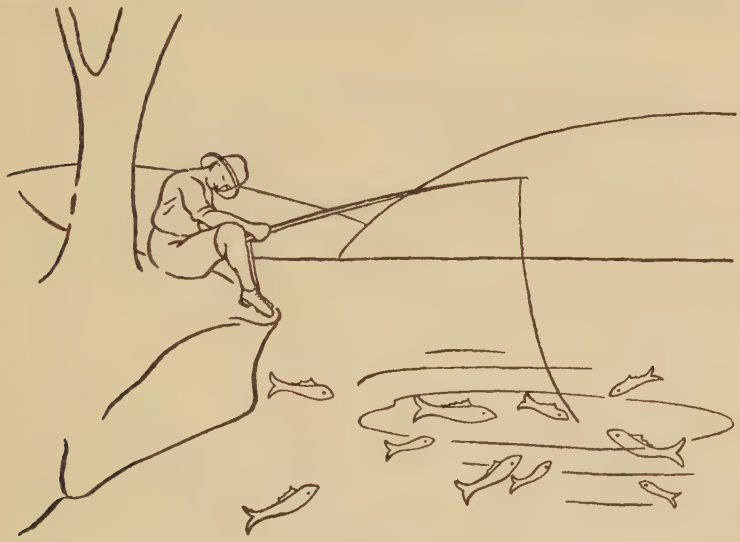

FIG. 8

the grass. . . ." While talking in this strain, the teacher may sketch on the board something to represent fields, grass, and flowers. Nothing particularly artistic is required, since the children's imagination will make good almost any artistic discrepancies. Each flower is represented by one of the number combinations. "Let us pick some of these flowers," says the teacher, "to take back with us. Perhaps we can send them to some child who cannot go out in the fields. Let us see who can pick a handful." Picking the flowers is, of course,

giving the right answer to the combinations. The element of contest can be introduced but the fun of the play is sufficient. A sketch of the field and the flowers is shown below.

Going to the lunch counter. This game depends upon the children's being acquainted with the lunch counter which many schools have. In order to play it successfully in schools that lack this convenience, the teacher should explain what a lunch counter is. Moreover, in order to play this game, the

FIG. 9

children must assume that the place where they are going to get all these good things to eat is in the basement.

The teacher may say: "Let us play that you have brought some money to school today and that when your work is done you may go down to the lunch room in our basement and buy something good to eat. You will have to be very careful on the stairs. If you fall some one will have to help you back to your room and you will miss your lunch."

The teacher either before or during the introductory talk about the game will sketch on the board a stairway leading to the lunch room. Each tread of the stairway will be represented by one of the number combinations. Each pupil should be permitted to go completely down the stairway, that is, he should attempt all the combinations represented.

He must "be careful on the stairs," and so he should not hurry. From time to time during the play some or all of these combinations may be changed. A sketch of the stairway is given on this page.

Card game. Cards bearing the number combinations on which drill is desired are placed along the blackboard. The teacher calls upon a pupil and gives him a certain combination to "get," that is, to say in its complete form. The

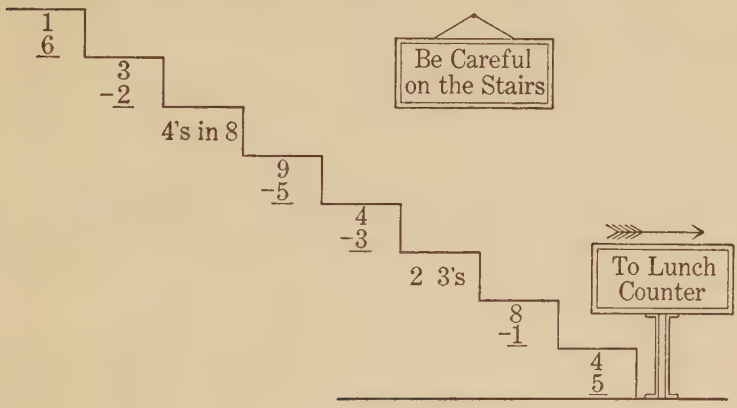

FIG. 10

pupil who gets a combination keeps the card that shows it, and the pupil or side obtaining the most cards within the time limit wins.

A guessing game. The teacher writes certain combinations on the board. One child leaves the room; another points to a combination; the first child returns and guesses what the combination is. Thus: "Is it 3 take away 2 are 1?" "Is it 1 and 6 are 7?" If he guesses correctly within, say, three chances he may have another turn.

Games in the matching and reading of numbers should be continued. Some of these have been previously suggested. The games which follow have also been found to be useful.

Tag game. Sides are chosen and each pupil has a tag pinned to his shoulder. The tags are marked with numbers. One side begins tagging the other side and captures a player by calling the number on his tag. After an interval, say of one minute, the other side may tag and capture their opponents by calling the tag numbers. The side having the largest number of players at the end of the game wins.

Messenger game. Numbers are put on small cards and given to some of the pupils. These are house numbers or addresses. Other pupils are given envelopes marked with the same numbers. These "letters" must be delivered to the right addresses. The pupils who have the small cards are the people who are to receive the letters. They hold their house numbers in their hands so that they can be seen. The messengers go about to the various houses trying to find the correct addresses. When a messenger finds the house number corresponding to the number on his letter, he delivers the letter to the pupil holding this number.

A counting game. A pupil comes to the front of the room and is blindfolded. The teacher silently indicates another pupil, who begins to count. It should be arranged beforehand to have the counting begin either at 1 or at any other number, and the number with which to end should also be decided upon. The first pupil must guess who is counting. He may have three guesses. Then the procedure is repeated with another blindfolded pupil. If desired, the successful guesses may be scored.

Another counting game. A pupil closes his eyes or is blindfolded and counts to 20 or to any number agreed upon. Another pupil hides a piece of chalk. When the first pupil finishes counting, he begins to search for the chalk. If he has not found it in a short time, the class tells him where it is. Then the procedure is repeated.

As drill devices some of the foregoing games have much less value than others. The fishing game, for example, is slow and

only a few children can participate. The same may be said of the flower game and of the game we have called "Going to the lunch counter." [1] But drill value is not the only value a game may have. If it has a high quality of interest it may be well worth while. Its importance may properly consist in making the number work interesting. As for the objection to some of these games that "numbers do not grow on flowers or appear on stones or attach themselves to ladders," we have no sympathy with such a point of view. The make-believe world is real to the child — not objectively real, but real in the sense that every normal child experiences it — and we have intentionally invoked this spirit of fancy both in our games and in our problems.

[1] An almost indefinite number of similar games may be used: leaves falling from a tree and marked for "gathering" by the pupils, potatoes similarly marked to be "put in the basket," apples to be picked, cookies to be eaten, and so on.

CHAPTER X

THE THIRD GROUP OF COMBINATIONS

The key combinations of the third group are as follows:

$$\begin{array}{ccccc} 2 & 1 & 2 & 1 & 2 \\ \underline{3} & \underline{5} & \underline{4} & \underline{8} & \underline{5} \end{array}$$

Since none of these is a double, there are four associated combinations in each of these teaching units, or twenty combinations in all. The average difficulty of each of the five teaching units and of all of them taken together is given in Table I on page 21. It is there shown that the average difficulty index is 43.4. Accordingly, the difficulty of Group III is about 3 points higher than that of Group II. The difficulty of each of the 20 combinations of Group III is given in percentage form in Table II on page 25. Enough has been said as to the teaching of the number facts. For suggested methods of presentation and drill see especially Chapters III and VII.

Verbal Problems

In regard to problems, a slight advance may now be considered. This advance relates to the *subtraction* problems. You will recall that we have hitherto advised a type of problem in which the take-away idea is made evident. All the subtraction problems which we have thus far presented have described the definite withdrawal of a part of a given whole; and the pupil has been clearly asked to find what was left.[1] In other words, the "cues" for subtraction were made very

[1] A few problems to be solved by measurement and comparison (Chapter VIII) are exceptions.

obvious. This cannot be continued indefinitely. Life does not always present the need for subtraction in this clearly defined manner. Of course, the teacher will have to decide in the presence of her own class when subtraction problems of a different character may properly be introduced. We have introduced them in the problems beginning on page 111, but if the children are not ready for this advance, these problems should be deferred until they are ready.

New types of subtraction problems. In the problems on page 115 number 5 under the combination "Two and five" reads: "There were 7 little sisters; 2 wore pink dresses and the rest wore blue dresses. How many sisters wore blue dresses?" This illustrates the related subtraction combination "7 less 2 are 5." But 2 are not taken away or withdrawn from 7, and the requirement is not to find how many were left after 2 were taken away. The teacher may well illustrate this problem on the board with blue and pink chalk. Or it may be illustrated by having seven girls come to the front of the class and by imagining that two of them wear pink dresses and the rest blue dresses. Imagination may be helped by pinning colored papers on each girl. By such means it may be objectively shown that "the rest" with blue dresses are five in number; in other words, that the result is the same as if the two who wore pink dresses were to walk away or be "taken away" from the seven with whom we began.

Several problems of this kind should be given, problems in which a total number is presented as made up of two components, of which one is known and the other required. Number 8 under "Two and five" on page 115 is a variation of this type. Observe that in this problem the word "more" is a cue to *subtraction.* This point should be developed with care. The idea should be made clear by the use of objective material that knowing one part of a number we must find how many more are in the number by subtraction. Thus,

if 7 of Little Boy Blue's cows got into the corn and if he had driven out 5 of them, we must find how many more he is to drive out by finding 7 less 5.

Another kind of subtraction problem occurs when we know the number left and wish to find the number withdrawn. In many cases this is the natural situation. If, for example, a number of pigeons are on the roof of a house and some of them fly away, we are much more likely to know the number left than the number that flew away. A boy selling ginger ale at a wayside stand probably knows how many bottles he had at the beginning of the day. To find out how many he has sold he counts those he has left. Among the subtraction problems following, there are a few of this kind, for instance, number 12 under "One and five" and number 8 under "One and eight."

A still different type of subtraction problem is illustrated by number 6 under "Two and four." The problem is: "Joe has 6 marbles. Tony has 2 marbles. How many more marbles has Joe than Tony?" Again the word "more" is used as a cue for subtraction. This time, however, it is not directly evident that we have a total group of "6" and a known component of it, "2." Indeed, the real point of attack is to *convert* this problem into that type. The teacher may illustrate this by actual marbles or by drawing them on the board. In either event a row of six marbles may illustrate what Joe has. Similarly a row of two may illustrate Tony's marbles. The picture (or better still the actual objects) will present this appearance:

Joe's marbles [••] ••••

Tony's marbles ••

It may now be shown that it takes two of Joe's marbles to equal, or be the same as, the marbles Tony has. In other words, before we can find out how many *more* marbles Joe has than Tony we must "take away" two of Joe's marbles to represent

the two that Tony has. In still other words, Joe's six marbles may be thought of as a whole composed of two marbles such as Tony has and *the rest* of his marbles, or the number he "has left."

This type of problem should be carefully presented with numerous illustrations. Problems 8 and 9 under "Two and four" are also of this kind.

Summary as to teaching problems. In using the problems which follow, the teacher is urged to take account of certain facts already presented. (1) Do not use the problems in the order in which we present them; mix them up. (2) Supplement the problems by others using the same situations. (3) Further supplement them by inventing new situations, especially those which have local meaning. (4) Still further supplement them by having pupils make problems of their own, at first giving them rather full directions. (5) Keep a record of good problems. (6) Give sufficient time and thought during the class period to the problem work; do not expect or desire children to work rapidly. A few problems thoughtfully handled are far better than many problems "fired at" the children. (7) Make the problems real, using such devices as sketches on the board, illustrative objects, and dramatization. (8) In presenting subtraction problems be sure to illustrate this operation not only by the take-away type but also by the type requiring the finding of the other part, or component, of a number when one part is given and the type requiring the finding of how much more one number is than another.

Illustrative Problems

Two and three

1. Baby Jane has 2 upper teeth and 3 lower teeth. How many teeth has Baby Jane?

2. Jack has 2 red pencils and 3 blue ones. How many pencils has Jack?

3. Mrs. Roberts bought 3 pint boxes of berries. When Mr. Roberts came home he brought a quart more. How many pints of berries did Mrs. Roberts then have?

4. Mary had 3 picture books. Her sisters gave her 2 more. How many picture books had Mary all together?

5. John can jump 3 feet. William can jump 2 feet more than John. How many feet can William jump?

6. Henry had 5 marbles and gave away 2. How many marbles had he left?

7. There were 5 little birds sitting in a tree; 2 flew away. How many were left?

8. Mrs. Pearl and Mrs. Robbins bought five pint boxes of berries. Mrs. Robbins said she would take a quart of them. How many pints did that leave for Mrs. Pearl?

9. Five little chickens stood in their feeding pan. "Cluck! Cluck!" called their mother, and 3 jumped out. How many chickens stayed in the pan?

10. There are 5 oranges in a dish; 3 of them are small, the others are large. How many oranges are large?

One and five

1. David spent 1 cent for a balloon and 5 cents for an ice-cream cone. How many cents did he spend?

2. A boy scout rode 1 mile and walked 5 miles. How many miles did he go all together?

3. Joe gave 5 cents at Sunday school and 1 cent to the organ man. How many cents did Joe give away?

4. A boy's shoes cost 5 dollars and his cap 1 dollar. How many dollars did his shoes and cap cost?

5. Edith has to sew 6 buttons on her dress. She has put on 1. How many more must she sew on?

6. There were 6 pencils in a box. The box fell on the floor and 1 pencil rolled out. How many pencils stayed in the box?

7. Father has to drive 6 miles. When he has driven 1 mile, how many more miles will he have to drive?

8. Mrs. Duffy had half a dozen oranges. She gave Henry one orange and used the rest for breakfast. How many did she use for breakfast?

9. Amy has to write 6 words. She has written 5. How many does she still have to write?

10. The children of our class want to learn 6 new songs before Christmas. When they have learned 5, how many more songs will they have to learn?

11. Mother had 6 buns and gave 1 to Fred. How many buns had she left?

12. There are 6 people in one family. There are only 5 at home. How many are away?

13. Betty put 6 beads on a string. She dropped the string and 5 fell off. How many beads were left on the string?

14. Lucy and Sadie were measuring their pencils. Lucy's was 5 inches long and Sadie's was 6 inches long. How many inches longer was Sadie's pencil than Lucy's?

Two and four

1. There were 2 children at the blackboard and 4 more went there. How many children were at the blackboard then?

2. On his way to school David passes 2 brick houses and 4 wooden houses. How many houses does he pass?

3. In the baker's window were 4 long loaves of bread and 2 round loaves. How many loaves of bread were in the window?

4. Some boy scouts went on a trip. They walked 4 miles and rode 2 miles. How many miles did they go?

5. A man has to dig 6 post holes. When he has dug 2, how many more post holes will he have to dig?

6. Joe has 6 marbles. Tony has 2 marbles. How many more marbles has Joe than Tony?

7. Mr. North bought half a dozen gas stoves. He has sold all but 2 of them. How many has he sold?

8. Susie had 6 cents and Anna had 4 cents. How many more cents had Susie than Anna?

9. Dan read 6 pages and Sam read 4 pages. How many more pages did Dan read than Sam?

10. Jean needed half a dozen eggs to put in a cake. She had 4. How many more did she need?

One and eight

1. A boy caught 1 large fish and 8 small ones. How many fish did he catch?

2. Frank rode 1 mile on a bicycle and 8 miles on a bus. How many miles did he ride?

3. There were 8 passenger cars and 1 baggage car in a train. How many cars were there all together?

4. Helen read 8 pages in one book and 1 page in another. How many pages did she read?

5. Rose had a piece of ribbon 9 inches long. She cut off 1 inch and used the rest for a badge. How many inches did she use for a badge?

6. Anna wants 9 children to come to her birthday party. She has invited 1. How many more children does she want to invite?

7. James wants 9 rabbits. He has 1. How many more rabbits must he get?

8. There were 9 apples in the basket. Now there are only 8. How many apples are gone?

9. There were 9 cows in a field in the morning. In the evening there were 8. How many more cows were in the field in the morning than in the evening?

10. Henry jumped 9 feet and Paul jumped 8 feet. How many feet more did Henry jump than Paul? How many inches is that?

Two and five

1. Baby May is learning to walk. She walked 2 steps yesterday and 5 today. How many steps has Baby walked?

2. Jane put 2 quarts of berries into the basket and John put 5 quarts into it. How many quarts did they both put into the basket?

3. A little boy had 5 rabbits and bought 2 more. How many rabbits did he then have?

4. There are 5 people in our family. If we have 2 visitors, how many people are there at our house?

5. There were 7 little sisters; 2 wore pink dresses and the rest wore blue dresses. How many sisters wore blue dresses?

6. Henry spent 7 cents; 2 cents were for marbles and the rest for soap. How many cents did he spend for soap?

7. Suppose you had put 7 pints of sand in a pile and had then filled a quart measure from the pile. How many pints of sand would you have left in the pile?

8. Seven of Little Boy Blue's cows got into the field of corn. He drove 5 of them out. How many more must he drive out of the corn?

9. William needs a box of paints that costs 7 cents. He has 5 cents. How many more cents does he need?

10. The postman left 7 letters at our house; 5 of them were father's and the rest were mother's. How many of the letters were mother's?

11. Helen's doll was 5 inches tall and Betty's was 7 inches tall. How much taller was Betty's doll than Helen's?

The Progress Test

After the combinations of Group III have been taught, practiced, and applied in problems, the success of the learning to this point should be ascertained. The usual progress test should be given. If the pupils have not mastered the work, more time should be spent on it.

The following test is submitted as a sample of what the teacher may use. As far as possible this test should be given in mimeographed form and written answers required. Observe that on each part of the test two verbal problems appear. So strongly do we believe that all practice and progress testing should include verbal problems that we venture to bring them in even as early as this in the teaching of arithmetic. The combinations in and of themselves have little value compared with their value as instruments for the solution of problems.

A class record sheet should be constructed for each part of the test. Use a form similar to the one given in connection with the test on Group I (see page 52). Remedial instruction and drill should be guided by the record as shown on this form.

Test on the Combinations of the First Three Groups

PART I

2	6	1	8	9	5
2	−1	5	−1	−8	−2
1	4	3	1	6	7
3	1	−2	2	−5	−5
1	4	4	4	3	6
6	4	5	2	−1	−4
3	5	4	9	4	9
3	−1	−2	−1	−3	−5
7	9	2 1's	2 3's	2's in 4	4's in 8
−6	−4	$\frac{1}{2}$ of 2		$\frac{1}{2}$ of 6	

1. Jane had 2 cents and Mother gave her 3 more. How many cents did she have then?

2. Dick had 4 marbles and lost 1. How many marbles had he left?

PART II

2	1	3	5	2	2
1	8	2	1	3	−1
1	3	1	5	8	6
7	1	4	−2	−4	−1
1	2	2	5	7	4
1	5	4	2	−1	−1
5	7	8	5	6	8
4	1	1	−3	−3	−7
6	7	2 2's	2 4's	1's in 2	3's in 6
−2	−2	½ of 4		½ of 8	

1. Tom had 8 apples and gave away 4. How many apples had he left?

2. Mary spent 1 cent for candy and 7 cents for soap. How many cents did she spend?

CHAPTER XI

THE ZERO COMBINATIONS

According to the table on page 20, the zero combinations constitute the fourth teaching group. There are 38 of them, 19 in addition and 19 in subtraction, as follows:

ADDITION

Direct	0	0	0	0	0	0	0	0	0	0
	0	1	2	3	4	5	6	7	8	9
Reverse		1	2	3	4	5	6	7	8	9
		0	0	0	0	0	0	0	0	0

SUBTRACTION

Direct	0	1	2	3	4	5	6	7	8	9
	−0	−0	−0	−0	−0	−0	−0	−0	−0	−0
Reverse		1	2	3	4	5	6	7	8	9
		−1	−2	−3	−4	−5	−6	−7	−8	−9

THE IMPORTANCE OF TEACHING THE ZERO COMBINATIONS

Not until lately has it been evident how difficult the zero combinations are. Taking all of them together and including the corresponding subtraction facts with the addition facts, their average difficulty on a percentage basis is represented by 6.06.[1] This means that, on the average, about 6 per cent of the children from the third to the eighth grade made errors on these combinations. It is therefore evident that the

[1] This figure was computed from Clapp's data.

mastery of these combinations cannot be left to incidental learning. They must be taught.

The old faith in the "45 combinations" is not justified. We have already shown this to be true for the reason that when the combinations are regarded as 45 in number, direct and reverse facts, such as $3 + 6$ and $6 + 3$, are erroneously regarded as the same. We now have a further reason for discarding the notion that in either addition or subtraction the basic facts may be limited to 45. This reason is that the zero combinations are not included. Yet every one of these 38 combinations is found to have an appreciable difficulty; and some of them have much greater difficulty than combinations which no one has ever thought of disregarding (see Table II, p. 25). It has been found that hundreds of elementary-school pupils cannot add zero to a number and that still more of them cannot subtract zero from a number.

Separate Presentation of All Zero Combinations not Needed

It is unnecessary for the teacher to present the zero combinations by 38 separate teaching acts. There are four general principles, two in respect to addition and two in respect to subtraction, which should be taught as generalized statements. For this purpose a few of the zero facts should be carefully presented as specific instances. On the other hand, in drill work every one of the 38 zero combinations should be practiced.

Addition combinations. The teacher should therefore present several of the addition facts involving zero, as these facts illustrate two principles. The object is to teach the principles. The first of these is that if any number is added to zero, the answer is that number. Equivalent expressions may be used for this, as "Zero and any number are the number itself."

The second general fact to be taught is the reverse of the first; namely, "If zero is added to any number, the answer is that number" or "Any number and zero are the number."

Subtraction combinations. Likewise in the case of the corresponding subtraction facts one need not teach each separate combination, although drill on every combination is imperative. Again, two facts are essential and the necessary ground may be covered by teaching these two facts rather than the 19 facts represented by the combinations themselves. The first fact is "Any number less zero is itself." The second is "Any number less itself is zero." Equivalent and more colloquial expressions for the same ideas may be used. For example, the first idea may be expressed, "If you take away zero from a number you have the number left." The second may be expressed, "If you have a number and take away the same number you have zero left."

Using Concrete Material

Direct addition. In teaching the zero combinations concrete material may be used as in the case of any other combinations. In teaching a direct addition fact, such as $0 + 4 = 4$, the following procedure will illustrate what we mean. A boy may be brought to the front of the class.[1]

Teacher. Tom, how many marbles have you in your pocket?

Tom. I haven't any.

Teacher. Philip, come stand beside Tom. How many marbles have *you* in your pocket?

Philip. I have four marbles.

Teacher. [*To the class*] How many marbles have Tom and Philip together?

[1] It is assumed, of course, that the teacher has prepared for what follows by seeing that Tom and Philip are properly equipped.

The marbles may be exhibited and counted if necessary. There will be little difficulty in securing the answer "Four." The teacher should then emphasize the idea that 4 marbles added to no marbles give 4 marbles, and the further idea that Tom and Philip together have the same number of marbles that Philip had alone. The combination $\begin{array}{r} 0 \\ +\,4 \\ \hline 4 \end{array}$ should be written and repeated. Again, it should be pointed out that the answer is the same as the number you add to zero. Then, instead of teaching the reverse fact at once, the teacher should present such other combinations as $\begin{array}{r} 0 \\ +\,2 \\ \hline \end{array}$, $\begin{array}{r} 0 \\ +\,7 \\ \hline \end{array}$, and $\begin{array}{r} 0 \\ +\,9 \\ \hline \end{array}$. This procedure will lead to the first general statement already mentioned, namely, that zero and any number are the number itself. All the combinations of this kind should be immediately used by way of applying this principle.

Reverse addition. In teaching the addition combinations in which zero is the *second* number, the handling of the concrete material will be reversed. In other words, if Tom and Philip are going to display their marbles, Philip with his 4 marbles will begin and will be followed by Tom, who has no marbles. Again, several specific facts should be taught leading to the statement that any number and zero are the number. This statement should then be tested with all the combinations to which it applies.

Direct subtraction. Concrete applications of the subtraction of zero from a number will readily suggest themselves. For example, Philip and Tom with their marbles are standing side by side. It is now understood that there are 4 marbles in all. Tom walks away. He has no marbles; he therefore "takes away" zero marbles. How many are left? Philip may show his four and the class will see that there are four left. Likewise, they may easily be brought to see that the number

Philip has left is precisely the number which they both had in the beginning; in other words, that if you take nothing from a number you have the number left. Additional combinations will be used to permit this generalization and to apply it.

Reverse subtraction. Finally, the teacher may illustrate the combinations in which a number is taken from itself. If it is understood that Tom and Philip standing together have 4 marbles in all and if Philip with his 4 marbles walks away, it will be made evident that there are none left. Or, to vary the type of material, suppose that 6 girls are brought before the class and stand inside a circle drawn on the floor, where they may be counted by the class.

TEACHER. How many girls are there in the circle?

PUPIL. Six.

The teacher may now state that six girls are to be taken away from the circle. A pupil may count them as the teacher takes them from the circle.

TEACHER. How many girls were taken away from the circle?

PUPIL. Six.

TEACHER. How many are now left in the circle?

The pupil's answer will undoubtedly be equivalent to "zero." At the same time the idea may be developed that, at least in this case, a number taken from itself gives zero. Through the presentation of similar facts, such as $\underline{-\,4}$, $\underline{-\,7}$, and $\underline{-\,9}$, this idea may be made general.

DRILL ACCORDING TO DIFFICULTY

Try to distribute your drill among the 38 zero combinations so that none is neglected. Remember, too, that in general the combinations in which large numbers occur are harder than those which are confined to small numbers. Give more attention to the "larger" combinations. Another point to bear in

mind is the difference in difficulty between the different kinds of zero combinations. The two kinds of addition facts are easier than the two kinds of subtraction facts. As between the two kinds of addition facts there is very little difference. Adding zero to a number, for instance $\begin{array}{r}3\\ \underline{0}\end{array}$, is apparently just a little easier than adding a number to zero, as $\begin{array}{r}0\\ \underline{3}\end{array}$. The two kinds of subtraction facts, however, not only are harder than the addition facts but differ widely from each other. Taking a number from itself, for example $\begin{array}{r}3\\ \underline{-3}\end{array}$, while generally harder than the addition facts involving zero, is not nearly so hard as taking zero from a number. The outstanding fact regarding the difficulty of the zero combinations is the high difficulty of *all* facts like $\begin{array}{r}3\\ \underline{-0}\end{array}$. Leaving $\begin{array}{r}0\\ \underline{-0}\end{array}$ out of account, there are nine of these (see page 118), four of which are *harder than any combination thus far taught.* The remaining five are nearly as hard. In fact, they are surpassed in difficulty by only one of the number facts hitherto presented. Accordingly, special attention and frequent drill must be given to those zero combinations in which zero is taken from a significant number.

The Use of Problems

Do not fail to apply the zero combinations in concrete problems. Such problems present some difficulties of construction. The main trouble is that we do not use the word "zero" in ordinary language. We cannot, therefore, except in an artificial way, use the same terminology in verbal problems that we do in combinations. This fact, however, has its compensations. It permits a fuller and better teaching of the idea of zero. You will miss a real opportunity if you do not in your

handling of problems involving zero do at least two things, namely, drill on specific combinations and teach the important and ill-understood idea of zero. Consider the following problem illustrating the combination $2 + 0$. "Grandpa's hat lay on the floor. Two little kittens came along and looked into it. There was nothing in it [zero kittens in it]. So they jumped in and went to sleep. How many kittens did Grandpa find in his hat?" Ask how many kittens were in Grandpa's hat at first and direct attention to the words which tell this fact. Get the answer in the arithmetical form. "Zero kittens" as a verbal expression is of course rather clumsy; but in sober mathematical truth, the number of kittens in Grandpa's hat before these two interesting wanderers arrived *was* zero. The pupils should know this and use the proper term. The condition is one which permits the pupils to obtain a notion of the meaning of the idea "just not anything," the idea we name zero.

Summary

The points we have made concerning the teaching of the zero combinations may be restated as follows: (1) Do not leave these combinations out. They must be taught. Though they are inherently easy when understood, they are often missed. Insufficient teaching and drill are the probable causes. (2) It is not necessary to teach each one separately. These are the only combinations which may be easily covered by a few general facts. Teach these general facts, using some of the combinations as teaching data and others as illustrations and applications. (3) Distribute your drill. Do not neglect any of the zero combinations. (4) Drill most on those involving the largest numbers and on the type "a number less zero," as $3 - 0$, and $8 - 0$. (5) Use many concrete problems not only in order to apply the zero combinations but also in order to make real the meaning of zero.

Illustrative Problems

The problems below are offered to illustrate the application of the zero combinations. Only a part of the combinations are used. As is often the case with verbal problems, some of these are capable of being dramatized and thus rendered more vivid. Such, for example, are problems 4, 6, 8, 13, 16, 21, and 24.

1. (0 + 0) Brother Fox went fishing. First he sat on a rock and fished a long time, but he caught no fish. Then he sat on a log and fished a long time, but again he caught no fish. How many fish did Brother Fox catch?

Note

Teacher. How many fish did Brother Fox catch while he was sitting on the rock?

Pupil. He didn't catch any (or words to that effect).

Teacher insists on the *number* of fish and obtains or tells, if necessary, the fact that there were zero fish.

Teacher. How many fish did Brother Fox catch while he was sitting on the log?

Pupil. [*Perhaps after an interval*] Zero fish.

Teacher. And zero fish and zero fish are how many fish?

Pupil probably gives answers equivalent to "no fish," which may be revised, if it is thought desirable, to "zero fish." Before the problem is abandoned, its connection with the combination $\underline{0}^{0}$ should be established.

2. (0 + 0) Mother sent Dan and Donald with a basket to pick up apples in the orchard. Dan said, "These apples are no good." So he put no apples into the basket. Donald said, "I can't find any apples." So *he* put no apples into the basket. Then they took the basket back to mother. How many apples were in it?

Note. This problem may easily be dramatized. A series of questions may also be asked similar to those given above under problem 1.

3. (0 + 3) There were no ducks in the duck pond. Then 3 ran in. How many ducks were then in the pond?

4. (0 + 5) An automobile stood by the roadside. There was no one in it. Then 5 people got in and went away. How many people went away in the automobile?

5. (0 + 7) We have no goldfish in our globe. Tomorrow we shall put in 7. How many goldfish will there be in our globe then?

6. (0 + 8) The hand-organ man and his monkey were near the school. The monkey held out his cap to the children. There was not a cent in it. But the children put in 8 cents. How many cents were there then in the monkey's cap?

7. (0 + 9) Last evening there were no pansies in my garden. In the night 9 pansies came out. How many were there this morning?

8. (2 + 0) Two sparrows were in the bird house. They would not let any of the sparrows who were outside come in. How many sparrows were there in the bird house?

9. (4 + 0) Ted found 4 eggs in one nest and none in the others. How many eggs did Ted find all together?

10. (6 + 0) Arthur read 6 pages of his storybook yesterday, but he has read none today. How many pages has he read all together?

11. (7 + 0) Elsie missed 7 days of school one month, and no days the next month. How many days did she miss in both months?

12. (8 + 0) Walter picked 8 quarts of cherries in the morning, and none in the afternoon. How many quarts did he pick that day?

13. (0 − 0) It was April Fool's Day. Sadie made a grab bag, but put no prizes in it. Harold grabbed but took out no prize. How many prizes were left in the bag?

14. (3 − 0) There are 3 cars fastened to Billy's engine and no cars fastened to Harry's. How many more cars are fastened to Billy's engine than to Harry's?

15. (4 − 0) Richard has put 4 dollars in his bank and has taken nothing out. How many dollars has he in his bank?

16. (6 − 0) One day 6 chickens got into Walter's garden. Walter ran out and cried "Shoo!" but none of the chickens went away. How many chickens stayed in the garden?

17. (7 − 0) There were 7 bunches of grapes hanging high above the ground. A fox came along and tried to get them. He jumped and jumped but got none. How many bunches of grapes were left hanging above the ground?

18. (8 — 0) Nora put 8 muffins on the breakfast table. No one took any of them. How many muffins were left on the table?

19. (9 — 0) Eva had 9 candies and Robert had none. How many more candies had Eva than Robert?

20. (1 — 1) A little boy had 1 cent. He bought a 1-cent balloon. How many cents had he left?

21. (2 — 2) Lucy and Ella were sent to bring 2 cabbages from the garden. Ella brought 2. How many did Lucy bring?

22. (4 — 4) Robert peeped into a nest and saw 4 little birds. The next day he peeped in again and saw 4 little birds. How many little birds had gone away?

23. (5 — 5) There are 5 school days in a week. Florence went to school 5 days last week. How many days was she absent?

24. (6 — 6) Mother told Nellie and Maud to dust the 6 dining-room chairs. Nellie dusted 6 of them. How many did Maud have to dust?

25. (7 — 7) There were 7 children playing on the beach in the morning, and 7 in the afternoon. How many more children played on the beach in the morning than in the afternoon?

26. (8 — 8) Mabel wants to buy an 8-cent rubber ball. She has 8 cents in her pocket. How many more cents does she need?

27. (9 — 9) The organ man's monkey got 9 pennies in his cap. The organ man took out 9 pennies. How many pennies were left in the cap?

The Progress Test

Up to this point a total of 102 number facts have been treated. Among these are 4 multiplication facts, 4 division facts, 4 partition facts, and 38 addition and subtraction facts involving zero. The 102 facts are presented below for check purposes in three parts. The arrangement of the material is such that Part I contains the easiest combinations; Part II, those of intermediate difficulty; and Part III, those of greatest difficulty. While these tests are being given, the combinations

which have been permanently posted for reference should be concealed. As has been suggested in connection with previous "progress tests," children should be examined if possible by presenting the material in mimeographed form and having them write their answers on the mimeographed sheets. If this is not practicable, the teacher may provide herself with mimeographed sheets, head each one with the name of the pupil she is about to examine, and enter upon it his responses to her oral questions. Or she may write the test on the board or dictate it to the class. What is required is a record from each child on every combination. A class record sheet similar to the one already recommended (see page 52) should be used for each of these three tests. For example, the column heads for the first test (easy combinations) will be $\underline{0}$, $\underline{3}$, $\underline{8}$, etc. Thus there will be 34 columns with such heads, plus two more for the verbal problems and two wider columns, one at the left for the pupils' initials and the other at the right for totals. The whole form can be put on ordinary letter-size paper ($8\frac{1}{2}$ by 11 inches) by making the narrow columns five to the inch. Square-ruled paper of this size with lines one fifth of an inch apart may be purchased at any stationery store, and the labor of ruling be saved. The line totals of the record sheet will show the pupils who need attention while the column totals will show the facts on which pupils are failing.

$$\begin{matrix} 0 & 1 & 1 \end{matrix}$$

The purpose of these progress tests is to inventory the abilities of the children to date. They permit the teacher to find out in what way each pupil needs additional attention. This additional attention should be given before any further work is attempted. Only in case a pupil is clearly too backward to justify the expenditure of time necessary to bring him to the desired point of proficiency should he be allowed to fall short of virtually 100-per-cent accuracy on these combinations.

Test on the Combinations of Groups I to IV

PART I. EASY COMBINATIONS

0	1	1	7	1
0	3	8	1	4
0	5	4	5	3
−0	4	0	−1	−2
2	1	3	1	6
2	1	1	5	0
1	3	4	3	0
7	3	1	2	8
2	6	4	5	4
1	−1	4	0	5
1	2	2	7	3
6	0	5	−7	0

2 3's 2's in 4 1's in 2 $\frac{1}{2}$ of 4

1. A little boy spent 2 cents for a pencil and 2 cents for candy. How many cents did he spend?

2. Jane had 3 apples. She gave away 2. How many apples had she left?

PART II. MEDIUM COMBINATIONS

3	6	5	4	9
−3	−6	1	2	−9
2	0	0	1	4
4	1	9	0	−4

PART II. MEDIUM COMBINATIONS (CONTINUED)

0	0	0	5	9
2	7	3	2	−8
0	4	1	8	2
4	−2	−1	0	3
8	8	5	5	0
1	−1	−2	−3	6
0	7	1	9	8
5	0	2	−1	−4

2 1's 2 4's $\frac{1}{2}$ of 2 $\frac{1}{2}$ of 6

1. John had just 1 rabbit. He sold 1. How many rabbits had he left?

2. May had 5 cents and earned 2 cents more. How many cents did she have then?

PART III. HARD COMBINATIONS

6	5	4	7	4
−5	−2	−1	−2	−0
5	2	8	9	9
−5	−1	−7	−0	−4
3	7	9	6	2
−1	−5	0	−0	−0
7	6	9	8	8
−1	−4	−5	−8	−0

PART III. HARD COMBINATIONS (CONTINUED)

6	6	7	5	7
-3	-1	-6	-0	-0
4	2	6	3	1
-3	-2	-2	-0	-0

4's in 8 2 2's 3's in 6 $\frac{1}{2}$ of 8

1. Tom wants to buy a pencil box that will cost 7 cents. He has 6 cents. How many more cents does he need?

2. There were 9 boys in the room; 5 went out. How many boys were left in the room?

CHAPTER XII

THE FIFTH GROUP OF COMBINATIONS

This group of five teaching units concludes the work with the easy combinations, the 55 in addition whose sums are less than 10 and the 55 in subtraction whose minuends are less than 10. The five key combinations of Group V are:

3	2	3	2	3
4	6	5	7	6

Since each of the five corresponding teaching units consists of four number facts, there are twenty in all. The difficulty of these combinations, though less than that of most of the combinations whose sums or minuends are greater than 10, is nevertheless appreciably more than that of the combinations of Groups I to IV. The evidence is that, on the average, each of these twenty number facts is missed by 8 or 9 per cent of the pupils in Grades III to VIII. This surprising amount of error means to the teacher that these combinations must be especially well taught and continuously practiced.

Summary as to Teaching

We have begun with the easier combinations of this group. The idea of arrangement according to difficulty, as you have seen, also applies to the entire range of combinations from those of Group I to those we are now considering. This means, in general, that pupils reach the hard combinations when, to some degree, growth and habituation permit them to do the work with the greatest likelihood of success.

Again, we have kept apart the combinations having common elements, thus seeking to avoid interferences in learning. On the other hand, in order to avail ourselves of the mutual support of certain related combinations, we have provided for teaching the subtraction facts in connection with the corresponding addition facts. Moreover, we have called for the introduction of the reverse facts immediately after the formal presentation of the direct facts. In other words we have kept apart the things which, if brought together, would create confusion and delay; and we have put together the things which belong together and may be economically learned in connection with one another.

We have also adopted certain devices for reducing the likelihood of mistakes. This is in recognition of the undoubted seriousness of error. The pedagogy of an earlier day knew this in a vague way, but the evidence arising from more recent investigation has made the earlier notions more definite. An error once made is likely to be made again. It will die, or almost die, with long enough disuse, but by far the most economical way is never to let it occur.

Finally, we have adopted means, both in the teaching and in the arrangement of combinations, to discourage counting. In our proposed teaching method we have suggested the use of counting for its legitimate purpose, namely, for finding out how many there are in a group thought of as a whole. We have counted totals and separate parts of totals, but never totals regarded as made up of parts or parts regarded as making a total. If we are dealing with $3 + 6 = 9$, we may present 9 first and count it without having any notion of its parts. Or we may first present 3 and then, after withdrawing it from view, we may present 6. In this case the 3 and the 6 are merely thought of as such, and not as parts of a larger whole. We have likewise sought to discourage counting by the arrangement of the combinations. In the first place, we do not

begin with an orderly arrangement of the combinations involving 1, followed by those involving 2, 3, etc. In the second place, we have adopted as the addition combinations to be objectively presented those in which a larger number is added to a smaller one except in doubles like $2 + 2$ or $3 + 3$.

Summary as to Drill

We have said above that the difficulty of the number facts of Group V means that they must be well taught and continuously practiced. We have just indicated some of the points which in our judgment should be kept in mind to the end that these facts may be "well taught." It is not so easy to provide that they be "continuously practiced." At least such provision in its finer details is not easy. We have, however, sought to make this continuous practice easy by introducing (1) certain plays and games, (2) the number chart, (3) drill cards, and (4) numerous concrete problems.

The more specific question as to how many times each combination should be practiced under given conditions of pupil ability, attention, interest, etc., is one which is particularly difficult, probably too difficult and complex ever to be answered in simple terms. Some of the grosser shortcomings with reference to the distribution of practice among combinations may, however, be avoided by recognizing certain principles.

1. No combination is so easy that it requires no practice.

2. The more difficult a combination is the more it requires practice.

3. Practice on a given combination must begin at the moment it is first taught; it must be repeated after a *short* interval, and then repeated after successive intervals which may be made longer and longer as time passes. The intervals should be as long as they can be without allowing the forgetting to become serious. Ultimately they should become

so long that the chance occurrences of life may be relied upon to provide sufficient practice.

4. At each recurrence of practice the object should be to bring the correct response to at least the same readiness and certitude as existed at the time of the original teaching with its immediate repetition, a time when the fact in question was thrown into high relief.

5. "Little and often" is the rule in drill. If you have ten minutes a day to devote to drill you will get better results by dividing it into two five-minute periods than you will by using the ten minutes all at one time. Periods of two or three minutes "chinked in" at different times in the day will be found advantageous.

6. Drill must include verbal problems, for this is the way life presents number relations. One does not add 3 and 5 merely as an exercise in addition. One has a problem to solve. While the abstract combinations will often be used for drill purposes, their meaning and application will be better apprehended if they are exemplified in verbal problems with considerably greater frequency than is usually the case in primary teaching.

7. Keep out of a rut. Do not drill on addition at one period and subtraction at another. Mix the addition and subtraction facts. Do not neglect the eight multiplication and division facts and the four partition facts ($\frac{1}{2}$ of 2, $\frac{1}{2}$ of 4, etc.). Do not have a bias toward certain combinations. Watch yourself in this respect. The only bias you should have is in the direction of drilling most on the hard combinations.

Illustrative Problems

In respect to verbal problems the same teaching devices, cautions, and suggestions which have been previously offered may be consulted again (see pages 34, 80, and 108). We append hereto another list of problems. While these problems

directly exemplify the particular combinations of Group V, the situations with which they are concerned are in many cases equally applicable to any of the combinations. Accordingly the problems are offered here not merely nor chiefly for their own value. They are offered largely for their suggestive value in the formation of other problems.

Three and four

1. Horse Dobbin worked 3 hours in the morning and 4 hours in the afternoon. How many hours did Dobbin work that day?

2. John skated 3 miles Monday and 4 miles Tuesday. How many miles did John skate in all?

3. In a house there are 4 rooms downstairs and 3 rooms upstairs. How many rooms are there in the house?

4. A man went to market and bought 4 pounds of beef and 3 pounds of pork. How many pounds of meat did he buy?

5. Jean had 7 beads in her hand and dropped 3 of them. How many beads were left in her hand?

6. Dick had 7 rabbits and sold 3. How many rabbits had he left?

7. Lillian is 7 years old. Elizabeth is 4. How many years older is Lillian than Elizabeth?

8. The dressmaker had to make 7 dresses. She finished 4. How many more dresses did she have to make?

Two and six

1. At a boys' camp there were 2 bathing suits hanging on a clothesline. The boys hung up 6 more. How many bathing suits were then on the line?

2. Allen raised 2 watermelons and 6 muskmelons in his garden. How many melons did he raise all together?

3. Mr. Smith has 6 black horses and 2 white ones. How many horses has Mr. Smith in all?

4. At the circus Tom saw 6 small monkeys and 2 large ones. How many monkeys did he see?

5. There were 8 boys in a boat; 2 jumped out. How many boys were still in the boat?

6. Some boys caught 8 fish, but let 2 of them go. How many fish did they keep?

7. There were 8 children at Mary's house; 6 of them were visitors, and the others lived there. How many children lived at Mary's house?

8. Richard's hens laid 8 eggs. Richard sold 6 of the eggs. How many did he keep?

Three and five

1. Joseph spent 3 cents for an orange and 5 cents for a cake of chocolate. How many cents did he spend?

2. Henry's hat cost 3 dollars. His shoes cost 5 dollars more than his hat. How many dollars did his shoes cost?

3. There were 5 small boys on one side and 3 large boys on the other, having a snowball battle. How many boys were in the snowball battle?

4. There were 5 children skating on the pond, and 3 more children came. How many children were then skating on the pond?

5. There were 8 black crows sitting in a tree, and 3 flew away. How many were left?

6. Mr. Cooper earns 8 dollars a day. Mr. Osburn earns 3 dollars less than Mr. Cooper. How many dollars does Mr. Osburn earn?

7. Frank can jump 8 feet. His little brother can jump 5 feet. How much farther can Frank jump than his brother?

8. Ella wrote 8 words on the blackboard; 5 were correct. How many words were wrong?

Two and seven

1. Alice planted 2 rows of pansies. Her mother planted 7 rows. How many rows of pansies did they both plant?

2. Robert paid 2 cents for a fishhook and 7 cents for a line. How many cents did he pay for the hook and line?

3. There were 7 automobiles parked in front of the church; later 2 more parked there. How many automobiles were then parked in front of the church?

4. There were 7 children on a street car, and 2 more got on. How many children were then on the car?

5. There were 9 cows in a pasture; 2 were called out. How many cows remained in the pasture?

6. Mr. Simpson, the milkman, had 9 pint bottles of cream in his wagon. Mrs. Hudson said she wanted a quart. How many pint bottles of cream did Mr. Simpson have left?

7. Father started to market at 7 o'clock in the evening and got home at 9 o'clock. How many hours was he gone?

8. Clara had to write 9 words. She has written 7. How many more words has she to write?

Three and six

1. James picked 3 quarts of cherries in the morning and 6 quarts in the afternoon. How many quarts did he pick that day?

2. Three boys and 6 girls came to school in a bus. How many school children were in the bus?

3. Harry spent 6 cents for cards and 3 cents for stamps. How many cents did he spend for both?

4. John earned 6 dollars last summer. Henry earned 3 dollars more than John. How many dollars did Henry earn?

5. Puppy had a rope that was 9 feet long. He chewed off a piece that was 3 feet long. How many feet long was the piece that was left?

6. Yesterday there were 9 words on the blackboard. Today there are only 3 of them. How many have been rubbed out?

7. There were 9 children at the picnic; 6 of them were girls. How many were boys?

8. A man had 9 bushels of apples and sold 6 bushels. How many bushels had he left?

The Progress Test

The addition combinations whose sums do not exceed 9, together with the corresponding subtraction combinations, constitute one large unit of the pre-third-grade work. There

are, in all, 110 of these combinations; and in the foregoing pages they have been treated in five groups. We have also introduced 12 multiplication, division, and partition facts involving 2 as a multiplier or quotient or as the denominator of the fraction $\frac{1}{2}$. In order that the teacher may know how well these combinations have been learned, we introduce here the usual progress test. It is divided into three parts, each consisting of 40 combinations. The combinations $0 + 0$ and $0 - 0$ are omitted. A score sheet for each of these tests should be constructed, similar to the one given on page 52. It will be convenient to use square-ruled paper with lines one fifth of an inch apart. The paper should be turned so that the column headings will run the length of the page.

Test on the Combinations of Groups I to V

PART I. EASY COMBINATIONS

2	1	2	1	1	5
2	7	1	6	3	4
1	3	6	2	1	4
1	3	− 1	0	8	0
3	4	4	2	7	5
1	1	4	5	1	− 1
4	1	3	5	7	1
3	5	2	0	− 7	4
3	6	0	4	3	3
− 2	0	8	5	0	− 3
3	2	6	0	0	8
6	4	2	2	4	1
2 2's		2 1's		2 4's	2 3's

1. Sadie had 3 cents and earned 2 cents more. How many cents had Sadie then?

2. There were 7 apples in a dish; I took out 7. How many apples were left in the dish?

3. Billy had 6 neckties and got 2 more as Christmas presents. How many neckties did Billy have then?

PART II. MEDIUM COMBINATIONS

7	0	6	0	0	4
2	5	−6	1	7	−2
8	7	5	0	0	1
−1	0	1	9	3	−1
5	5	1	4	1	5
3	−2	2	2	0	2
8	5	9	9	2	4
0	−3	−1	−9	7	−4
9	6	3	2	3	0
−8	3	5	3	4	6
8	6	5	3	7	6
−4	−5	−5	−1	−1	−3
3's in 6		1's in 2		2's in 4	4's in 8

1. Mary went to look for violets. The first day she found none; the next day she found 5. How many violets did she find on both days?

2. Jack had 4 rabbits. His father gave him 2 more. How many rabbits did Jack have all together?

3. On Margaret's birthday her mother baked 8 little cakes for her. She gave 4 of them to her friends. How many cakes did Margaret have left for herself?

PART III. HARD COMBINATIONS

4	5	2	7	6	6
−3	−2	−1	−5	−4	−1
2	4	8	2	9	9
−2	−1	−7	6	0	−5
7	6	8	7	9	6
−6	−2	−6	−2	−0	−0
9	7	8	9	8	5
−3	−3	−8	−6	−2	−0
3	4	9	9	2	8
−0	−0	−4	−7	−0	−0
7	8	1	7	9	8
−0	−3	−0	−4	−2	−5

$\frac{1}{2}$ of 4 $\frac{1}{2}$ of 6 $\frac{1}{2}$ of 8 $\frac{1}{2}$ of 2

1. Tom had 9 cents and spent 6 cents for car fare. How many cents had he left?

2. Jane had 6 Easter eggs; 2 were red, and the rest were green. How many were green?

3. There were 7 boys in the schoolroom. The teacher sent 4 of them to get some books. How many boys were in the room then?

CHAPTER XIII

MEASURING, ORDINALS, ROMAN NUMERALS, SIGNS, VOCABULARY

Review measuring and estimating, using as units *inch, foot, pint,* and *quart.* Also review *nickel, dime, quarter, half dollar,* and *dollar.* Apply these units in problems.

Teaching the Unit Yard

The *yard* is sufficiently common to justify teaching it in the pre-third-grade work. Do this with an actual yardstick. Show that the foot rule may be laid upon the yardstick three times, and that therefore three feet are the same as a yard. Show, too, by the inch markings on the yardstick, that there are 36 inches in a yard. Show that 18 inches are half a yard, and give practice in estimating half a yard.

Give exercises in estimating distances with the yard as the unit. Have these estimates tested by measuring. Measure with the yardstick distances such as the length and width of the schoolroom, the teacher's desk, the length of the blackboard, and the height of the door. It will be at once evident that things are not just one yard long, or two or three yards long. For example, the space between the door and the window is *this much* more than two yards. Then measure this remaining distance. They may find it amounts to 15 inches and they may then realize that between the door and the window the distance is really 2 yards and 15 inches.[1]

[1] Whether or not the foot as a unit will be used in connection with this work depends upon the ability and maturity of the pupils. In the above situation, the distance may be recorded as 2 yards, 1 foot, 3 inches.

Verbal problems may be made involving a limited knowledge of foot and yard, although far more problems will become available when higher-decade addition and subtraction have been undertaken and the basic facts of multiplication and division have been taught. Even now, however, a few problems may be introduced. A situation in which one yard is to be added to one foot and the answer is required in feet is appropriate. Similarly 2 feet, 3 feet, 4 feet, 5 feet, or 6 feet may be added to 1 yard. Again, 1 yard may be subtracted from 4, 5, 6, 7, 8, or 9 feet. Since the pupils know the value of two 3's they may tell how many feet there are in two yards, also, from the corresponding division fact, how many yards there are in six feet.

Ordinals

By this time it is likely that the pupils will have use for the ordinal numbers. In any event it will not be difficult to create a situation in which this need will be felt. It is recommended, then, that the ordinal numbers — first, second, third, and so forth — be introduced. Have the children arrange materials and count them by ordinals. Have them find out which item of a series — which boy, girl, box, or book — is the sixth, tenth, fifteenth, and so on.

Two things must be provided for: (1) The objects must be arranged *in order*, because an ordinal number, as its name suggests, indicates the order or position of a particular thing in a definite arrangement. One cannot apply ordinal numbers to objects unless the objects are *arranged*. (2) The pupils must understand that an ordinal number refers to a single thing and not to a group of things. The fifth book from the right in the row on the teacher's desk is a particular book. It does not tell how many there are in a group of books. The third child in the row is an individual named Susan, not a group of three children.

Roman Numerals

Practice in teaching Roman notation varies greatly, but all are agreed that pupils should know the first twelve numerals. The great majority of published courses of study indicate that this teaching is to take place during the pre-third-grade period. An interesting discussion may be held concerning the early Romans and their way of writing numbers. The teacher may show that we still use the Roman way in numbering chapters of books, in numbering volumes in sets of books, and in showing the hours on the clockface. Thus the children may be made to see that they will need to know such of these numbers as are in commonest use. Later it will probably be advisable to go beyond twelve in presenting the Roman numerals. At present, however, this seems to be unnecessary.

Show the following series and, if practicable, leave it permanently displayed for reference and for use in drill:

1	I	one	5	V	five	9	IX	nine
2	II	two	6	VI	six	10	X	ten
3	III	three	7	VII	seven	11	XI	eleven
4	IV	(or IIII) four	8	VIII	eight	12	XII	twelve

New Forms of Statement

Hitherto we have expressed addition and subtraction facts in vertical form. Since, however, horizontal forms are often used, children should be instructed in their meaning. This involves the use of the plus and minus signs and the sign of equality. The minus sign has already been introduced, although its use has been limited to the vertical statement.

Addition. Present a number fact in vertical form and have it read downward in the usual way; that is, $\begin{array}{r} 3 \\ 4 \\ \hline 7 \end{array}$ is read "three and four are seven." Then show that the same fact may be written

$3 + 4$ are 7 where the sign + means "and." Give it its name "plus." Then show that we have another mark or sign to tell us that one thing is of the same amount as or equal to another. This sign is =. It is read "equal" or "equals." We know that $3 + 4$ taken together is the same as 7, or equals 7. We may therefore write $3 + 4 = 7$. This means the same as $\begin{array}{r} 3 \\ 4. \\ \hline 7 \end{array}$ It is read "three plus four equals seven." Have the pupils practice reading and writing the horizontal form.

Subtraction. Similarly, present a subtraction fact in vertical form, as $\begin{array}{r} 5 \\ -\,3. \\ \hline 2 \end{array}$ If this is still being read "five take away three are two," the teacher may now show that the sign − is also read "less" or "minus." The same fact may be presented in horizontal form, $5 - 3 = 2$, and read "five less [or minus] three equals two." Let the pupils practice the reading and writing of such forms.

Vocabulary

Technical words. Each school subject has its own peculiar vocabulary. It not only uses special word forms, but it also uses common forms with special meanings. For example, outside of arithmetic the child seldom meets such words as *add*, *cost*, *half*, *inch*, and the like. Moreover, he encounters old words in new senses. His previous knowledge of *make* helps but little in understanding "two pints make a quart." The home use of the word *foot* gives no hint of *foot* as a unit of measure. A boy's experience, both auditory and tactile, of *pound* affords no suggestion of *pound* as a unit of weight. These are not so much old words with new meanings as they are *new words*.

A list of technical and partly technical words such as children will have use for in pre-third-grade work is given next.

Technical and Partly Technical Words

above
across
add
after
afternoon
again
all
all together
almost
already
also
another
answer
at (price)
away
bank
before
behind
besides
big
both
bought
bushel
buy
cent
change
cost (*noun*)
cost (*verb*)
count (*verb*)
day
dime
dollar
down
dozen
each
enough
evening
every
example
far
feet
few
fifth
first
foot
fourth
gallon
half
high, higher
hold
hour
in all
inch
in front of
large
last
left
length
less
lines
little
long, longer
low, lower
many
market
measure
middle
mile
minute
money
more
morning
mouth
much
narrow
near
nearly
nickel
none
noon
nothing
now
number
once
only
order (*verb*)
paid
pair
pay
penny
piece
pint
postage
pound
price
quart
rest (remainder)
row
ruler
second (*ordinal*)
sell
short, shorter
side
small
sold
some
soon
stamp
strip
take away
tall
then
third (*ordinal*)
time
toward
under
up
week
weight
whole
wide
yard
year
yesterday

Some of the units named in the foregoing list, such as *hour*, *day*, and *year*, have not yet been formally taught. Nevertheless, these words are among the commonest in the language and such use as is made of them in problems appropriate to

this stage of the pupil's development will present no real difficulty. Most of the children already use *hour*, *day*, and *year*. Some certainly use *bushel*, *mile*, and *pound*. They do not know, nor is this the time to teach them, that the hour has 60 minutes, the day 24 hours, and the year 12 months. The important thing at this point is to be sure that the children have a sufficient idea of the meaning of these and similar words to enable them to understand the problems in which they occur. It is assumed that the problems are presented orally.

These "technical and partly technical" terms are by no means all the words which children must know in their first-grade and second-grade number work. While the problems in this book are intended to be presented orally, and while therefore the idea of a reading vocabulary does not apply, nevertheless, the children must know the meaning of the words we use in our directions, explanations, and problems. When, a little later, these children are called upon to read their textbooks, they will do so much more easily if they have associated the right meanings with the *sound* of the words they are then called upon to read. Moreover, it is not too much to expect that in the second grade some of the problems may be read from the blackboard or from mimeographed sheets.

Inevitably any large number of problems, such as those presented in this book (there are more than eleven hundred of them), must draw on a rather wide vocabulary, if they would escape the dullness of too frequent repetition. Many different situations must be described and appropriate words must be used. In these situations persons buy, sell, earn, save, spend, go walking and riding, cook and sew, work and play. Certain action words must therefore be used. A still greater number of words are required to name the persons who buy and sell, measure and count, and do all the different things which we describe. There must likewise be names for

the things which are bought and sold, lost and found, had "in all," and had "left."

Clearly, therefore, the vocabulary even of pre-third-grade arithmetic may be rather extensive. The problem material in this book utilizes about twelve hundred different words. About 170 of them, including a few phrases like *how many* and *in front of*, are classed as technical or partly technical words. A selection of these is given on page 146. The remainder are words needed in the writing of the problems. A selection from this vocabulary is given below. It is to be understood that some words used in our problems, either in previous pages or in pages yet to come, are used so infrequently that the teacher may merely tell the children what they mean, if they do not know. Generally, however, these words will be well known to the pupils and will require no treatment at all. For example, *armchair*, *baggage*, *baseball*, and *bookcase* occur but once in our problems. They are likely to be known to most of the children. We do not include them in the subjoined list because the teacher does not need, at least so far as using this book is concerned, to be certain that every child is formally introduced to them.

As to the following list, however, we advise that when these words occur a few moments be spent in making certain that all the children know their meaning. Their frequency of use makes this desirable. At the same time, it is probable that most of the terms will be well known.

Nontechnical Words

absent	bag	bee
animal	baker	beet
any	barnyard	berries
aunt	basket	bicycle
automobile	beans	birthday
autumn	bear (*noun*)	black (color)

block (play)
block (street)
blouse
blue
boarder
boat
book
bottle
bowl
box
branch
breakfast
bring
broom
brother
brought
brown (color)
bun
bunch
bus
button (*noun*)
cabbage
cake
can (*noun*)
candle
car
card
chair
checkers
cherries
chick
chicken
children
chocolate
circus
city
class (*noun*)
clean (*verb*)
closet (book closet)
cloth
coffee
concert (*noun*)
cookies
corn
cotton
cousin
cow
crate (*noun*)
cricket
crow
curtain (*noun*)
dandelion
dinner
domino, dominoes
dressmaker
drive (*verb*)
drove (*verb*)
duck (*noun*)
ear (of corn)
earn, earned
egg
evening
fair
family
farmer
fence
field
finish (*verb*)
fish
flag
flew
flock (*noun*)
flour
flower
folks
fox
friend
frog
fruit
garden
gasoline
gather
geese
gift
glass
globe
goldfish
grade (*noun*)
grape
grasshopper
gray
green
grocer
groceries
handkerchief
hat
help (*verb*)
hen
hill
hole
ice
invite
jack-o'-lantern
jacks (play)
jar (*noun*)
jelly
just
kite
kitten
knife
lace
lake
lamb
laundry

leave (*verb*)
leaves (*noun*)
lemon
lemonade
lesson
letter
lettuce
load
loaf
lost
lunch
marble
march (*verb*)
meat
meeting (*noun*)
melon
milk
miss (*verb*)
neighbor
nest
night
nut
o'clock
office
old, older
orange (color)
orange (fruit)
orchard
organ (music)
package
page
pansy, pansies
parade
park (*verb, noun*)
party
pass (*verb*)
pasture
peach
peanut
pear
pen
pencil
people
pet
picnic
picture
pie
pink (color)
plan (*verb*)
plant (*noun*)
plant (*verb*)
platform
plum
pocket
pond
pop corn
porch
post card
postman
potato
present (here)
present (*noun*)
prize
problem
pumpkin
pupil
rabbit
radish
raisin
raspberries
receive
red
remain
ribbon
ride (*noun*)
road
roadside
robin
room
rose
sack
sandwich
save
school
score
seashore
sew
sheep
shelf
shoes
silk
sing (*verb*)
snowball
soap
soldier
song
soup
sparrow
spelling (*noun*)
spend, spent
spool
spoon
spring
squirrel
stamp (*noun*)
stand (*noun*)
star
station
store (*noun*)
storekeeper
strawberries
street
string (*noun*)
sugar

summer
supper
swim
table
tablet
take
tea
team
thread
ticket
together
tomato
took
tool
tortoise
towel
town
toy
train (*noun*)
trip (*noun*)
trolley (car)
tulip
turtle
uncle
upstairs
vacation
valentine
vegetable
violet (flower)
visit
visitors
wagon
white
window
winner
winter
woods
word
wren
yard (land)
yeast cake
yellow

The opportunities which the arithmetic period affords for increasing the usable vocabulary of pupils should be seized. Not only may new words be introduced but words half known or known to but few members of the class may be made familiar. There is constant use in problem work for such classes of words as vegetables, fruits, toys, animals, flowers, birds, things to wear, games, containers, family relations (cousin, brother, aunt, and the like), vehicles, utensils, quantitative adjectives, and verbs which tell what these things do or what is done with them. Children should be encouraged to "name all the fruits they can" or to "tell the vegetables the farmer sells" or to answer the question "What do we put things into?" meaning containers such as bags, baskets, boxes, bottles, pails, buckets, jars, crates, cartons. The pupils should not only understand these words when some one else uses them, but they themselves should be able to use them. The making of problems by the pupils, an activity which should have a large place in our plans, affords the best opportunity for the pupils to show their mastery of these words. When a child actually uses a word correctly, he offers functional evidence that he knows its meaning.

CHAPTER XIV

TELLING TIME

Teaching children to tell time should not be undertaken until the children realize a need for knowing what time it is. This should be developed conversationally during a period of several days before the actual teaching takes place. It might be approached somewhat as follows, although what we are about to say is intended to be merely suggestive.

"What time does your father stop work on Saturdays?" A number of children will no doubt answer "At noon" or "At 12 o'clock."

"On other days when does your father stop work?" Various answers will be given: "At 4 o'clock," "At 5 o'clock," "At half past four," and so on.

"At what time do you have dinner [or supper]?" Such replies may be expected as "Six o'clock" or "Half past six."

"At what time does your father begin work in the morning?" This will give rise to another set of answers.

"One day I passed a railway station, but there was no one there except the men who worked at the station. Another day I passed by and saw many people there and others hurrying to get there. Why were these people coming together at the station?" Such answers may be expected as "To meet a train" or "A train was coming" or "It was time for the train."

"How did these people know when to go to the station?" Some children may be expected to reply to the effect that they looked at their clocks or watches.

You can easily show that each man or woman has work to do and that this work must be done at a particular time. Many

people have to work with other people, and these others would have to wait if the work were not done at the right time. The children may be encouraged to bring in lists of activities related to the school or the home, to places of business, to the church, or to other aspects of the life about them. Most of these things must be done with reference to time. Getting up in the morning, eating meals, having music lessons, going to school and to other places — all these are done at particular times. The mail carrier comes at a regular time, stores are opened and closed, factories and offices are run with the same dependence upon the clock.

Discuss these matters as far as may be necessary to bring about a lively realization of the need of time-telling. Show that, without knowledge of time, serious losses and accidents may take place.

It will likewise be interesting to show how much more important it is now than it used to be to tell time *exactly*. Long, long ago, however, people felt the need of knowing the time of day. They did not know how to make watches or clocks, but they used other means. They had one way of telling time which we still have, although no one can be very exact in using this way. Perhaps some boy who has lived in the country where he did not hear whistles or trains has learned about the sun-and-shadow. This will be an interesting topic for language study. The important task now is to arouse interest. Work in a little mystery and awaken curiosity, always, however, sticking to the facts of the case.

"When Little Boy Blue woke up, if his haystack threw a long shadow away off toward the west, what time of day was it? If Boy Blue crawled out and after rubbing his eyes saw a little, fat shadow right down by his feet, what part of the day was it? But one of his cows came out of the corn just as he was looking about, and her shadow was on the east side of her. Her shadow-legs looked so long and her shadow-body

seemed to be so high on these legs that Boy Blue knew right away that it must be evening."

From this, children may be brought to observe the positions and lengths of their own shadows as they go to and from school, morning, noon, and afternoon. They may notice the shadow of the flagpole, of the schoolhouse, of the gateposts, and of many other objects. They may be able to make imaginary stories involving the use of sun-and-shadow. Do not let them think so much about the shadow as to forget the sun and its position at different times.

Here is the place to tell about the sundial. Look up its history. Do the same for the notched candle and the hourglass.

Perhaps you will say this is using up your precious recitation periods day after day, and that you have not yet begun to teach the telling of time — a topic required by your course of study. The time, however, is far from wasted and it has values greater than those immediately pertaining to arithmetic. Indeed it is quite likely that if you were to keep on a bit longer, increasing the children's interest in the subject, you might not have to "teach" telling time at all.

You are now almost at the point at which cast-off alarm clocks and handless watches are likely to show up unaccountably on your desk. If they do not, hard-hearted mothers have cleaned house too thoroughly.

If you have a school clock in your room, occasionally look at it long and intently as if not quite certain whether or not it were time to do the next thing. If your wrist watch lies on your desk, do not be severe if some little soiled and errant finger pokes it around to a more readable position; or, if it is on your wrist, never mind if heads turn bottom side up in order to get a better view of it.

Let us assume that by this time the children want to learn to tell time. Provide yourself with a large clock dial upon which the numbers are plainly written, preferably in Arabic

numerals.[1] Study the dial as to the position of the numbers. It would be well to spend two or three periods on this important matter. Draw a circle on the blackboard. Place the numbers on it as they appear on the clockface. Draw another circle beside the first and put the number 12 in its proper position.

"Where shall I place 6 in this circle?" (indicating the second one). With the figures already in position on the first circle, it will be easy to secure such answers as "At the bottom," "Opposite 12," "Right below 12," "Halfway round the clock." We may next place the 3 and 9, bringing out the idea that 3 is halfway between 12 and 6 and that 9 is halfway between 6 and 12, on the other side. Let the children point to 3 with their right hands and to 9 with their left. Let them answer such questions as "What number goes here?" (indicating the place). "Where do you put 12, 6, 3, 9?" Let a pupil place a pencil or pointer at 12 and move it around to the right. Let him keep on going around in order to get the feeling of the clockwise movement. Then tell him to stop at 6 or 3 or 9 or 12. Repeat this procedure with other children.

Ask the pupils to look at the schoolroom clock and notice what number stands next after (that is, to the right of) 12. Here the practice which we have suggested of moving the pencil or pointer clockwise will prove to be helpful. Thus the 1 may be placed and by similar procedure the 2, 4, and 5 may be placed. The 3 and 6 have already been placed.

Thus far we have been going away from 12. We now begin to come toward it. It is well to point out this fact, for it is used when, instead of saying "nine thirty-five," we say "twenty-five minutes before ten" (or "to" or "of" ten).

[1] If, however, there is trouble in securing a clock dial with Arabic numerals, teach the first twelve numbers according to the Roman notation. If this is to be done, bring the clock dial in long before the point we have now reached, namely, the point where the children are eager to tell time.

Either at this time or at the next period, place the remaining numbers *starting at* 12. Thus 11 is the first number *before* 12, and 10 is the second number before 12. Drill on the placing of all the numbers. Let the children point to the position of a number called for by the teacher or by another pupil. Let them likewise tell the numbers when the teacher or pupil points to a given position.

It is now time to use a large clockface with hands. Place the hands so that they both point to 12. Be sure that everyone understands that this means 12 o'clock. Move the long hand of the clock all the way around and point the short one directly at one of the numbers. Ask what time this means. Some of the pupils will be able to tell from previous knowledge. See that they all learn that it is 2 o'clock when the long hand is at 12 and the short hand is at 2, that it is 4 o'clock when the long hand is at 12 and the short hand is at 4, and so on. Let the long hand go around the dial each time the position of the short hand is changed. As you do so, you may say such things as this: "Every time the short hand goes from one number to the next this long hand takes a journey all the way around the circle. It has so far to go that it must move much faster than the lazy short hand. It takes just one hour to make the trip." Be sure that when the short hand is pointing exactly at a number the long hand is at 12.

You now need a real clock for purposes of demonstration. Set the clock at 12. Move the hands slowly until one o'clock is shown. Again point out the fact that the long hand has gone all the way around while the short hand has gone only to 1.

"If we had waited for the clock to do this itself, how long would it have taken?" Tell the children if they do not know. At this point something should be said as to the nature of the unit "one hour," as, for example, "That would be long enough for us to go home for lunch and get back."

The following are some of the questions to be asked and answered: How far did the long hand go in an hour? How long did it take the long hand to go around the clock? How far did the short hand go? (From 12 to 1, or from 6 to 7, or, in general, from one number to the next.) How long did it take the short hand to go from one number to the next? How far did the long hand go while the short one was passing from one number to the next?

Several forms of practice now present themselves. A clockface without hands may be drawn on the blackboard. One child may suggest a time and another place the hands. For the present this must be limited to "even hours." Let the children now "go to sleep" (putting their heads down). They may dream about the biggest clock at home. Jane may tell what she saw in her dream (location of the hands), and Billy may tell what time it was in Jane's dream. The children may dream again. Joe may go to the board and draw his dream on the clockface.

Make a large clockface on the floor. Alice is tall; she may be the long hand. Dot is short; she may be the short hand. It is now about 10 o'clock. Let's play it is *just* 10. Dot stands at 10 a little toward the center of the circle. Alice stands at 12 on the circumference of the circle. Now Alice and Dot may move around in the same way that the clock hands move. We want it to be 11 o'clock. Dot must move very slowly and Alice much faster.

Have different children act out this exercise. Let some children call for the time, by hours only, and let others demonstrate it by positions.

In a similar manner teach half past with practice in reading the time and in placing the clock hands. It is easy to show the meaning of "half" in this connection. We are assuming, however, that the children already know the meaning of one half as it has been developed in Chapter VIII.

It is not worth while at this juncture to try to teach in any developmental way the fact that it takes the long hand 5 minutes to pass from one number to the next. Simply tell it to the children and make numerous applications of it. Let 12 be the starting point. Use a regular clock and set it at an even hour, say 10 o'clock. Give the children something to do for 5 minutes and then call "Time." Let them notice the position of the long hand on the clock at this moment. The idea is for them to get the "feel" of 5 minutes. Let the children understand that it is now 5 minutes past 10. Since we have already shown that it takes 5 minutes for the long hand to pass from one number to the next and since the children have already learned to count by 5's, there will be little difficulty in having them read time by 5-minute intervals from the even hour up to half past the hour.

Show that if the long hand passes the halfway mark, it cannot get any farther away from 12, but must begin to come toward 12. Set the hands of your clockface at half past two and have the children tell what time it is, then move the minute hand forward to 7. Let the children count by 5's from 12 *the shortest way* to the place where the minute hand now is. "Then how many minutes will it be before the minute hand will reach 12?" Answer: "25 minutes." "But in 25 minutes, when the long hand has come up to 12, what time will it then be?" If the children have difficulty in answering this, move the minute hand on to 12 and have the children realize that in 25 minutes it will be 3 o'clock. "Then how many minutes is it before 3?" Answer: "25 minutes." "So, we see, it is 25 minutes before 3. Sometimes we say 25 minutes *of* 3 or 25 minutes *to* 3." Do the same thing with the minute hand at 8 (20 minutes before the hour), at 9 (15 minutes before the hour), etc.

Now use different hours and all the 5-minute points. Again have drill both ways, that is, in reading the time and in setting

the clock from dictation. Incidentally tell the children that instead of saying "15 minutes after 9" we often say "quarter after 9." Show, too, that instead of saying "15 minutes before 9" we may say "quarter before 9." Use "of" and "to" as well as "before."

Apply the new knowledge at once. Appoint a monitor to notify you when it is time for each activity according to your program. Have a different monitor for each item. A great many ways will suggest themselves for utilizing the ability of the children to tell time. Of course, it will very soon become apparent that telling time to the nearest 5 minutes is not sufficiently accurate. We are not suggesting in this pre-third-grade work that children be taught to tell time to the minute. Nevertheless, the interest of the children should be your guide. With some classes it will be evident that the more accurate telling of time should be taught.

The Calendar

Most published courses of study which mention the teaching of the calendar do so in connection with the work of either the first or the second grade. It is suggested that this topic be presented as a continuation and extension of telling time. It can easily be shown that if we want to know the time at which something happened several days ago or the time at which we intend to do something two or three days hence, it will not be enough to know the clock time. Indeed, the clock time may be unimportant. It may be enough to know that a thing happened last Wednesday or that we are planning to do something next Friday.

First, be sure that the children know the names of the days of the week *beginning with Sunday as the first day of the week.* From this it may easily be shown that there are seven days in one week. Teach the children to read and write the names of the days of the week.

Procure a large calendar and show the pupils the page for the current month. Have an informal discussion on the meaning of the arrangement of the numbers and the abbreviations at the top of the columns. Let us suppose that the page of the calendar for the current month is as given next below.

1927		NOVEMBER			1927	
Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

Point out from this page of the calendar the abbreviations for the days of the week. Whether to teach these abbreviations to the point where the children can reproduce them in writing is a question for the teacher to decide in the light of the ability of the class and the time at her disposal. In any event the children should be able to identify the abbreviations when they see them. With reference to the month of November, 1927, as shown above, such questions as these should be asked and answered: How many days are there in November? On what day of the week is the first day of November? the last? the tenth? the seventh? the twenty-fifth? etc.[1] How many Wednesdays are there? How many Sundays? How many Tuesdays? Thanksgiving Day comes on the twenty-fourth. What day of the week is that?

Show from the calendar the application of today, yesterday, and tomorrow. Ask such questions as What day of the week

[1] These numbers on the calendar are really ordinals and should be treated as such. They indicate the number of the day in a series.

is it today? What day of the month? Point to today on the calendar. What day of the week and month will it be tomorrow? Point to tomorrow. What day of the week and month was it yesterday? Point to yesterday. If the answers do not carry over into the next month or back into the previous month, one may ask: What date will it be a week from today? two weeks from today? What date was it a week ago today? two weeks ago today? What date will next Sunday be? What date was it last Tuesday? and so on.

Having discussed the representation of the current month on the calendar, show the pupils the next month. Ask similar questions about it. Then show one or two other months and have the children learn the months of the year in their order beginning with January as the first month. In this connection it will be shown that there are twelve months in the year. Show that the current year is 1927 (or whatever it may be). It is not recommended that the full significance of 1927 be given. Simply call it "nineteen twenty-seven" and show that this is the name of the year. Now teach the children to write dates, sometimes using the day of the week as well as the day of the month; for example, Wednesday, September 7, 1927. As in the case of the abbreviations for the days of the week, the question of teaching the abbreviations for the months will be decided by the prevailing circumstances.

CHAPTER XV

DRILLS AND GAMES. III

Some of the following devices for drill can be used at various stages. Most of them can be used to best advantage when a relatively large number of combinations have been taught and the children have advanced in their counting, reading, and writing of numbers at least to 100. The time at which to apply these devices depends largely upon the ground already covered and the aptitude of the pupils.

The postmaster. Each pupil is provided with a number representing his home, or his "address." He holds this number where it may be seen. The teacher then selects a pupil to be postmaster. She gives him envelopes bearing the same numbers which she has already given the pupils. The postmaster is to deliver the letters to the proper persons as represented by the house numbers — number 28 to the pupil holding number 28, number 56 to the person holding number 56, and so on. If he makes a mistake, he loses his right to be postmaster, and the pupil who discovers his mistake may take his place.

This game may be varied by having either the envelopes or the addresses in the form of combinations. It is understood that while the children are playing games in which a knowledge of the number facts is required, the combinations posted in front of the room for reference remain in full view.

A guessing game. TEACHER. I am thinking of two numbers that make nine.

PUPIL. [*Writing on the board the number fact*

$$\begin{array}{r} 5 \\ 4 \\ \hline 9 \end{array}$$

] Are you thinking of "five and four are nine"?

TEACHER. No, I am not thinking of "five and four are nine."

PUPIL. [*Writing on the board the number fact* $\begin{array}{r} 6 \\ 3 \\ \hline 9 \end{array}$] Are you thinking of "six and three are nine"?

TEACHER. Yes, I am thinking of "six and three are nine."

The pupil who guesses the combination the teacher is thinking of takes the teacher's place and the game continues. The subtraction combinations should be brought in to an even greater extent than the addition combinations. This is because, as we have seen, they are harder. For example, the teacher may tell the pupils at the outset that only the take-aways are going to be used, or she may have a way of bringing the subtraction facts forward in the same exercise with the addition facts. Thus, she may announce that she is thinking of two numbers that make four. These numbers may be taken additively or subtractively. For example, they may be "three and one" or "seven less three." Another variation, an excellent one, is illustrated by the teacher or a pupil saying, "I am thinking of two numbers that make eight; one of them is three." The other number is to be guessed, with a dialogue similar to that just indicated.

A card game. Cards with single numbers are placed in the chalk tray. Combinations, most of them subtraction combinations, without answers are written on the blackboard. The teacher points to one of these combinations. A pupil goes to the cards and selects the one on which the correct answer to the combination is written. This game can be adapted for seat work; it can also be used as a contest.

A ball game. The pupils form a circle. Each one is given a number. The teacher calls "8 less 3," and at the same time bounces a rubber ball in the circle. The pupil who has the number 5 catches the ball and takes the place of the teacher. He then calls another combination and bounces the ball. Of course only a limited number of pupils can be in the circle.

Bringing home the cows. The farmer has a great many cows, so he gives them numbers instead of names. These numbers are represented by combinations. For example, if a cow's number is 6, this 6 may be represented by $\begin{matrix}4\\2\end{matrix}$ or $\begin{matrix}7\\-1\end{matrix}$ or 2 3's. The cows in this picture eat grass all day, but as the sun goes down the children go out to bring them home. Each cow has

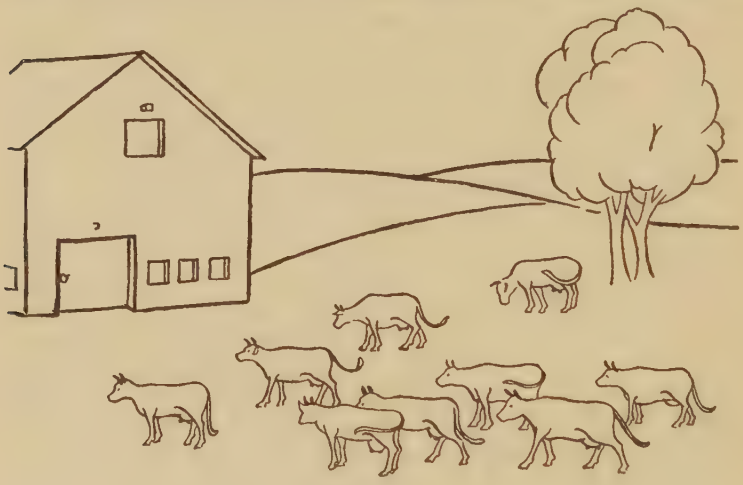

FIG. 11

to be called by its right number before it will come. The teacher, after making a sketch similar to the one shown, gives the setting just suggested and marks each cow with a combination. She then selects a pupil to be the farmer's boy. He is to start after the cows and bring in all he can. If he calls one by the wrong number, some other child will have to try. The teacher points to the combination and the child calls the answer only. If it is right, he has been successful in bringing home the cow.

Hopscotch. The teacher draws on the board a large rectangle and divides it into smaller rectangles. Any desired

number of these smaller rectangles may be made. In them are written the combinations which need practice, especially the large addition facts and the subtraction facts. Pupils play hopscotch by starting at a certain point and "going as far as they can." They may go up one column of rectangles and down the next one. Each combination stated correctly permits them to move on to the next rectangle. This game may be played with or without scoring. If it is scored, each combination correctly answered counts 1.

Relay race. The teacher writes a combination at each end of the blackboard, for example:

$$\begin{array}{r} 5 \\ -\,3 \\ \hline \end{array} \qquad\qquad 4\text{'s in } 8 =$$

Two lines, each of the same number of pupils, are formed. One line stands in front of the combination on the right, the other in front of the one on the left. At the word "Go," the first pupil in each line runs to the board, writes down the answer to the combination which his line is facing, and beside this combination writes another. The figures on the board may then look like this:

$$\begin{array}{rr} 5 & 7 \\ -\,3 & -\,4 \\ \hline 2 & \end{array} \qquad\qquad \begin{array}{r} 5 \\ 2 \\ \hline \end{array} \quad 4\text{'s in } 8 = 2$$

As soon as the first pupil in each line has completed this task he runs back to his line and touches the second pupil, who is then allowed to run to the board, insert the answer to the last combination, and write another one. Meanwhile the first pupil passes on to the foot of his line. The procedure may continue until all the pupils in one of the lines have had a turn. If a pupil makes a mistake or writes a combination that has not been taught, one of the succeeding pupils in his line is expected to make the necessary correction. The line which finishes first wins the relay race.

Card game. The teacher puts a row of combination cards along the blackboard tray. Two pupils are chosen and begin at opposite ends to "capture" the cards by giving the correct answer. The pupil who captures the greater number of cards in a given time wins.

The game of silence. The teacher writes several combinations on the board. She then erases one and writes a pupil's name on the board. This pupil then repeats the combination with its answer. This game requires close attention on the part of the pupils. It also has the advantage of enabling the teacher to select the hard combinations and the pupils who need drill.

Number match. The pupils choose sides or are divided into two groups by the teacher as in a spelling match. The first pupil on one side is given a combination. If he answers it, he remains standing, and another combination is given to the first pupil on the other side. If he misses it, he takes his seat and it is given to his opponent. This procedure continues until time is up. The side having the greater number of pupils standing at the close of the match wins. This game is valuable if worked right. As a mere drill device, it is useless. The ones who need the drill do not get it. But the game does show both *to the teacher and to the pupils* the ones who need drill. The pupils may then be permitted to drill one another in order that their side may win next time.

The circle game. The teacher draws a circle and writes numbers along the circumference. At the center she writes a number which is to be added to or subtracted from each number along the circumference. If the number at the center is to be subtracted, the minus sign should be put in front of it. The teacher may point rapidly to the numbers on the circumference, calling on a pupil for each response, or she may call on the pupil to go around the circle. This game may be varied by telling, rather than writing, the number that belongs

in the center of the circle. The ability to deal with unseen numbers is important as a preparation for column addition.

The lost geese. The teacher selects a pupil to be the little old woman who has lost her geese (or the little old man, if a boy is chosen). Some of these geese are white and some are gray. The rest of the pupils are formed in a circle and each pupil is given a number. The little old woman walks around the inside of the circle and, stopping in front of a pupil, says, "Have you seen my geese?" To which the pupil addressed replies, "How many geese did you lose?" The little old woman answers by giving two numbers, for example, "I lost 3 white geese and 5 gray geese." Whereupon the pupil in the circle whose number is 8 cries out, "I saw your 8 geese."

The game can be played so as to afford drill on the subtraction facts by having the little old woman say, "I had 6 geese and now I have only 2." The pupil whose number is 4 will say, "I saw your 4 geese." Or, if the pupils' numbers are pinned on them so that all can see them, the one having the right number may simply say, "I saw your geese." His number shows whether or not he was the one to reply.

CHAPTER XVI

COLUMN ADDITION

Owing to the fact that the order of presenting the foregoing combinations was determined by facts other than the size of the sum (or minuend) there has been no time until the present when we could say that all the combinations up to those having specified sums had been taught. Since, however, in the beginning we divided all the basic combinations into those whose sums were less than 10 and those whose sums were 10 or more (calling the first the easy combinations and the second the hard combinations), and since by this time we have provided for the teaching of all the facts whose sums are less than 10, it is now possible to do some things which we could not have done earlier.

We may, for example, introduce the addition of *more than two* numbers; in other words, column addition. There is, of course, no real reason why a child should wait until he has mastered *all* the basic combinations before he puts more than two of them together. Many situations naturally call for the combining of more than two numbers. We have felt this as a limitation upon the construction of problems. Not more than *two* things could be bought at a time. Not more than *two* children could pool their treasures. Only *two* events could be mentioned. Why should we not assist our pupils to meet a greater variety of needs by teaching them column addition as soon as they can learn it easily? Accordingly we recommend that the addition of three one-place numbers now be introduced, provided, of course, that the sums do not exceed nine.

If this limitation is made, none but the easy combinations will be involved, and there will always be two of them in each column. One will be a "seen" combination, the other an "unseen." In the column

$$\begin{array}{r} 3 \\ 2 \\ \underline{4} \end{array}$$

the first combination (assuming downward addition) is $3 + 2$; and it is "seen" because both its terms are visible. The second combination is $5 + 4$. It may be called an "unseen" combination because one of its terms, the 5, is not visible.

If we were to introduce columns of *four* one-place figures, we should find that *three* combinations were involved. If there were five figures, there would be four combinations; in other words, always one less than the number of figures in the column.

We shall, however, limit ourselves for the present to three figures in the column and consequently to two consecutive combinations. Moreover, both these combinations will be of the "easy" kind, because up to this point no others have been taught. Later, when pupils have learned more number facts, the addition of three one-place numbers will be analyzed into types and carried to completion (see Chapter XXIV). Meanwhile, the type with which we are now concerned, that in which both combinations are *easy*, may be called Type A.

The "easy" number facts may be applied in other ways than in column addition. For example, the child who has learned $2 + 7$ may learn $12 + 7$ or $22 + 7$ at once. He need not wait until he has learned the "hard" number facts. Hence we may teach *higher-decade addition without bridging the tens* before we teach the harder basic facts. Chapter XVII enters fully into this matter. Similarly, the *higher-decade subtraction without bridging*, described in Chapter XVIII, may

be taken up as an application of the easy basic subtraction facts. Finally, *any* addition without carrying and *any* subtraction without borrowing may now be done, provided the pupils can read, write, and understand the numbers involved. Chapter XX is devoted to this topic.

We have introduced all these topics *before* presenting the hard basic combinations; that is, those whose sums or minuends exceed nine (Chapter XXI). It may not be wise for you to adopt this order. Different classes should be handled differently. You may find that your pupils have early and imperative need for the combinations whose sums are more than nine. You may also find that they do not need the concentrated drill on the easy combinations which the full use of this chapter and Chapters XVII–XX will afford. Under such circumstances, do not hesitate to begin teaching the hard combinations at an early date. In that case adopt the teaching order given in Chapter XXI and intersperse the new basic combinations with the different types of drill and application set forth in the present chapter and the four chapters immediately following it.

Teaching Column Addition

Suppose the teacher presents on the board the following column:

$$\begin{array}{r} 2 \\ 2 \\ \underline{1} \end{array}$$

Any one of a number of methods of treatment may be adopted. If the children appear to profit by the appeal to concrete objects, it is easy for the teacher to resort to such an appeal in this instance. The method of hidden addition will apply admirably. Two pieces of chalk identified as two and hidden behind the chalk box, followed by two more similarly identified, and finally by one more, will, when

brought to view and counted, reveal the fact that there are five pieces of chalk.

At the foot of the column which the teacher has written on the board the figure five may now be written. The pupils may be shown that this answer can be quickly and easily obtained from what they already know.

TEACHER. Beginning at the top, let us say how many two and two are.

PUPIL. Four.

TEACHER. But this is not all we have in this column. We have one more. How many are four and one?

PUPIL. Five.

The teacher should go over the column once or twice more. She should show that the first two numbers are like the numbers on the cards which have been posted in front of the room. Just as two and two on those cards give four, so two and two in the column addition give four. We do not write this four because we are not through adding the column, but we must not forget it. Then we have four and one, which we know to be five. The teacher should then give other columns:

1	3	1
4	3	7
2	1	1

Downward addition preferred. In each case let the addition be *from the top downward.* Throughout the entire pre-third-grade work teach and practice downward addition to the exclusion of upward addition. There are many reasons for this, but first it should be pointed out that *consistency* is highly desirable. At this stage of teaching and practicing column addition and for some time to come, adding in one direction to the exclusion of the other is better than allowing the pupils to add now one way and now the other.

The advantage of consistency is of the same nature in this case as in the case of teaching subtraction facts. As we have already pointed out, one may teach these facts by naming the minuend first; for example, "seven less five are two." Or one may follow the plan of naming the subtrahend first, "five *from* seven are two." But because *either* method is practicable does not mean that *both* are desirable with the same children. Much unnecessary confusion, as every practical teacher knows, has been caused by mixing two or three procedures before one of them has been thoroughly learned.

Moreover, if you can be sure that your pupils will add one way and only one way, you can make your drill material more effective. A great deal has been written about distributed drill. The idea is that the basic number facts should all be practiced to some extent and that the difficult ones should be practiced more frequently, according to their difficulty. This desirable condition cannot be attained so far as column addition is concerned unless one knows which way the children are going to add. You may know, for example, that your class needs more drill on $4 + 3$ and $3 + 5$. You may give practice columns in which these combinations occur, such as the following:

$$\begin{array}{cc} 4 & 1 \\ 3 & 2 \\ \underline{2} & \underline{5} \end{array}$$

But these columns will not bring the desired combinations into play unless the child adds downward. You can, of course, be equally sure of your results if you are certain that the child will add upward. We therefore urge that for the time being the choice between upward and downward addition be made and adhered to.

We believe, however, that preference should be given to the downward direction. In the first place, it is the natural movement in reading. Vertical street signs are never read

upward. House numbers whose digits appear one under the other are read from the top down. For this reason we have provided in previous chapters for the downward rather than the upward reading of combinations. The children, if they have been taught as we have suggested, already regard $\frac{2}{3}$ as "two and three." When another number is added to the combination it is natural to keep the same direction. In the second place, a series of numbers is always written downward. No one ever writes a column of figures from the bottom up. In writing the basic combinations one writes the upper figure first. The children are already doing this. The fact, therefore, that in reading and writing numbers we move from the top down strongly suggests that we move in the same direction when we add.

There is another reason for downward addition; it brings the eye and the hand to the point where the sum is to be written. A certain per cent of errors in column addition is due to writing the wrong figure for the answer or putting it in the wrong place. The child is much less likely to forget the answer he has obtained if he writes it instantly without having to hold it in mind while finding its place. This becomes an important question as the length of columns increases and as the numbers to be added increase in size so that carrying is required.

If we teach upward addition the number of movements of hand and eye is greater than it is when we teach downward addition. Most column addition involves numbers of more than one digit and calls for carrying. When we teach addition involving carrying, we are practically obliged to allow the children to write the number to be carried. This, we admit, raises the whole question of the use of "crutches." In this case, however, we believe that a "crutch" is justified. A principle more fundamental than the use of a device after-

wards to be discarded is at stake. It is the principle of introducing a child to a new and difficult process in such a way as to make his success most likely. We cannot afford to train in error; yet this is what we do when we permit several difficulties to present themselves upon the introduction of a new topic. One new difficulty at a time brought sharply into the focus of attention and surmounted before the next difficulty is permitted to come into view is good pedagogy.

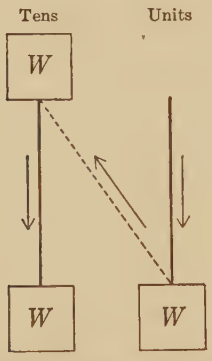

FIG. 12. Eye and hand movements in adding two-place numbers with carrying. Downward movement

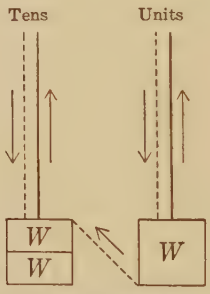

FIG. 13. Eye and hand movements in adding two-place numbers with carrying. Upward movement

We have said that when carrying is involved, upward addition requires more movements of the eye and hand than downward addition. Fig. 12 shows the movements in the case of downward addition for a column of two-place numbers. The solid lines represent movements made while adding. The dotted line represents movements without adding; that is, eye and hand sweeps. The *W's* represent something to write.

In Fig. 12 observe that there are three up-and-down movements. First the child moves with eye and hand down the units' column and writes the unit figure of the sum. He then moves with eye and hand to the top of the tens' column where he writes the number to be carried. Finally, he moves

down the tens' column, adding as he goes, and writes the sum at the foot of this column.

In Fig. 13, however, which shows a picture of the movements in upward addition for the same type of example, it will be observed that there are four up-and-down movements. The child goes up the units' column, adding as he goes. He then comes back to the bottom of the same column where he writes the unit figure of the sum. At this point the procedure is optional. The number to be carried may either be written at the bottom or at the top of the tens' column. Fig. 13 assumes that it is written at the bottom. If it is written at the top of the tens' column, two more additional up-and-down movements are required, making six instead of four as indicated in Fig. 13. Assuming then that the number to be carried is written at the bottom of the tens' column, the third movement is upward through the column as the addition takes place. Finally, the fourth movement is back to the bottom of the tens' column where the sum is written.

Observe that in column addition with carrying, the downward movement strongly favors the writing of the number to be carried at the top of the next column, while upward addition encourages writing this number at the bottom of the next column — the alternative being two extra movements. Generally, however, there is *no room* at the bottom of the next column. Naturally children of the second and third grades write large. In fact they should do so. Their best writing instrument is a soft pencil. It is, therefore, practically impossible for them to write a legible number at the foot of a column. To require this is to encourage error. On the other hand, there is plenty of room at the top of the next column. The downward adders, as pointed out, may write the figure at the top of the column without additional movements of eye and hand. The upward adders can accomplish this only by using two extra movements. Of course there are some who

insist that children should be required from the beginning to carry without writing the number to be carried. We believe that this is a mistaken notion. But even if this requirement is made, the downward process gets the work done for a two-column example with three movements instead of four. Every extra movement is an opportunity for error.

It is not surprising, therefore, that the experimental evidence, at least so far as accuracy is concerned, favors the downward movement in addition. Cole[1] shows that for college students downward addition, although slower than upward addition, was more accurate. Recently we have obtained some independent evidence on the question. In five out of six controlled experiments in the teaching of upward and downward addition, the pupils who were taught downward addition did better than their classmates of equal ability who were taught upward addition. Probably the figures obtained from college students are of little value in this connection, but those which refer to the actual teaching of third-grade children are significant.

Let us return now to the matter immediately in hand; namely, the teaching of the simplest case of column addition, or three one-place numbers whose sum is less than ten. Several points should be brought out in the teaching.

1. Assuming downward addition and previous downward movement in the basic number facts, the teacher should not fail to make the obvious connection. The top figure of the column together with the next figure under it is just like the ordinary number fact with which the pupil is already familiar. For example, in

$$\begin{array}{r} 1 \\ 4 \\ \underline{2} \end{array}$$

[1] Lawrence E. W. Cole, "Adding Upward and Downward," *Journal of Educational Psychology* (February, 1912), Vol. III, pp. 83–94.

the pupil's reaction to $\begin{array}{r} 1 \\ \underline{4} \end{array}$ should be just the same as in the basic combination $\begin{array}{r} 1 \\ \underline{4} \end{array}$. In other words, he must *think* "one and four are five."

2. The answer to the first combination, 5, in the column to which we are referring, must *not be forgotten.* This necessity for holding in mind a number which is not visible and which is not to be written constitutes a new difficulty. Moreover, it is a difficulty characteristic of all column addition.

3. The next lower number must now be added to the unseen result of the first combination. This necessity, likewise characteristic of all column addition, of uniting an unseen number with a seen number must be carefully provided for. Some of the games and devices which we have already suggested are valuable in this connection. For example, the game in which the teacher writes numbers on the circumference of a circle with a number at the center may be varied for this purpose by omitting the number at the center of the circle. The teacher may tell the children this number and direct them to add to it each of the numbers on the circle as she points to them.

4. The answer to the "unseen" combination is the answer to the example. If the pupils are engaged in written work, this answer, of course, is to be written directly under the column.

When a zero appears in the column, have the pupils treat it as part of a combination rather than omit it. The short cut of disregarding the zeros in column addition should come later. At present, the pupils need drill in handling zero in a combination. Accordingly, in the column

$$\begin{array}{r} 6 \\ 0 \\ \underline{3} \end{array}$$

the pupil should, for the present, think "6 and 0 are 6; 6 and 3 are 9," *not* "6 and 3 are 9."

Column Addition as Drill

Column addition will provide admirably for extensive drill on the addition combinations. It has one capital characteristic of good drill: it provides repetition from a new point of view. Therefore the teacher is urged to present a large amount of column addition at this time and to continue to present column addition along with the teaching of new materials. Indeed, since column addition is one of the most difficult as well as one of the most useful of the arithmetical skills, there is scarcely a time during the elementary-school period when practice in adding columns may be abandoned. The new difficulties in column addition, however, must not be minimized. They are (1) the holding of the result of the first combination in mind; (2) the combining of this unseen number with a seen number; (3) the holding of attention upon the task in hand long enough to complete it; and (4) the writing of the answer (if writing is required). The chief difficulties in written work are copying the figures in a straight column and writing the answer directly under the column.

Practice Material

In order to assist the teacher in presenting column addition the following examples are offered. In them drill is well distributed among the fifty-five addition combinations which have been taught thus far. No combination has been neglected, and the harder combinations appear more frequently.

Although there are three hundred of these examples, it is not intended that they should all be used at this point in the pre-third-grade work. Use some of them now, and use others at various times later when it seems best to review this type of work. Very likely you may not need to use all these examples at any time.

Column Addition

Three one-digit numbers. Sums less than 10

a

1.	0	0	1	2	0
	2	7	0	7	8
	1	2	8	0	0
2.	4	5	3	1	1
	2	1	2	2	4
	2	3	4	5	0
3.	5	0	3	3	2
	3	6	0	5	0
	0	0	4	0	5
4.	0	2	2	1	8
	5	5	1	1	0
	2	1	3	4	1
5.	6	7	7	6	4
	0	1	1	2	1
	0	0	1	0	2

b

1.	6	1	2	0	0
	3	4	4	3	2
	0	1	0	1	3
2.	5	4	1	1	3
	0	0	3	6	3
	3	2	4	2	3
3.	0	0	2	2	4
	4	4	1	1	3
	3	4	5	4	2
4.	2	0	5	3	1
	0	4	2	2	0
	7	5	2	1	6
5.	5	9	0	3	6
	2	0	4	0	0
	0	0	2	4	1

c

1.	5	4	3	4	1
	1	1	2	0	2
	0	1	0	5	4
2.	2	1	4	2	1
	6	5	1	3	1
	1	0	3	0	2
3.	4	3	1	0	2
	0	2	3	3	0
	3	3	4	2	2
4.	3	5	0	1	0
	3	0	5	2	0
	1	4	2	6	9
5.	2	0	3	0	2
	3	1	1	5	4
	3	7	5	1	0

d

1.	0	2	3	2	7
	1	1	4	6	0
	8	6	1	0	2
2.	6	8	2	0	1
	1	0	2	5	2
	1	0	1	3	5
3.	0	0	3	1	5
	4	1	1	2	0
	3	4	5	4	3
4.	1	1	1	0	4
	2	0	3	5	1
	0	1	1	1	0
5.	0	1	5	4	3
	1	3	4	2	1
	2	0	0	2	3

e

1. 4	1	1	3	0
2	4	7	3	7
3	2	0	2	2
2. 2	0	2	3	5
4	3	0	2	0
3	5	6	4	2
3. 1	2	4	1	1
2	1	2	0	0
6	4	2	4	5
4. 6	0	3	4	2
1	5	0	0	3
2	0	1	3	1
5. 1	0	5	7	5
2	3	1	1	3
1	3	3	1	0

f

1. 2	5	5	4	1
0	4	2	3	5
7	0	1	1	2
2. 3	2	3	1	1
4	3	1	2	4
1	2	5	3	3
3. 3	2	4	7	4
2	5	1	2	3
2	1	4	0	0
4. 2	3	3	1	1
7	4	1	4	2
0	2	2	4	4
5. 5	0	0	6	7
2	3	6	1	1
0	6	2	1	0

g

1. 2	1	3	5	1
5	3	4	1	0
2	4	2	3	7
2. 4	3	1	3	1
3	0	6	4	2
1	5	2	1	4
3. 4	8	4	2	7
5	0	2	1	0
0	1	3	5	2
4. 3	2	3	9	1
0	1	2	0	0
6	4	2	0	8
5. 3	0	5	4	5
2	3	1	0	1
1	3	2	3	1

h

1. 0	1	0	4	1
2	3	2	0	5
5	2	6	4	3
2. 2	0	7	4	1
1	2	1	2	6
6	7	0	1	1
3. 2	3	1	4	6
1	5	3	2	1
3	1	3	3	2
4. 1	7	2	3	5
2	2	4	4	3
2	0	2	1	1
5. 7	4	2	0	6
1	3	1	2	0
1	0	4	4	2

i

1. 0	0	1	2	3
5	1	3	3	6
2	3	5	4	0
2. 3	0	0	1	2
1	8	4	3	5
5	1	0	2	1
3. 4	0	1	1	3
0	5	7	2	2
5	3	0	5	2
4. 4	2	5	4	2
3	1	2	2	1
1	2	1	3	3
5. 6	7	1	3	2
1	0	4	1	0
1	2	2	4	6

j

1. 1	0	7	0	1
0	2	0	3	4
8	7	1	3	4
2. 0	3	2	2	0
4	2	4	6	6
3	4	1	1	2
3. 4	5	1	3	1
5	0	3	4	5
0	3	2	0	2
4. 5	6	3	4	3
2	2	1	1	1
2	1	2	1	3
5. 2	4	1	3	1
5	2	4	4	6
1	3	2	1	0

k

1. 1	0	7	4	0
8	3	2	3	5
0	5	0	1	2
2. 4	5	2	2	5
1	2	4	1	0
2	1	1	3	4
3. 1	1	3	2	0
6	3	1	4	4
2	2	5	3	2
4. 4	2	8	2	1
4	3	1	0	6
0	3	0	5	1
5. 3	2	1	4	0
1	7	5	2	4
4	0	1	3	5

l

1. 9	1	0	2	3
0	1	3	0	2
0	7	6	7	4
2. 5	3	2	0	5
0	1	3	3	0
4	1	2	4	3
3. 0	4	2	1	0
2	1	4	5	4
6	4	3	2	1
4. 2	1	1	7	3
3	0	5	0	2
3	6	1	2	3
5. 4	2	3	1	3
1	1	3	2	0
3	2	1	2	5

Problems Involving Column Addition

The following problems are offered to illustrate the addition of three one-digit numbers whose sums are less than ten. Some of these problems are really rather extended descriptions of situations. Some of them will be more effectively presented in dramatized form than as a part of regular class routine. It would be rather futile, for example, merely to read the first problem, the one about the duck, the hen, and the robin to the children. But if one pupil impersonates self-contented Mrs. Puddleduck, another Mrs. Cacklehen in her despair, and still another fussy Mrs. Robin, the meaning of the problem will be conveyed — and something else too.

The teacher would do well to read some of the suggestions we have already given concerning the treatment of verbal problems, suggestions the burden of which is that such problems are to be really taught rather than thrust at the children. It is further suggested that in treating these problems pupils be expected, at least part of the time, to write the column of numbers which the problem exemplifies. After having written the column, they will of course be expected to write the answer in its appropriate place.

The drill on the basic combinations afforded by these problems is distributed so that no combinations are neglected except a few which involve zero. The difficulty of working these zero combinations into concrete problems has already been referred to, and it is believed that the problems here presented give sufficient recognition to these combinations *as types*. In distributing the drill it has been assumed that the pupil will add the figures in the order in which they are presented in the problem. This will likewise mean that when he writes the column corresponding he will write the first number at the top of the column, the second next, and the third at the bottom.

1. MRS. PUDDLEDUCK. [*Waddling across the barnyard*] Good morning, Mrs. Cacklehen. How many eggs have you saved up in your nest?

MRS. CACKLEHEN. Oh, Mrs. Puddleduck, not a single one, for someone takes my egg away every day. Not one, not one! [*Shaking her head sadly*]

MRS. PUDDLEDUCK. I have eight, for I don't make much of a nest, and I cover my eggs with grass, so they are hard to find.

MRS. ROBIN. [*Turning her ear and listening intently*] Chirrup! Chirrup! Oh, I must fly away and finish my nest quickly, for I too have not a single egg. Not one, not one! [*Flaps her wings and flies lightly away.*]

How many eggs have they all together?

2. On the top shelf of Brother Bear's closet stood 1 big jar of honey, on the middle shelf were 8 little jars, and on the bottom shelf there was none. How many jars of honey were there in Brother Bear's closet?

3. Mistress Mary went out to see how her garden grew. She found 1 lily, no roses, and 7 pansies. How many flowers did she find?

4. There were 3 little houses on a little street. In the first house 1 lady lived all alone, in the next 5 people lived, and in the last no one lived. How many people lived in the 3 little houses?

5. John went to the pond one day and saw 2 gray geese swimming. He went the next day and saw none. The day after that he saw 7. How many gray geese did he see?

6. The children wanted to find the plums that were in their pie. Jack found 5 in his, Jim found 0 in his, and Tom found 1 in his. How many plums did they all find?

7. Harry, Peter, and William were playing a bean-bag game. Harry got 3 points, Peter got 5, and William got none. How many points did they have all together?

8. Betty, Louise, and Hannah were also playing the bean-bag game. Betty got 1 point, Louise 6, and Hannah none. How many points did they have all together?

9. Betty helped her mother by darning stockings. In her own there were 2 holes, in Robert's there were 6, and in Baby Lu's there was none. How many holes were there for Betty to darn?

10. The crayons were all gone from the blackboard tray. The children found some old boxes in the closet. They looked in one and found 4 crayons, in another and found none, in another and found 2. How many crayons did they find?

11. Tom, Sam, and Jim went out to look for pine trees. Tom found 2, Sam did not find any, and Jim found 4. How many pine trees did they all find?

12. Anna sewed 1 button on baby's dress, 3 on brother's blouse, and 2 on father's shirt. How many buttons did Anna sew on?

13. A mother wren and 4 little wrens were in a birdhouse. Soon the father wren went in. How many wrens were then in the birdhouse?

14. William went to visit his cousin. He walked 1 mile to town, then rode 6 miles in a bus and 1 mile in his cousin's automobile. How many miles did William go?

15. A little tree had its first ripe apples. The morning breeze blew, but none fell off. The evening breeze blew and 3 fell off. The night wind blew and down fell 4. How many apples fell?

16. Ella learned 3 new words Monday, 3 Tuesday, and 2 Wednesday. How many new words did she learn?

17. Ralph spent 1 cent for a pencil, 1 cent for a pen point, and 5 cents for a tablet. How many cents did he spend?

18. Harry and Dick and little Jane went to the garden to pick beans. Harry picked 3 quarts, Dick picked 3 quarts, and little Jane picked 1 quart. How many quarts did they all pick?

19. There are 4 people in Lucy's family. When her aunt and 4 cousins visit them, how many people are there at Lucy's house?

20. The postman left 2 letters for mother, 2 for sister, and 5 for father. How many letters did he leave?

21. Some boys were playing Indians. One had 5 arrows, another had 2, and another had 2. How many arrows did they all have?

22. Ella paid 2 cents for a pen, 5 cents for post cards, and 2 cents for stamps. How many cents did she pay for all?

23. Philip's pets are 2 kittens, 1 puppy, and 6 chickens. How many pets has Philip?

24. When Kitty had her party, 6 automobiles came and stood in the street near her home. Soon 2 more came, and then 1 more. How many automobiles were then in the street?

25. In a bus 1 man sat in front to drive, 2 men sat on one side, and 6 children on the other side. How many people were in the bus?

26. In Frank's grocery basket were 2 pounds of rice, 6 pounds of beans, and 1 pound of coffee. How many pounds of groceries were in the basket?

27. Harry has 6 white rabbits, 1 black one, and 2 brown ones. How many rabbits has he?

28. There are 3 eggs in Mrs. Robin's nest, 1 in Mrs. Wren's, and 4 in Mrs. Sparrow's. How many eggs are there in all the nests?

29. Ben was playing tenpins. The first time he knocked down 4, the next time 3, and the next time 1. How many pins did he knock down?

30. Some children were playing on the ice: 1 child was pulling a sled; on the sled were 3 children; 4 children were skating. How many were playing on the ice?

31. Alice picked 4 quarts of berries one day, 3 quarts the next, and 1 the next. How many quarts did she pick all together?

32. Tom sold 4 tickets, Jane sold 3, and William sold 2. How many tickets did they all sell?

33. One evening 4 little crickets were singing a cricket song; 2 more joined in, and then 3 more. How many were then singing the cricket song?

34. Wallace went out to look for eggs. He found 3 in a hole in the haystack, 2 in the barn, and 3 in the henhouse. How many eggs did he find?

35. One spring morning Fred saw 1 bluebird, 4 robins, and 2 wrens. How many birds did he see?

36. Last summer James earned just enough money to pay for a cap, a pair of shoes, and a sweater. The cap cost 1 dollar, the shoes 5, and the sweater 3. How many dollars did James earn?

37. On the hill there were 4 red sleds, 1 black one, and 2 green ones. How many sleds were there on the hill?

38. A boy was practicing walking. One day he walked 1 mile, the next day 2 miles, and the next day 4 miles. How many miles did he walk in all?

39. I saw 3 letter carriers come out of the post office, then 2 more came, followed by 3 more. How many letter carriers did I see come out?

40. Walter planted 3 rows of corn, 1 row of beets, and 5 rows of beans. How many rows did he plant in all?

41. In the baker's window there were 2 raisin pies, 2 apple pies, and 4 pumpkin pies. How many pies were in the window?

42. Uncle John had 2 cows in the barn, 3 in the yard, and 4 in the field. How many cows had he?

43. There were 2 ladies, 2 men, and 3 children in an automobile. How many people were in the automobile?

44. Some boys went to gather nuts. Joe got 2 quarts, Henry got 1, and Walter got 1. How many quarts did they all get?

45. In a garden there were 3 red roses, 5 pink roses, and 1 white rose. How many roses were there in the garden?

46. Mr. Gray planted 5 apple trees, 1 plum tree, and 3 peach trees. How many trees did he plant?

47. The teacher, 5 boys, and 3 girls were playing a game. How many people were playing the game?

48. Kate is learning to swim. One day she swam 2 yards, the next day 3 yards, and the next day 4 yards. How many yards did she swim all together?

49. Susie bought a piece of candy for 1 cent, a yeast cake for 3 cents, and a box of crackers for 5 cents. How many cents did she spend?

50. In a store window there was a Teddybear that cost 2 dollars, an express cart that cost 3 dollars, and a pair of roller skates that cost 2 dollars. How many dollars would it take to buy them all?

51. There were 3 little turtles sunning themselves on a log. Soon 2 more crawled up, and then 2 more. How many turtles were there on the log then?

52. Dobbin ate 2 quarts of oats in the morning, 2 at noon, and 3 at evening. How many quarts of oats did he eat that day?

53. Some boys were shooting arrows at a tree. Herbert hit it 8 times, Robert once, and Frank not at all. How many arrows hit the tree?

54. Jimmy Skunk was hunting for bugs. He looked under a dead leaf and found 1. He dug under an old log and found 8. He looked into the dry sand and found none. How many bugs did Jimmy Skunk find?

55. Mother Hen was chasing a grasshopper. "Cluck, cluck!" she called, and 8 baby chicks ran after her. Again she said "Cluck, cluck!" but no more chicks came. "Cluck, cluck, *cluck!*" she said, and then 1 more came. How many chicks came to Mother Hen?

56. Mr. Squirrel and Mrs. Squirrel and 4 baby squirrels were sleeping on a sunny branch. How many squirrels were on the branch?

57. Helen was counting the spots on the dominoes she had in her hand. On the first there were 0 spots, on the next 1, and on the next 7. How many spots were there on all her dominoes?

58. Over in the oak woods stood some old hollow trees. In the first hollow tree 2 squirrels were living. In the next 7 squirrels were living, and in the next no squirrels at all were living. How many squirrels were living in the hollow trees?

59. Brother Bear went fishing, but he caught no fish. Brother Rabbit went, and caught 8. Brother Fox went, and caught 1. How many fish did they all catch?

60. One day Henry sold 6 quarts of cherries; the next day he sold none; and the third day he sold 2 quarts. How many quarts of cherries did he sell all together?

61. The first week of school our class had 6 tardy marks, the second week it had 1, and the third week it had none. How many tardy marks did our class then have?

The Progress Test

At the conclusion of the work on column addition of the kind set forth in this chapter it is recommended that the pupils be given a test such as the one presented below. This test is divided into two parts, each consisting of fifteen abstract examples and two verbal problems. The material in the two parts affords a test of all the fifty-five addition combinations thus far taught. The distribution of occurrences of these combinations is such that none appears more than twice.

It is recommended that Parts I and II be given on different days, that if at all practicable each part be mimeographed separately, and that the pupils be required to write the answers in the proper places on the mimeographed sheets. (Observe that answers to verbal problems are to be written in the squares marked "Ans.") Of course the test can be put on the board, but the labor of copying it is greater than one likes to impose upon such young children. Perhaps if the test is written on the board it should be taken in three parts rather than in two. The verbal problems may be dictated to the pupils, or read by them as their ability in reading seems to warrant.

A class record sheet, if made to accompany this test, will have a column for each example or problem and a line for each pupil (see page 52). A sheet should be made for each part. The analysis afforded by such a sheet should be supplemented by interviews with the pupils. The point is that each example or problem involves not only the peculiar difficulties of column addition — such as holding the unseen sum in mind, keeping the attention concentrated, and writing the answer correctly — but also a knowledge of two combinations. As to the combinations, we cannot be sure when a child fails whether he has failed on one combination or the other. In the example

$$\begin{array}{r} 2 \\ 1 \\ \underline{5} \end{array}$$

if the pupil gets 9 as an answer we cannot be sure, again limiting ourselves to a consideration of the combinations, whether he made a mistake on $\overset{2}{\underline{1}}$ or $\overset{3}{\underline{5}}$. The way to determine this, after having located the particular columns on which the pupils have made mistakes, is to take the pupils individually and have them add these columns aloud.

Test on Column Addition of Type A

PART I

2	1	2	0	3
1	3	3	8	4
$\underline{5}$	$\underline{0}$	$\underline{1}$	$\underline{0}$	$\underline{2}$
3	0	1	1	0
6	1	4	0	5
$\underline{0}$	$\underline{6}$	$\underline{4}$	$\underline{8}$	$\underline{4}$
0	3	0	2	1
2	1	3	0	2
$\underline{2}$	$\underline{4}$	$\underline{3}$	$\underline{7}$	$\underline{5}$

1. Harry brought home 4 cows, William brought home 2, and Jim brought home 3. How many did they all bring home?

Ans. ____

2. Mother ate 1 egg for breakfast, father ate 2, and little Polly ate 1. How many eggs did they all eat?

Ans. ____

PART II

5	0	4	4	1
2	7	2	5	7
1	0	1	0	1
4	3	5	6	1
3	0	0	0	1
2	2	3	1	4
0	1	0	2	0
4	5	0	6	6
1	3	9	1	2

1. We learned to spell 2 words yesterday. Today we are learning 5, but tomorrow we shall learn none. How many new words does that make all together?

Ans. ____

2. An apple cost 2 cents, an orange 3 cents, and a top 4 cents. How much did they all cost?

Ans. ____

CHAPTER XVII

HIGHER-DECADE ADDITION

In our decimal system numbers appear in *decades*. Those from zero to nine are in the lowest decade. Those from ten to nineteen are in the next decade; the twenties constitute the next decade, and so on. All the decades above the lowest — that is, from the 'teens to the nineties inclusive — may be called *higher decades*. Any number, therefore, from ten to ninety-nine inclusive is a higher-decade number.

The Nature and Value of Higher-Decade Addition

We are often called upon to add a one-digit number to a higher-decade number. Not only do many numerical facts come to us in this form, but column addition also constantly calls for combinations of two digits with one digit. So also does carrying in the process of multiplication. We usually add such numbers by a single act of combining rather than by a more detailed process. Moreover, we do this mentally rather than in writing. For example, if we have to add 23 and 6, we know that 3 and 6 are 9; that we start in the twenties; and that the result must be in the same decade, since the sum of 3 and 6 is *less than 10*. We therefore think 29 at once instead of thinking "3 and 6 are 9; write 9; 2 and 0 are 2; write 2; answer, 29." Similarly, if we have to add 23 and 8, we remember that 3 and 8 are 11; note that we begin in the twenties; and realize that the result is going to be in the next higher decade, since the sum of 3 and 8 is 10 or more. Accordingly, we reach the sum, 31, by a direct

process. In fact, if one has good control of the number facts, there is practically no *process* at all. Addition such as we have been describing should be performed much the same as addition of the basic one-digit numbers, namely, by a single direct response.

This kind of addition we shall call "higher-decade addition." It is sometimes called "addition in series" or "addition by endings." As to the numbers involved, it is finding the sum of a two-digit number and a one-digit number. As to the procedure, it is a direct combining of the numbers as they are given rather than a detailed process.

Higher-decade addition is not a form of column addition. We do not want the child in adding 36 and 2 to add 6 and 2 first and then "bring down" the 3. We want him to know "36 and 2 are 38" *as a fact.* Do not, therefore, write $\begin{array}{r} 36 \\ \underline{2\,,} \\ 38 \end{array}$ but rather $36 + 2 = 38$, thus making systematic use of the signs for "plus" and "equals" presented in Chapter XIII. The horizontal form of expression is preferable. Neither in column addition nor in multiplication, the processes in which higher-decade addition is most used, does the two-place number appear either above or under the one-place number. In fact, the two-place number does not appear at all. Manifestly, we cannot in written work avoid presenting it to the eye, but by presenting it together with the one-place number in horizontal form, we at least avoid putting in the child's hands a crutch which he will not be able to use. The test form should be "36 and 2 are?" or "$36 + 2 = ?$". Much oral work should be done in higher-decade addition; likewise in games where the two-place number is invisible and the one-place number is in view.

A word as to the order in which the addends should be written and said. The two-place number should usually pre-

cede the one-place number. This is because it always comes first in column addition and in carrying when multiplying. Under other circumstances, however, we may have the order reversed; that is, "2 + 36" as well as "36 + 2" should occasionally be used. When the one-place number is written first, the pupils should be taught to reverse the numbers before trying to give the answer. In this connection the teacher should bring out the fact, already abundantly evident in the basic combinations, that the order in which two numbers are added is immaterial so far as the answer is concerned.

The Analysis of Higher-Decade Combinations

Higher-decade addition may be divided into that which involves bridging the tens and that which does not involve bridging the tens. The combination 48 + 4 involves bridging the tens, that is, going into the next higher decade; but 23 + 6 does not involve bridging the tens.

Now, when higher-decade addition does not involve bridging the tens, the sum of the unit figures is always less than 10. All combinations whose sums are less than 10 have now been taught. Accordingly, all the basic facts that can ever occur in higher-decade addition without bridging have been provided for. It appears, therefore, that there is a natural division in higher-decade addition (bridging and not bridging) and that this division corresponds precisely to the division we have made between the easy and hard basic combinations. This chapter is limited to a consideration of the higher-decade addition *which does not involve bridging the tens.*

Like column addition, as treated in the last chapter, higher-decade addition permits of drill on the basic combinations[1] while at the same time it involves a new point of view. It is therefore especially valuable as drill material.

[1] Strictly speaking, this drill is not directly on the basic combinations. It does not repeat them literally. It repeats them as they appear in higher decades.

Higher-decade addition is something more, however. Arguments have been presented to show that it is sufficiently different from related basic combinations so that proficiency in the latter does not guarantee proficiency in the former.[1] These higher-decade combinations must be specifically learned. Of course this learning is greatly facilitated by knowledge of the related basic combinations. Moreover, the facts in higher-decade addition run in series; and this also makes learning easy. One such series is $16 + 3$, $26 + 3$, $36 + 3$, and so forth. If a knowledge of 6 and 3 does not guarantee a knowledge of 16 and 3, it at least greatly lessens the labor of acquiring that knowledge. Moreover, if a child has learned not only 6 and 3 but also 16 and 3, it is certainly not much of a task to learn the rest of the series. In fact, as each successive item in a series is learned the remaining items in that series are no doubt more easily mastered.

Of higher-decade combinations whose sums are less than 100, there are in all 765. This includes both those which involve bridging and those which do not. Each of the nine one-digit numbers may be added to each two-digit number from 10 to 90 inclusive.[2] This makes 81 times 9, or 729, combinations. Then there are 36 more, representing the numbers which may be added to 91, 92, $\cdots$, 98 without exceeding 99. The total, then, is 765. Of these higher-decade facts, 405 do not involve bridging the tens; and it is only these 405 that concern us in this chapter.

The 135 combinations that are most useful in column addition. The higher-decade combinations, whether with or without bridging, are not all equally useful. Their greatest use is in

[1] W. J. Osburn, Corrective Arithmetic, pp. 10–25. Houghton Mifflin Company, Boston, 1924.

[2] Combinations in which the one-digit number is zero are disregarded. No account is taken of such combinations as $36 + 0$ or $48 + 0$. Higher-decade addition finds its chief use in column addition and in carrying in multiplication. In either case such combinations as we have just mentioned are not thought of. If zero appears in a column we omit it. It cannot appear at all in carrying.

column addition and in carrying in multiplication. Now, most of the columns we have to add are short. Seldom do their sums exceed forty; and unless they do we have no need for addition in decades above the thirties.

Let us then make special provision in our arithmetic teaching for all higher-decade combinations whose sums are less than 40. There are 225 of them, 9 each for the numbers from 10 to 30 inclusive, and 36 more for the numbers from 31 to 38 inclusive. And among these let us provide *in this chapter* for those which do not bridge the tens. There are 135 of them. Each of the numbers from 1 to 9 may be added to 10 without bridging into the 20's. Similarly, each of the numbers from 1 to 8 may be added to 11, those from 1 to 7 may be added to 12, and so on. This provides for 45 combinations based on the 'teens; and the same is true for the 20's and 30's; total, 135.

The 52 additional combinations used in multiplication. In addition to these 135, there are 52 more combinations to which we must give special attention in this chapter. This is because of their frequent use in multiplication. In that operation we do not have to carry — that is, add — anything to any two-digit number unless that number is a product in the multiplication table. We may have to add 5 to 42, as in the example $\begin{array}{r} 68 \\ \times\ 7 \\ \hline \end{array}$. But, we never have to add 5 to 41, for example, because 41 does not occur in the multiplication of whole numbers.

Thus, we must be sure to provide for $42 + 5$. With respect to $41 + 5$, however, we may content ourselves with less practice, relying upon the transfer from $1 + 5$ fortified — as it will be if we give special attention to the facts used in multiplication — by drill on $21 + 5$ and $81 + 5$.

How to identify the combinations used in multiplication. Every primary teacher should know precisely what the higher-decade addition combinations are that are used in multiplication. A

large proportion of the mistakes children make in multiplying are made in the carrying process. The teacher should be equipped to forestall these errors. She cannot do so unless she knows the combinations used.

Ten is the smallest higher-decade number. It is also a product in the multiplication table. To it we may have to add, or "carry," in multiplication 1, 2, 3, or 4. For example, in $\begin{array}{r} 56 \\ \times\ 2 \end{array}$ we have $10 + 1$; in $\begin{array}{r} 24 \\ \times\ 5 \end{array}$ we have $10 + 2$; in $\begin{array}{r} 27 \\ \times\ 5 \end{array}$ we have $10 + 3$; and in $\begin{array}{r} 29 \\ \times\ 5 \end{array}$ we have $10 + 4$. One cannot, however, make a multiplication example in which a number as large as 5 will be added or carried to 10. Note in this connection that 5, the limiting number in carrying, is also the larger of the two factors of 10. After 10 the next larger multiplication-table product is 12; and to it we may have to add, or carry, 1, 2, 3, 4, or 5. Five is the largest number we shall have to add. Note that 5 is one less than 6, the larger factor in 12 (regarded as equal to 6×2). Similarly, it will be found that to 14, the next larger product, we shall have to add 1, 2, 3, 4, 5, and 6, 6 being one less than 7, the larger factor in 14. This rule holds generally.

Accordingly, we may make a table of the "higher-decade addition combinations, used in multiplication, which do not involve bridging the tens." Such a table is presented as Table III. Note that it is divided by a dotted line. The combinations above this line have sums less than 40; and there are 63 of them. They are included in the 135 which, as we have already shown on pages 194–195, comprise *all* the higher-decade combinations whose sums are less than 40 and which do not involve bridging. Since these 63 are already included in the 135 which are so common in column addition, we may as well leave the 63 out of account. They will be cared for, but the 52 combinations below the dotted line in Table III

TABLE III. HIGHER-DECADE ADDITION COMBINATIONS, USED IN MULTIPLICATION, WHICH DO NOT INVOLVE BRIDGING THE TENS

10 + 1	12 + 1	14 + 1	15 + 1	16 + 1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	18 + 1
	5	5		
20 + 1	21 + 1	24 + 1	25 + 1	27 + 1
2	2	2	2	2
3	3	3	3	
4	4	4	4	
	5	5		
	6			
28 + 1	30 + 1	32 + 1	35 + 1	36 + 1
	2	2	2	2
	3	3	3	3
	4	4	4	
	5	5		
		6		
		7		
40 + 1	42 + 1	45 + 1	48 + 1	54 + 1
2	2	2		2
3	3	3		3
4	4	4		4
5	5			5
6	6			
7				
56 + 1	63 + 1	64 + 1	72 + 1	81 + 1
2	2	2	2	2
3	3	3	3	3
	4	4	4	4
	5	5	5	5
	6		6	6
			7	7
				8

must be given special attention. They occur constantly in multiplication; and, their sums being over 40, they must be considered in addition to the 135 already provided for.

Summary of the analysis. This whole matter is summarized in Table IV. Only the 405 combinations "without bridging" are to be regarded in this chapter, because those involving bridging demand a knowledge of combinations not yet taught. Within the 405 the 135 whose sums are less than 40 are to be given special attention, as are the 52 additional combinations used in multiplication.

TABLE IV. HIGHER-DECADE ADDITION

	WITHOUT BRIDGING	WITH BRIDGING	ALL
Very common			
a. Combinations whose sums are less than 40	135	90	225
b. Additional combinations used in multiplication	52	35	87
Not as common as the above; that is, sums 40 or more and not used in multiplication	218	235	453
Total	405	360	765

You can easily tell when a combination is one of the 135. Its sum is less than 40 and is in the same decade with the two-digit addend; such as $34 + 4$ or $17 + 2$. A combination like $41 + 6$ does not belong to the 135 because its sum is not less than 40; and $28 + 5$ does not belong to the 135 because, although its sum is less than 40, it involves bridging. You can also tell when a combination is one of the 52 by studying Table III (below the dotted line). The conditions are that its sum is 40 or more, its two-digit addend is a product in the multiplication table, and its one-digit addend is less than the larger factor of this product.

Teaching Higher-Decade Addition

It is suggested that the first higher-decade combination be presented in connection with a problem. For example: "Harry had 24 cents and earned 3 cents more. How many cents did he have then?"

Take real or toy cent pieces. Have a child count 24 of them, and perhaps have another child verify his count. Let these represent the 24 cents which Harry had at first. Now conceal this amount and let 3 cents be counted out as the number of cents he earned. Conceal these with the 24 and then show the total number. The children may count and report that there are 27 cents. Therefore 24 cents and 3 cents are 27 cents.

Teacher. Then twenty-four and three are how many?

Pupil. Twenty-seven.

Teacher. How many are four and three?

Pupil. Seven.

Teacher. How many are *twenty*-four and three?

Pupil. Twenty-seven.

Teacher. How many would *thirty*-four and three be?

Probably a bright child will answer "*Thirty*-seven."

Similarly, the teacher, largely for the sake of the "swing," may ask for *forty*-four and three, *fifty*-four and three, and *sixty*-four and three. The point should be brought out that 24 and 3 are 27, 34 and 3 are 37, etc., because 4 and 3 are 7.

Teacher. How much are four*teen* and three?

Pupil. Seventeen.

The four facts whose sums are less than forty may now be written on the board in horizontal form beginning with the basic fact:

$$4 + 3 = 7$$
$$14 + 3 = 17$$
$$24 + 3 = 27$$
$$34 + 3 = 37$$

These facts should be read by several pupils, each one reading the entire series. The object is to have the children *infer* the higher-decade facts as extensions of the corresponding basic fact.

The teacher may now take up another series, such as $1 + 4$, $11 + 4$, $21 + 4$, $31 + 4$, and work toward the same kind of inference. This should be continued until the children show their understanding of the principle involved. As a test, one child may give a basic fact and another may build up the related series. It is suggested that a variation of the idea of "telling the whole story" be employed. Thus the whole story for $3 + 6$ would be "$3 + 6 = 9$; $13 + 6 = 19$; $23 + 6 = 29$; and so on."

When the process is reversed and a single higher-decade combination is given, such as $22 + 6$, the pupils should be taught to select the right basic combination, $2 + 6$. If in the effort they are making to meet the new demands they forget the answer to $2 + 6$, they have the posted combinations to help them. Having thought "$2 + 6 = 8$" they should be able to make the inference "$22 + 6 = 28$."

All the higher-decade combinations whose sums are less than 40 should be given considerable drill. The number chart will be found very helpful. The work will be within the second, third, and fourth columns. With respect to the remaining columns, you may well underscore in the chart the numbers which are products in the multiplication table. Nothing need be said at the time as to the reason for this, and indeed if you do not need this visual aid yourself there is no reason why the figures need be underlined. You should, however, have in mind that the addition of such numbers as will not lead to a bridging of the tens should frequently be made to the multiplication-table products, that is, to 40, 42, 45, 48 (49 cannot be included because any addition would bridge the tens), 54, 56, 63, 64, 72, and 81.

It is not intended that the 218 combinations (see Table IV) which are "not as common" should be omitted from the drill, but the 187 very common facts should receive the most drill.

Practice Material, Higher-Decade Addition

Below are given several sets of examples involving higher-decade addition without bridging the tens. Some of them should be done orally and some in writing. In Section I the sums are less than 40. In Section II the sums are 40 or more and the combinations are those used in multiplication. In Section I the amount of practice afforded on each of the 135 combinations is practically the same. Each of them occurs either three or four times, and these occurrences are so distributed that the same combination never appears twice in the same set. If the combinations having the same basic fact, such as $12 + 5$, $22 + 5$, and $32 + 5$, are regarded as the same, each combination occurs not less than ten nor more than twelve times. This means that each basic fact is practiced in Section I from ten to twelve times. Section II, consisting of 160 examples, presents each of the 52 combinations used in multiplication three or four times.

Section I. Sums less than 40

a

$10 + 5$	$13 + 2$	$31 + 7$	$11 + 7$	$13 + 6$
$1 + 26$	$31 + 8$	$23 + 1$	$30 + 6$	$34 + 3$
$30 + 7$	$23 + 6$	$31 + 3$	$37 + 1$	$5 + 23$
$13 + 3$	$12 + 6$	$4 + 22$	$17 + 2$	$4 + 33$
$36 + 3$	$13 + 4$	$2 + 11$	$21 + 8$	$17 + 1$
$33 + 5$	$34 + 1$	$13 + 5$	$38 + 1$	$31 + 5$
$8 + 11$	$20 + 6$	$34 + 2$	$20 + 7$	$37 + 2$
$33 + 6$	$7 + 10$	$22 + 3$	$33 + 1$	$20 + 8$
$10 + 8$	$34 + 4$	$22 + 6$	$11 + 3$	$31 + 1$
$9 + 30$	$22 + 5$	$11 + 1$	$23 + 4$	$30 + 8$

b

10 + 9	34 + 5	11 + 4	20 + 5	4 + 31
23 + 3	22 + 7	3 + 26	31 + 2	7 + 12
6 + 10	11 + 5	25 + 3	10 + 2	3 + 35
35 + 1	30 + 2	15 + 3	10 + 4	6 + 21
24 + 4	16 + 2	20 + 3	30 + 1	4 + 14
26 + 2	13 + 1	2 + 22	11 + 6	33 + 2
20 + 9	31 + 6	22 + 1	21 + 7	23 + 2
12 + 5	15 + 2	10 + 3	21 + 3	24 + 1
25 + 2	20 + 4	21 + 5	7 + 32	12 + 1
12 + 3	20 + 1	21 + 4	10 + 1	1 + 21

c

12 + 2	30 + 4	1 + 36	32 + 6	21 + 2
3 + 36	15 + 1	24 + 5	32 + 1	30 + 3
32 + 2	15 + 4	36 + 2	30 + 5	2 + 24
25 + 1	14 + 2	5 + 32	16 + 3	14 + 1
37 + 1	30 + 9	22 + 1	31 + 7	5 + 23
12 + 4	2 + 27	18 + 1	25 + 4	14 + 3
24 + 3	5 + 14	28 + 1	32 + 3	35 + 4
35 + 2	16 + 1	27 + 1	20 + 2	4 + 32
33 + 5	31 + 3	17 + 1	6 + 31	13 + 6
22 + 5	21 + 7	13 + 2	33 + 6	1 + 11

d

23 + 2	34 + 1	10 + 7	23 + 4	10 + 6
33 + 1	10 + 9	3 + 25	38 + 1	20 + 8
35 + 3	26 + 1	13 + 3	20 + 5	12 + 5
17 + 2	2 + 26	10 + 3	23 + 1	33 + 4
24 + 1	34 + 3	11 + 6	34 + 4	35 + 1
5 + 13	22 + 3	30 + 8	11 + 5	5 + 10
34 + 5	10 + 2	5 + 31	12 + 6	11 + 4
8 + 31	22 + 4	31 + 4	2 + 15	11 + 7
13 + 1	21 + 3	6 + 30	10 + 8	22 + 2
11 + 8	6 + 22	33 + 2	30 + 2	6 + 20

e

11 + 3	23 + 3	15 + 3	34 + 2	31 + 1
21 + 6	2 + 37	13 + 4	31 + 2	25 + 2
21 + 8	20 + 9	21 + 5	7 + 32	30 + 3
28 + 1	20 + 3	32 + 3	30 + 1	35 + 4
2 + 36	21 + 4	30 + 5	10 + 1	2 + 24
7 + 22	10 + 4	20 + 7	3 + 33	26 + 3
30 + 7	11 + 2	7 + 12	20 + 4	23 + 6
12 + 1	3 + 24	14 + 5	24 + 4	16 + 2
4 + 14	32 + 2	3 + 12	15 + 4	20 + 1
21 + 1	2 + 35	12 + 2	16 + 1	30 + 4

f

27 + 1	20 + 2	36 + 1	6 + 32	21 + 2
32 + 5	18 + 1	3 + 16	25 + 4	14 + 1
33 + 5	17 + 1	13 + 6	9 + 30	31 + 7
13 + 5	30 + 8	10 + 5	10 + 9	38 + 1
31 + 8	4 + 31	11 + 7	26 + 2	23 + 1
25 + 1	12 + 4	14 + 2	2 + 27	32 + 4
14 + 3	32 + 1	5 + 13	15 + 1	24 + 5
22 + 5	13 + 2	1 + 11	34 + 1	4 + 23
34 + 5	5 + 31	11 + 4	1 + 26	20 + 5
13 + 1	30 + 6	2 + 22	34 + 3	11 + 6

g

11 + 8	33 + 2	20 + 6	23 + 3	2 + 34
30 + 7	7 + 12	23 + 6	20 + 9	31 + 3
6 + 33	23 + 2	10 + 7	10 + 6	22 + 3
12 + 6	3 + 35	13 + 3	12 + 5	22 + 4
10 + 8	24 + 1	34 + 4	35 + 1	22 + 6
22 + 7	20 + 7	3 + 26	37 + 2	31 + 2
6 + 31	37 + 1	22 + 1	23 + 5	21 + 7
11 + 5	33 + 1	3 + 25	20 + 8	10 + 2
15 + 2	2 + 17	10 + 3	33 + 4	21 + 3
30 + 2	3 + 11	3 + 15	31 + 1	10 + 4

h

3 + 33	21 + 6	4 + 13	25 + 2	11 + 2
24 + 4	20 + 3	4 + 14	20 + 1	10 + 1
27 + 2	25 + 4	36 + 3	24 + 5	30 + 3
24 + 2	16 + 1	20 + 2	25 + 1	5 + 32
12 + 3	21 + 4	21 + 1	30 + 4	6 + 32
30 + 5	2 + 35	27 + 1	32 + 4	14 + 2
20 + 4	21 + 8	21 + 5	33 + 2	32 + 1
12 + 2	7 + 32	36 + 1	21 + 2	24 + 3
5 + 14	32 + 3	36 + 2	2 + 32	28 + 1
14 + 1	12 + 1	2 + 16	30 + 1	35 + 4
12 + 4	1 + 18	14 + 3	15 + 1	15 + 4

i

33 + 5	31 + 6	13 + 6	22 + 1	31 + 7
5 + 13	11 + 5	10 + 5	25 + 3	38 + 1
31 + 8	15 + 2	7 + 11	10 + 3	23 + 1
11 + 8	30 + 2	20 + 6	3 + 15	34 + 2
30 + 7	20 + 4	23 + 6	21 + 5	3 + 31
7 + 21	13 + 2	2 + 23	34 + 1	10 + 6
10 + 2	31 + 5	3 + 35	26 + 1	12 + 5
21 + 3	6 + 30	24 + 1	34 + 3	35 + 1
10 + 4	20 + 7	6 + 21	37 + 2	25 + 2
33 + 6	33 + 1	3 + 13	33 + 4	22 + 6

j

33 + 3	21 + 8	17 + 1	11 + 1	9 + 10
37 + 1	30 + 9	10 + 7	4 + 23	20 + 8
11 + 3	23 + 3	13 + 4	2 + 31	23 + 5
17 + 2	2 + 26	34 + 4	11 + 6	31 + 1
20 + 5	13 + 1	30 + 1	3 + 26	20 + 9
34 + 5	22 + 4	4 + 31	10 + 8	22 + 2
22 + 5	3 + 22	30 + 8	12 + 6	11 + 4
22 + 7	11 + 2	12 + 7	7 + 32	30 + 3

Section II. Sums 40 or more. Combinations used in multiplication

a

40 + 1	45 + 1	63 + 2	48 + 1	81 + 2
81 + 1	63 + 1	64 + 1	72 + 2	40 + 4
42 + 2	54 + 4	40 + 2	56 + 2	42 + 1
40 + 3	54 + 3	72 + 3	42 + 3	42 + 4

b

64 + 2	63 + 4	72 + 4	40 + 5	81 + 5
72 + 5	42 + 5	63 + 5	64 + 4	45 + 3
63 + 3	81 + 3	81 + 4	64 + 3	45 + 2
81 + 6	81 + 7	72 + 6	54 + 1	40 + 6

c

40 + 7	42 + 6	56 + 3	54 + 2	81 + 8
56 + 1	40 + 1	45 + 1	63 + 2	48 + 1
72 + 1	45 + 4	64 + 5	63 + 6	72 + 7
54 + 5	42 + 2	54 + 4	40 + 2	56 + 2

d

81 + 2	81 + 1	63 + 1	64 + 1	72 + 2
40 + 4	64 + 2	63 + 4	72 + 4	40 + 5
42 + 1	40 + 3	54 + 3	72 + 3	42 + 3
42 + 4	63 + 3	81 + 3	81 + 4	64 + 3

e

81 + 5	72 + 5	42 + 5	63 + 5	64 + 4
45 + 3	40 + 7	42 + 6	56 + 3	54 + 2
45 + 2	81 + 6	81 + 7	72 + 6	54 + 1
40 + 6	72 + 1	45 + 4	64 + 5	63 + 6

f

81 + 8	56 + 1	40 + 1	45 + 1	63 + 2
48 + 1	81 + 2	81 + 1	63 + 1	64 + 1
72 + 7	54 + 5	42 + 2	54 + 4	40 + 2
56 + 2	42 + 1	40 + 3	54 + 3	72 + 3

g

72 + 2	40 + 4	64 + 2	63 + 4	72 + 4
40 + 5	81 + 5	72 + 5	42 + 5	63 + 5
42 + 3	42 + 4	63 + 3	81 + 3	81 + 4
64 + 3	45 + 2	81 + 6	81 + 7	72 + 6

h

54 + 1	40 + 6	72 + 1	45 + 4	64 + 5
63 + 6	72 + 7	56 + 1	54 + 4	72 + 4
64 + 4	45 + 3	40 + 7	42 + 6	56 + 3
54 + 2	81 + 8	54 + 5	56 + 2	64 + 3

Illustrative Problems

In the following problems all the basic combinations thus far taught, except the ten having zero as the second number, are represented in corresponding higher-decade addition. Each of the more difficult basic facts is represented twice.

1. All the children had to write some words. Ethel had 45 right. "I must do better than that," she said. So the next time her teacher told her she had 4 more right. How many words did Ethel have right that time?

2. Clara's hair ribbon was 27 inches long. She said she wished it were 2 inches longer. How many inches long did she want her ribbon to be?

3. Ray spent 10 cents for crackajack and 2 cents for a balloon. How many cents did he spend?

4. Ralph went to the store to buy a dozen eggs. He gave the grocer a quarter. The grocer said: "Eggs cost more than they did last week. I need 4 cents more." How much did the eggs cost?

5. A bushel basket holds 32 quarts. In the market Joe saw a bushel basket and 4 quart boxes full of nuts. How many quarts of nuts did he see?

6. Mother stayed at Aunt Sarah's 63 days. To go there and come back took 4 days. How many days was Mother away from home?

7. Jane had a dime and earned a cent. How many cents had she then?

8. One spring day 24 children and their teacher went into the woods to learn about birds. How many people went into the woods?

9. Ruth went to the store and bought a bag of sugar for 42 cents and a cake of soap for 6 cents. How many cents did they both cost?

10. Jim had sold 64 papers and was saying to himself, "I guess that will be all for tonight." Just then some men came along and bought 3 more. How many papers did Jim sell that night?

11. Susie had 63 cents in her toy bank. Brother Bill dropped in 3 cents more. How many cents did Susie then have in her box?

12. Cherries are picked into quart boxes. Then the quart boxes are put into a large box called a crate. A crate holds 24 quart boxes. Fred filled a crate and then set 3 more quarts on top of the crate. Then he said, "O father, see how many quarts of cherries I have picked." How many quarts *had* he picked?

13. Jack was going to have his birthday party in the grove. His mother made 36 sandwiches. Jack said, "O mother, I'm so hungry; put in just 2 more." So his mother did. How many sandwiches did Jack take?

14. In the camp over by the lake there are 3 men and 56 boys. How many people are there in the camp?

15. Last year Harry went to Sunday school 42 Sundays. Then he was sick and missed several times, but after he got well he went 4 Sundays more. How many Sundays did he go to Sunday school?

16. An old mother duck said she had 18 pretty children and 1 ugly one. How many children did she have?

17. At first there were 40 children in our class. One day 17 more came in because their school had been closed. How many were in our class then?

18. Polly was sitting in the shoe store while her mother tried on shoes. Polly counted 54 white shoe boxes on one side of the store. Then the shoe man put up 2 more. How many white shoe boxes were on that side of the store?

19. There were 31 pupils in a first grade the first day of school. The next day 2 new pupils came. How many pupils were then in the first grade?

20. James paid 42 cents for sugar and 3 cents for a yeast cake. How many cents did he pay for both?

21. Mr. Blake drove his automobile 32 miles. Then he stopped to get some gasoline. After that he drove on 7 miles more. How many miles did he drive?

22. At a Mothers' Meeting 27 school children were standing at the front of the room, ready to sing a school song. Then 1 child that was late came hurrying up, just in time to sing. How many children were there to sing the song?

23. Baby is 14 weeks old. In 2 weeks he will go away to the country with his mother and brother. How many weeks old will Baby be when he goes to the country?

24. Mrs. Black had 7 oranges in the house and bought a dozen more. How many oranges did she then have?

25. Many children in Miss Gray's class had been measured. So when Catherine went home from school she measured her dolls. Mary Jane was 1 foot and 2 inches tall. How many inches tall was Mary Jane?

26. Catherine found that her other doll, Priscilla Ann, was 1 foot and 4 inches tall. How many inches tall was Priscilla Ann?

27. Hal saw Walter carrying in the eggs. "How many eggs did you get today, Walter?" called Hal. "A dozen and 5," answered Walter. How many eggs was Walter carrying in?

28. Aunt Polly made a pretty new pillow to put in Betty's arm-chair. Betty measured her pillow and found that it was 1 foot and 3 inches long. How many inches long was it?

29. One day Henry and Uncle John went to town to sell pumpkins. One lady bought a dozen because she wanted to give some to her friends. The lady at the next house bought 6. How many pumpkins did both ladies buy?

30. On the station platform there were 33 cans of milk to be carried to the city. Tom and Uncle Ben drove up and put their 6 cans on the platform. How many cans were on the platform then?

31. A farmer put 3 bags of apples and 16 bags of potatoes on a truck. How many bags of both were there?

32. Mr. Green drove 72 miles in his automobile. Then he stopped to eat lunch. After that he drove 7 miles more. How many miles did he drive in all?

33. Philip's mother kept boarders. Philip went to the store and bought 72 oranges for his mother, and the grocer gave him 1 for himself. How many oranges did Philip get?

34. There were 21 children going to have a picnic. Then they remembered that 4 children had just come to live in a house near by. So they took them along, too. How many children were going to the picnic?

35. There were 14 boys building snow forts in the school yard; 4 more boys went out to help them. How many boys were then building snow forts?

36. Alice read 7 pages of her new book yesterday and 21 pages today. How many pages has she read?

37. A farmer had 63 pear trees and planted 6 more. How many pear trees did he have in all?

38. Allen lived in the country. He went to the city to visit his cousin Fred. Fred said, "Let's go out for a trolley ride. I'll take you to the tallest building in the city." Allen said, "All right." So they rode 21 blocks. Then Allen said, "Here it is. Let's get off." Fred said, "No, that's not it. We have to ride 3 blocks more." How many blocks did they ride?

39. Little Dick made a play church with 24 blocks. Then he put on 4 blocks for the steeple. How many blocks did it take all together to make the church?

40. One afternoon in the spring Jean came to visit Polly. She said, "I have seen 21 robins this spring." Polly said, "I have seen only 5." How many robins had Jean and Polly seen?

41. Over on the hillside stood 45 pine trees and 2 cedar trees. How many trees stood on the hillside?

42. The day before Easter there were 23 tulips blooming in Lena's garden. The next morning 4 more came out. How many tulips did Lena have for Easter?

43. Lucy put 33 beads on a string and thought that was all. Then she found 5 more to put on. How many beads were on the string when Lucy had finished?

44. Paul was going to carry his eggs to the store. He had 23 in a basket and his mother put in 1 more. How many eggs did Paul carry to the store?

45. Fanny helped Mrs. Clark, the dressmaker. She sewed on 32 buttons and was tired. Mrs. Clark said, "Sew on just 6 more and you will be through." So Fanny finished her work before she stopped. How many buttons did Fanny sew on?

46. Mrs. Green spent 64 cents for the things she wanted for making ice cream. Then she found she had not enough salt to put into her freezer, so she had to spend 5 cents for that. How many cents did she spend for her ice cream?

47. There were 54 people in a railroad car when the train stopped at the station. Nobody got out, but 5 people got on. How many people were in the car when the train went away?

48. One bright spring morning 2 teachers and 56 pupils went out to look for wild flowers. How many people were out looking for wild flowers?

49. Billy Brown went to see a moving picture. While he was there he counted the people. There were 72 who had come to see the picture and 5 boys who helped them find seats. How many people were at the picture show?

50. I saw 30 white geese and 6 gray geese. How many geese did I see?

51. George worked last summer, and when winter came he had saved enough money to buy a pair of shoes that cost 8 dollars and a coat that cost 20 dollars. How many dollars had he saved?

52. Going to Uncle Will's farm.

CHARACTERS: CLARA, her Mother and Father, UNCLE WILL, and AUNT MARY.

TIME: *Morning.*

MOTHER. [*Pretending to be putting a few last things in an imaginary bag*] Clara! [*Pause.*] Clara! [*In a calling voice*] We must start

now. Close your bag and put on your hat and wraps. Father will walk over to the train with us.

[*They all start, Father carrying Mother's bag and Mother carrying Clara's (or any other arrangement). The children walk to some assigned part of the room; Father helps them up an imaginary platform and into a seat (in a railroad car). They say "Good-by" and Father adds, "Don't get tired, Clara, you will have a long ride." He hurries out of the train, down the platform, goes home, and sits down to rest and think.*]

CLARA. Is it a long ride in the train, mother?

MOTHER. Yes, it is eighty-one miles.

[*At length they reach the station of Shady Dell and get off. In his buggy sits Uncle Will, who gets out and is glad to see them. He helps them in, pulls up his reins, and Dobbin trots off home. Aunt Mary appears.*]

AUNT MARY. Oh, I am *so* glad you are here at last, just in time for dinner. Clara, you have had a long ride, haven't you?

CLARA. Oh, yes, Aunt Mary. I rode eighty-one miles on the train.

AUNT MARY. Oh, my dear, eighty-one miles, and one mile with Dobbin and the buggy! Why, you must be hungry, for you have ridden eighty-two miles since breakfast!

[*Dobbin is left under the tree for the present, and they all disappear into the house.*]

53. Alfred went to the store to buy some groceries. "How much are they?" he asked. "They are 81 cents," said the grocer. "Oh, there," said Alfred, "I forgot! I want a pound of rice." "That will be 8 cents more," said the grocer. How many cents did all the things cost?

54. On Saturday 35 children and 3 teachers went to the park to see some baby lions. How many people went?

55. Some soldiers passed down our street. I counted 36 walking and 1 on horseback. How many men did I count?

56. Kate paid 25 cents for writing paper and 2 cents for a stamp. How many cents did she pay for both?

57. There were 2 big ducks and 27 little ducks in the pond. How many ducks were there in the pond?

58. Mary's big brother was 20 years old when he went away to war. He stayed away 3 years. How old was he when he came home?

59. Dora's father is 35 years old now. How many years old will he be in 1 year from now?

60. There are 15 houses in the block where Anna lives, and 3 new ones will soon be built. How many houses will there then be in the block?

61. Mrs. Brown had a dozen eggs in the house. Henry brought in 5 more. How many eggs did Mrs. Brown then have?

62. If a board is 1 foot and 2 inches wide, how many inches wide is it?

63. Walter's hens laid 13 white eggs and 5 brown ones. How many eggs did they lay?

64. At a Fourth of July picnic some boys shot off 11 little firecrackers and 6 big ones. How many firecrackers did they shoot off?

65. Mother said: "Bobbie, here are 30 cents. Run over to the store and get half a pound of tea. And here is a nickel more for you." How many cents did mother give Bobbie?

66. Forty little dandelions
Are shining in the grass.
Then four more dandelions came out.
Just count them as you pass.

How many dandelions are there?

67. Ruby's grandma is 63 years old. Her grandpa is 2 years older. How many years old is her grandpa?

68. Janice paid 40 cents for sugar and 9 cents for bread. How many cents did she pay for both?

69. Theodore picked 5 quarts of strawberries and Harriet picked 11 quarts. How many quarts did they both pick?

70. Harry went to the city to stay 21 days. His father wrote him saying that he might stay 6 days more. How many days could Harry stay in the city?

71. Stephen brought the eggs in from the barnyard and put 36 into a basket to send to town. He had 1 left. How many eggs did he find that day?

72. Philip put some large red apples on his stand by the roadside. On Saturday he sold 41 of them. On Monday he sold 8. How many did he sell all together?

73. If you ride 81 miles on the train and 7 miles in an automobile, how many miles do you ride all together?

74. Mother Duck sat on her nest 28 days and nearly all of her baby ducks had hatched out. Then she thought, "I will wait 1 day more for the others." How many days did she sit on the nest?

75. Carl told a lady that he had 15 quarts of nice ripe cherries to sell. The lady said she would buy them and 1 more quart. How many quarts of cherries did she want to buy?

76. Brother is 17 years old. In 1 year more he will go to college. How many years old will he be when he goes to college?

The Progress Test

As usual, in order to round off the work, we give a suggested test. It should be administered in two parts on successive days. The best way is to give the pupils mimeographed sheets and to have them insert the answers in the proper places. (Note the squares for the answers to verbal problems.) The verbal problems may well be read to the pupils while they follow the reading as they look at their own copies.

This progress test affords an opportunity to try out the pupils on all the basic addition facts thus far taught, except those in which zero is the second addend. This exception is rendered practically necessary because either in column addition or in multiplication such a combination as $\begin{array}{r} 21 \\ \underline{0} \end{array}$ is practically meaningless.

Do not forget to analyze the result by means of a class record sheet (see page 52) in which there is a column for

each item of the test and a line for each pupil. Two such sheets will be required, one for each part. Each sheet will have twenty-seven columns besides a column at the left for the pupils' names and one at the right for totals. Ordinary square-ruled paper with one-fifth-inch rulings will serve very well. When you insert the combinations at the tops of the columns, you will save space if you write them like this:

	40 + 7	21 + 6	15 + 3	56 + 2	etc.

Test on Higher-Decade Addition without Bridging

PART I

40 + 7 =	54 + 1 =	2 + 34 =	25 + 2 =
4 + 33 =	63 + 5 =	42 + 6 =	16 + 3 =
45 + 1 =	30 + 5 =	72 + 7 =	32 + 2 =
72 + 4 =	40 + 8 =	56 + 1 =	30 + 4 =
31 + 2 =	15 + 3 =	21 + 3 =	81 + 5 =
21 + 6 =	41 + 7 =	48 + 1 =	63 + 2 =

63 + 1 =

1. Helen bought a box of candy. She gave the storekeeper 25 cents. He said, "3 cents more, please." How many cents did the candy cost?

Ans. ____

2. Henry drew a line 1 foot long. How many inches long was the line?

Ans. ____

PART II

81 + 8 =	15 + 4 =	40 + 6 =	64 + 5 =
20 + 1 =	42 + 1 =	31 + 1 =	54 + 5 =
14 + 3 =	20 + 2 =	64 + 3 =	12 + 5 =
27 + 2 =	45 + 4 =	10 + 3 =	37 + 1 =
3 + 56 =	6 + 13 =	81 + 4 =	24 + 4 =
50 + 9 =	36 + 2 =	23 + 3 =	32 + 3 =
	33 + 5 =		

1. There were 37 children in our class. Today 2 new ones came in. How many children are there in our class now?

Ans. ____

2. Anna's mother sold 1 dozen and 4 eggs to Mr. Brown. How many eggs did she sell him?

Ans. ____

CHAPTER XVIII

HIGHER-DECADE SUBTRACTION

The same considerations which led us to recommend column addition and higher-decade addition before all the basic addition facts had been taught now lead us to recommend a similar extension of subtraction — an extension to certain examples having minuends of two digits. These examples are the kind which we ordinarily do mentally and by a single direct response, just as we respond to the basic combinations. There are two types, one in which the subtrahend is a one-digit number, the other in which the remainder is a one-digit number. Subtraction of this kind we shall call higher-decade subtraction. It is useful for its own sake. Life presents numerous opportunities and requirements in which such operations as $27 - 4$ or $48 - 42$ are called for. Moreover, the drill on the basic combinations which higher-decade subtraction affords is especially desirable since it involves repetition from a new point of view. Of course, at this juncture no examples or problems will involve basic combinations whose minuends exceed nine.

The argument for introducing this extension of subtraction at a time when only a part of the basic combinations have been taught has been suggested in the sections on column addition and higher-decade addition. It is the same argument which induces music teachers in these days to abandon the prolonged infliction of scales and velocity studies and to introduce instead at an early date the application of these fundamentals in attractive and useful form. It is the same argument which is leading you when you teach reading to prefer the

worth-while story, poem, or fable to an excessive treatment of the mechanics of reading. We see no reason why the pupil, having learned to read and write numbers to one hundred, should be debarred from applying any of these numbers. Is it desirable that for a long time — until all the basic combinations have been learned — such numbers as twenty-six, forty-two, and sixty-three should still be known merely as points in the series of numbers from one to one hundred?

Case I. Subtracting a One-Digit Number from a Two-Digit Number

The first case of higher-decade subtraction is that in which a one-digit number is taken from a two-digit number. Of course at this stage of the pupil's advancement there will be no bridging of tens, — no "borrowing," as we shall later call it. In other words, if the minuend is in the twenties, the difference will likewise be in the twenties; if the minuend is in the fifties, the remainder will likewise be in the fifties. Examples of Case I in which no bridging of tens occurs are

$$27 - 4 \qquad 48 - 2 \qquad 15 - 3 \qquad 38 - 8$$

There are a great many higher-decade subtraction facts of this kind; in fact, forty-five for each of the nine decades, or four hundred and five all together.[1] As time passes and as you continue to drill on these higher-decade subtraction facts, you will doubtless give some attention to all of them. It will be well, however, for you to give early attention to the lower decades, — say to the 'teens, twenties, and thirties. This is chiefly because small numbers occur so much more frequently in ordinary life. Moreover, after the combinations in the

[1] This neglects the taking of zero from a higher-decade number; for example, $15 - 0 = 15$, $62 - 0 = 62$, and the like. If these combinations were included, there would be 495 instead of 405; that is, 55 for each of the nine decades.

lower decades have been learned, the transfer to corresponding combinations in the higher decades is relatively easy. What we mean is this: A child may know that 7 less 4 are 3, without being able to tell us that 17 less 4 are 13, or that 27 less 4 are 23. But if he has been drilled on 17 less 4 and 27 less 4, thus having made what the psychologists call the transfer from the basic combination 7 less 4 to its application in the 'teens and twenties, he will probably be able to continue the application in still higher decades. Accordingly we are probably justified, on the basis both of social usage and the psychology of learning, in directing attention for the most part to the lower decades.

Case I of higher-decade subtraction is not so important as higher-decade addition, because it has no large use in other operations. Moreover, it is not so important as Case II, which, as we shall see, is useful in short division. But Case I is nevertheless sufficiently important to justify considerable attention. The life conditions are numerous which call for reducing a two-digit number by a one-digit number. Pupils should therefore be drilled in performing such operations not in writing but mentally and without breaking the process up into steps. When the combinations are written, they should be in horizontal form. In drill work use the number chart frequently.

Method of teaching. Begin with a problem. Be on the lookout for one to arise naturally — a situation in which the children, or some of them, really want to subtract as called for by Case I (without bridging). If a spontaneous situation does not arise, the next best thing is to introduce a problem which has some interest and which is easily understood. Suppose this to be the problem: "Edith had a toy bank with 28 cents in it. She needed a pencil that cost 3 cents, so her mother let her take 3 cents from her bank for the pencil. How many cents did she have in her bank then?"

Let real or toy money be used in counting out the 28 cents. Then have 3 of the 28 cents counted out and put aside. The rest is what Edith had left. A count shows that this is 25 cents.

TEACHER. Then twenty-eight less three are how many?

PUPIL. Twenty-five.

TEACHER. [*Writing* $28 - 3 = 25$] How many are 8 less 3?

PUPIL. Five.

The teacher writes just over the previously written fact, "$8 - 3 = 5$." She then builds up the "family" by calling for 38 less 3, 48 less 3, and so on. The relation of all the higher-decade facts of this family to the basic combination should be pointed out. If necessary, other problems calling for different higher-decade combinations may be brought up. In any event, several "families" should be developed. Children should have two kinds of practice: (1) in telling from a given higher-decade combination what the *complete* basic fact is and in deriving the answer; (2) in telling several of the higher-decade facts corresponding to a given basic fact. According to the first kind of practice the pupils will make such statements as this: "$37 - 4$ has $7 - 4 = 3$ in it, so $37 - 4 = 33$." According to the second kind of practice they will make such statements as this: "$9 - 2 = 7$, $19 - 2 = 17$, $29 - 2 = 27$, $39 - 2 = 37$."

Again, as in the case of higher-decade addition, children may "tell the whole story" about 9 less 2. Since as we shall see later in this chapter there is another story about 9 less 2, we may call the present one the first story. It will be the story in which the statements show a one-place number in the subtrahend. If the teacher prefers, she may have the children "name the members of the 9 less 2 family." Again, however, since 9 less 2 has another higher-decade series (Case II below), it will be necessary to call this the first family, reserving the term second family for Case II.

Case II. Two-Digit Numbers from Two-Digit Numbers, Differences Less than Ten

Case II of higher-decade subtraction is the one in which a two-digit number is subtracted from a two-digit number, the difference being less than ten. So far as this chapter is concerned there must be no bridging of tens; that is, both these numbers must be in the same decade, for example, 38 less 32. In examples of this sort you need not analyze the process to the point of having the pupils think "8 less 2 are 6" and "3 less 3 are 0." Let the difference between 38 and 32 be a clear inference *within the 30's* from the known fact, 8 less 2. As in the case of higher-decade addition and of Case I of higher-decade subtraction, have the pupils respond without analysis as they do with the basic combinations. The combinations, when written, should be in horizontal form.

In the experience of both children and adults the subtraction of two-digit numbers from two-digit numbers is exceedingly common. It is probably not unreasonable to hope that our pupils may ultimately be able to make all such subtractions mentally. This, of course, includes such difficult cases as $62 - 38$ and $84 - 67$, where the remainder is a two-place number. These, however, do not concern us yet. What we are now interested in is higher-decade subtraction of Case II within the same decade. Even when thus restricted there are a great many different examples; in fact, forty-five for each of the nine higher decades, or four hundred and five in all.[1]

Use in short division. Some of these subtractions are much more likely to occur in life than others. The same tendency to the use of small numbers to which we have formerly referred is in evidence here. Moreover, another fact should be mentioned. Later when the pupil has to do short division, he

[1] This does not include the taking of a higher-decade number from itself; for example, $15 - 15 = 0$, $62 - 62 = 0$, and the like. If such combinations were included, there would be 495; that is, 55 for each of the nine decades.

will continually be called upon to subtract one of the products of the multiplication table from a larger number. In the example $6\overline{)294}$ the child must subtract 24, one of the multiplication-table products, from 29.

Accordingly we see that the difficulty that all pupils have in subtraction as it applies in short division may be largely forestalled by familiarizing them with the particular subtractions which they are going to make. Let us, therefore, in selecting our drill material under Case II of higher-decade subtraction favor the combinations in which the subtrahend is a multiplication-table product. But this is not all. The minuend must also be considered. It must be such that the difference, that is, the answer, is *less than the larger factor in the subtrahend.* In the short-division example listed above, one has to subtract 24 from 29, and this yields a remainder of 5. This is the largest remainder it is possible to have when one is dividing by 6. Under no proper circumstances can we obtain a remainder as large as the divisor. Now, since *either* factor of a product may be a divisor, this means that the *larger* factor sets the limit to which the remainder can go, and that limit is always one less than this larger factor.

We may make one or two applications of this fact. In short division, 10 may be subtracted from 11, 12, 13, or 14, but not from any larger number. This is because the only way we can get 10 to subtract from *anything* in short division is either through multiplying a divisor 5 by a quotient 2, or by multiplying a divisor 2 by a quotient 5. If the divisor is 5 and a number larger than 14 appears as a partial dividend, 5 will "go" three times and an entirely different set of circumstances will follow. Again, in short division, 12 may be subtracted from 13, 14, 15, 16, or 17, but from no larger number. Observe that when it is subtracted from 17 the remainder is 5, which is one less than the larger factor in 12.

Table V gives all the higher-decade subtractions which occur in short division and which do not involve bridging the tens. One may, of course, go outside of these combinations. All the subtractions of two-digit numbers from two-digit numbers in the same decade are useful. The only point we wish to make is that the combinations listed in the table, amounting to a total of 115, are especially useful and that they should be favored.

Method of teaching. Case II of higher-decade subtraction may be taught so nearly like Case I that the teacher is referred to page 218 for a suggested method. Indeed, if the ability of the class permits, the two cases may be taught together. For example, when a number in the twenties or thirties is separated into two numbers, one of which is a one-place number, *either* of these two numbers may be the number "taken away." If the one-place number is taken away, Case I arises; if the two-place number, Case II arises. Suppose 36 objects have been counted and that 4 have been set apart. A count of the rest shows 32. This is Case I ($36 - 4 = 32$). At this point we may take away these 32 objects from the total, showing that 4 are left. If we do this, we are dealing with Case II ($36 - 32 = 4$). For every fact of Case I there is a corresponding fact of Case II, and the latter is the reverse of the former.

Series, or families, of Case II facts should be presented written, read, and repeated from memory. The pupils should identify the basic facts when higher-decade combinations of this case are given, and should infer higher-decade facts from given basic facts. The teacher is referred to page 219 of this chapter for suggestions as to "telling the whole story," or naming the members of a family.

TABLE V. HIGHER-DECADE SUBTRACTIONS USED IN SHORT DIVISION, NO BRIDGING OF THE TENS

11 − 10	13 − 12	15 − 14	16 − 15	17 − 16
12	14	16	17	18
13	15	17	18	19
14	16	18	19	
	17	19		19 − 18
	22 − 21	25 − 24	26 − 25	28 − 27
	23	26	27	29
21 − 20	24	27	28	
22	25	28	29	
23	26	29		
24	27			
29 − 28	31 − 30	33 − 32	36 − 35	37 − 36
	32	34	37	38
	33	35	38	39
	34	36	39	
	35	37		
		38		
		39		
41 − 40	43 − 42	46 − 45	49 − 48	55 − 54
42	44	47		56
43	45	48		57
44	46	49		58
45	47			59
46	48			
47				
57 − 56	64 − 63	65 − 64	73 − 72	82 − 81
58	65	66	74	83
59	66	67	75	84
	67	68	76	85
	68	69	77	86
	69		78	87
			79	88
				89

A Final Suggestion

Do not allow yourself to be oppressed by the refinements we have made in this and previous chapters. They will repay study and will assist you in your teaching. There is a great deal to be learned and applied in the teaching of arithmetic in the primary grades. Let no one delude you, and do not delude yourself, into supposing that this work is easy or that competence in it is to be attained without continuous study. Many searching analyses have been made by investigators and many more will be made. We know much more now than we did a few years ago, and our knowledge is likely to increase. It will not be easy for the teacher to learn even the most useful of these things; but those who do learn them and apply them will become expert in a new and superior sense. They will teach much more effectively. Moreover, they will recognize that these refinements and distinctions have always existed even when there was no one to point them out.

Drill Material on Case I

Below are given two hundred abstract examples and thirty-four verbal problems as practice material in Case I of higher-decade subtraction. Of course, not all the four hundred and five possible combinations of this case are presented. This does not appear to be necessary. The reduction in the amount of drill material is accomplished in two ways. In the first place, preference is given to the earlier decades. Of the two hundred examples, fifty-three are in the tens, fifty-three in the twenties, fifty-three in the thirties, and only forty-one in all the decades above the thirties. In the second place, the material is arranged so that the corresponding basic-subtraction combinations are about equally practiced. The ten basic facts which have zero for the subtrahend are disregarded.

The resulting higher-decade subtractions, such as $29 - 0$, $17 - 0$, and $30 - 0$, are rather absurd. This leaves forty-five of the fifty-five easy basic combinations to be represented by their higher-decade derivatives. Of these forty-five basic combinations, twenty-five appear four times and twenty appear five times in the two hundred examples given on the pages which follow.

The thirty-four problems serve no more than to show that there are natural and interesting situations which involve the kind of computation we have called Case I of higher-decade subtraction. In most instances the teacher may use these situations with different numbers. There is no reason, for example, why Grandfather Frog's singing class should have a membership of 27. But the chief reliance of the teacher for problem material will be the local and immediate environment of the pupils. A few problems growing out of actual experience will gain so much in vividness that they will serve as well as a much larger number of more remote problems. Among these thirty-four problems all the basic-subtraction combinations hitherto taught are represented except the relatively easy ones in which the subtrahend is zero or one or equal to the minuend.

Abstract examples

a			*b*		
$27 - 4$	$19 - 4$	$78 - 6$	$84 - 3$	$23 - 3$	$37 - 1$
$36 - 2$	$27 - 3$	$14 - 1$	$15 - 2$	$17 - 6$	$86 - 5$
$15 - 3$	$43 - 1$	$89 - 8$	$49 - 1$	$39 - 9$	$45 - 5$
$24 - 2$	$32 - 1$	$38 - 7$	$37 - 7$	$12 - 2$	$38 - 1$
$58 - 4$	$13 - 2$	$65 - 1$	$18 - 2$	$29 - 2$	$17 - 5$
$16 - 3$	$38 - 5$	$17 - 2$	$26 - 1$	$34 - 4$	$38 - 4$
$29 - 5$	$41 - 1$	$28 - 3$	$29 - 7$	$48 - 8$	$29 - 5$
$46 - 4$	$59 - 3$	$39 - 6$	$46 - 6$	$15 - 4$	$46 - 4$
	$47 - 5$			$28 - 3$	

c

24 – 1	17 – 3	19 – 9
15 – 3	68 – 5	27 – 2
49 – 4	14 – 2	22 – 1
25 – 5	36 – 3	37 – 7
18 – 7	34 – 4	46 – 1
36 – 2	78 – 6	59 – 3
25 – 1	27 – 4	21 – 1
39 – 6	13 – 1	39 – 8
	13 – 2	

d

65 – 2	38 – 8	46 – 4
17 – 1	17 – 6	23 – 3
32 – 2	29 – 2	28 – 4
39 – 7	34 – 3	37 – 5
26 – 6	28 – 1	19 – 9
18 – 2	59 – 5	34 – 2
49 – 1	16 – 5	25 – 1
25 – 4	18 – 3	36 – 3
	17 – 2	

e

88 – 7	18 – 6	29 – 4
12 – 1	15 – 5	13 – 2
26 – 2	19 – 6	21 – 1
35 – 3	23 – 1	36 – 1
44 – 4	39 – 3	17 – 1
57 – 3	17 – 7	39 – 2
24 – 1	29 – 9	24 – 3
37 – 4	68 – 5	16 – 6
	58 – 2	

f

18 – 1	35 – 4	17 – 2
22 – 2	79 – 7	68 – 4
35 – 2	14 – 2	39 – 5
26 – 5	26 – 3	16 – 4
38 – 8	37 – 5	28 – 7
49 – 1	29 – 9	37 – 3
33 – 3	48 – 3	25 – 5
17 – 6	35 – 1	16 – 2
	23 – 1	

g

24 – 1	27 – 1	36 – 6
19 – 3	69 – 6	19 – 7
37 – 4	37 – 7	25 – 4
78 – 6	19 – 8	13 – 2
18 – 5	22 – 1	39 – 1
26 – 1	58 – 2	26 – 5
34 – 4	39 – 4	28 – 8
29 – 2	35 – 3	14 – 3
	31 – 1	

h

18 – 1	22 – 2	49 – 1
39 – 9	65 – 1	36 – 3
27 – 6	33 – 1	24 – 2
13 – 3	18 – 7	35 – 5
47 – 2	59 – 5	29 – 7
28 – 4	38 – 3	17 – 4
37 – 5	15 – 2	38 – 5
16 – 4	26 – 2	39 – 4
	18 – 6	

Concrete problems

1. Grandfather Frog had a singing class. There were 27 frogs in it. One night they had a concert, but 2 little frogs had colds and could not sing. How many frogs sang in the concert?

2. There were 19 birds sitting on the telephone wire; 5 flew away. How many were left?

3. Alice and Carrie each had a new book. Alice said, "How many pages of your book have you read, Carrie?" Carrie said, "I have read 37 pages." "Oh," said Alice, "you have read 4 pages more than I have." Then Carrie knew just how many pages Alice had read. How many *had* she read?

4. Once 39 children planned to go on a school picnic, but 3 of them could not go. How many children went on the picnic?

5. Kate bought a pound of coffee and gave the grocer 45 cents. The grocer gave her 2 cents change. How many cents did the pound of coffee cost?

6. A mother duck *thought* she had 23 ducklings, but 1 was really a swan. How many ducklings *did* she have?

7. There were 39 candles on mother's birthday cake; 6 of them were white and the rest were pink. How many of the candles were pink?

8. Clifford had 37 cents and spent 5 cents for pop corn. How many cents had he left?

9. There were 39 pears on a tree. The wind blew off 7 of them. How many pears were still on the tree?

10. Mrs. Brown bought 36 spools of cotton. She sent 4 of them with some cloth to the dressmaker. How many spools did Mrs. Brown have left?

11. The postman had 29 letters in his bag. He left 2 of them at Mrs. White's house. How many letters were still in his bag?

12. In all, 27 people went on a sailing party; 6 of them were grown people. How many children went?

13. Joe and Uncle Ben had to drive 38 miles one evening. By and by Uncle Ben said, "Well, Joe, we have gone 6 miles already." "Yes, sir!" said Joe, "and now we have only ____ miles to go."

14. There were 28 dominoes in the box. Fred took out 4. How many dominoes were left in the box?

15. The grocer brought 24 oranges, and the children took 3 of them to school. How many of the oranges were left?

16. Mr. Stone had 39 sheep, but he sold 4 to his neighbor. How many sheep did he have left?

17. Gilbert wants to buy a bicycle that will cost 16 dollars. He has saved 5 dollars towards it. How many more dollars will he have to save?

18. There were 38 books on a library shelf, and some children took down 5. How many books were left for the other children?

19. There were 34 children in Miss Green's class. One day 2 stayed at home. How many were at school?

20. After Christmas Florence went into a toy store and asked the price of a pretty little doll. The toy man said, "Before Christmas it was 15 cents, but now I will take off 3 cents." How much did Florence have to pay for the doll?

21. James paid 28 cents for a ticket to the picture show; 3 cents were for war tax. How many cents would the ticket have cost if there had been no war tax?

22. Once 16 tiny fishes said they would go far, far away down the stream. But when it was time to start 3 of them were afraid and did not go. How many tiny fishes went down the stream?

23. Philip took out 38 newspapers after school. At supper time he had only 2. How many papers had he sold?

24. Katie made 27 cakes. She gave 3 of them to the children for their tea party. How many cakes were left for the family?

25. On a hillside 35 geese were eating grass; 4 of them were white and the others were gray. How many gray geese were there?

26. The Squirrel family had 29 nuts put away for the winter. They used 8 of them for their Christmas dinner. How many nuts did they have after Christmas?

27. There were 28 dominoes to be put into the box. Betty put in 7 of them and Bobbie put in the rest. How many dominoes did Bobbie put into the box?

28. Herbert picked 26 quarts of strawberries. He gave his mother 2 quarts and sold the rest. How many quarts did Herbert sell?

29. Mrs. Brown sent Harold to the store to buy a dozen bananas. She told him he might have 1 to eat. How many bananas were left?

30. There were 13 children playing tag in the school yard. The teacher called 2 of them in. How many were left to play tag?

31. Roy bought a dozen eggs and brought them home in a paper bag. When he reached home 2 were broken. How many whole eggs were left?

32. One day mother bought a bag of flour that weighed 49 pounds. The next morning she used 5 pounds in baking some bread and cake. How many pounds of flour were left in the bag?

33. Sister Ruth was going to make a coat for Emma's doll. "How long do you want it to be?" asked Ruth. Emma took her ruler and measured from Dolly's neck to the bottom of her skirts, and said, "Just 1 inch less than a foot." How many inches long was the coat to be?

34. Eleanor had a piece of ribbon that was 1 foot long. She cut off 2 inches to use for a badge. How many inches of ribbon were left in the piece?

Drill Material on Case II

We give herewith two hundred and fifty examples of higher-decade subtraction in which two-digit numbers are subtracted from two-digit numbers with remainders amounting to less than ten. We also give fifty-four problems in which the same kind of subtraction is called for. Preference is given to the lower decades and to situations such as arise in short division. All the basic subtraction facts which have thus far been taught are given in this material, the harder ones receiving the most attention.

Abstract examples

a

29 – 24	39 – 36	38 – 32	47 – 46	69 – 61	48 – 46
36 – 32	16 – 15	10 – 10	29 – 20	38 – 37	54 – 53
49 – 45	25 – 21	28 – 25	35 – 33	22 – 21	18 – 11
56 – 50	17 – 15	26 – 24	27 – 24	19 – 18	69 – 69
14 – 11	38 – 30	88 – 88	19 – 13	53 – 51	25 – 22
56 – 53	55 – 55	87 – 83	35 – 34	74 – 70	37 – 30
32 – 32	47 – 42	76 – 71	29 – 22	24 – 22	18 – 14
18 – 14	27 – 21	48 – 43	26 – 26	33 – 32	47 – 47
	59 – 57			51 – 51	

b

c

35 – 30	63 – 63	28 – 22	49 – 45	24 – 20	39 – 32
54 – 54	16 – 13	46 – 40	65 – 61	18 – 17	35 – 34
29 – 27	37 – 31	36 – 35	58 – 58	29 – 21	69 – 63
36 – 34	21 – 20	29 – 24	76 – 72	76 – 76	11 – 11
12 – 10	48 – 43	45 – 45	88 – 81	43 – 42	28 – 26
44 – 41	59 – 56	47 – 42	12 – 12	12 – 11	29 – 20
58 – 55	40 – 40	26 – 21	24 – 23	27 – 25	47 – 40
73 – 70	17 – 13	18 – 10	49 – 48	17 – 17	39 – 39
	38 – 34			15 – 13	

d

e

24 – 22	26 – 21	48 – 43	78 – 72	46 – 45	65 – 64
33 – 31	39 – 36	19 – 17	97 – 93	28 – 26	48 – 48
37 – 36	53 – 50	24 – 21	40 – 40	12 – 11	28 – 27
14 – 14	92 – 90	18 – 14	49 – 49	39 – 31	16 – 12
25 – 20	46 – 45	16 – 13	15 – 13	41 – 41	88 – 81
45 – 42	61 – 60	48 – 45	14 – 13	49 – 48	29 – 22
27 – 24	27 – 22	37 – 31	69 – 63	25 – 21	74 – 70
33 – 33	56 – 54	86 – 80	98 – 90	37 – 30	67 – 65
	39 – 34			46 – 41	

f

g

99 – 95	84 – 82	83 – 83
16 – 16	27 – 26	79 – 76
43 – 42	54 – 54	24 – 21
99 – 90	31 – 30	15 – 15
37 – 37	75 – 72	26 – 21
17 – 14	43 – 41	38 – 34
35 – 30	12 – 10	19 – 17
32 – 32	53 – 50	26 – 24
	16 – 10	

h

27 – 21	15 – 13	26 – 22
38 – 33	20 – 20	19 – 13
17 – 12	27 – 25	39 – 39
29 – 24	18 – 18	16 – 15
34 – 30	24 – 23	25 – 21
16 – 13	39 – 31	37 – 30
49 – 42	47 – 43	21 – 21
28 – 27	18 – 10	15 – 14
	18 – 12	

i

19 – 18	19 – 15	22 – 20
22 – 21	27 – 26	14 – 14
18 – 11	13 – 11	33 – 30
27 – 24	26 – 26	22 – 22
38 – 36	33 – 32	11 – 10
15 – 10	45 – 42	39 – 36
24 – 22	59 – 50	18 – 15
33 – 33	17 – 17	36 – 34
	19 – 17	

j

38 – 34	16 – 13	19 – 15
19 – 13	28 – 27	28 – 26
27 – 25	17 – 12	17 – 14
39 – 32	25 – 23	15 – 12
17 – 13	18 – 12	37 – 36
16 – 12	35 – 34	14 – 13
38 – 33	19 – 18	24 – 22
49 – 44	26 – 25	19 – 11
	29 – 20	

Concrete problems

1. Mother Turkey was walking through the tall grass with her 17 baby turkeys. She stopped to see if all her children were coming, and there were only 16. How many little turkeys had stayed behind?

2. Anna and Ella live in the same city and go to the same school. Next summer Anna will visit her aunt 68 miles away and Ella will visit her grandma 63 miles away along the same road. How many miles will Ella be from Anna next summer?

3. Alice and Mary were counting stars. Alice said she "had" 49. Mary said she was doing almost as well, for she "had" 42. Said Alice, "Oh! I have ____ more than you have."

4. Sarah and Lulu were jumping rope. Sarah jumped 67 times without missing and Lulu 64 times. How many more times did Sarah jump than Lulu did?

5. It is 67 miles to London town. We have traveled 63. How many more miles is it to London town?

6. There were 16 lemons in a box in the store. Esther bought 1 dozen of them. (A dozen is 12.) How many lemons were left in the box?

7. Harold and Louis said they were going to pick 36 quarts of strawberries before they ate dinner. When the dinner bell rang they had picked 32 quarts. How many more quarts did they have to pick before going to dinner?

8. On Monday Helen did all her music exercises in 58 minutes. On Tuesday she did them in 56 minutes. How many minutes less did she take on Tuesday?

9. The teacher called 69 boys out to drill in the school yard; 64 marched in line and the others were captains. How many were captains?

10. The man who sells flowers cut 38 roses and put them into a jar of water. A lady came and bought 36 of them. How many roses were left in the jar?

11. Mr. Brown and his little son Jim saw a pretty picture and wanted to buy it for mother's Christmas present. The price was 29 dollars. Mr. Brown said he would give 25 dollars if Jim would give the rest. Jim said he would. How many dollars did Jim give toward the picture?

12. A boy scout had to go to a place 18 miles away. A man took him 15 miles toward the place in his automobile. How many miles did the boy scout still have to go?

13. Roger is 17 years old. His sister is 12. How many years older is Roger than his sister?

14. There were 77 apples in baskets down by Alfred's garden. A man came along in his automobile and bought 72 of them. How many apples were left?

15. Mr. Allen's family and Mr. Cole's family started on a long drive in their automobiles. Mr. Allen drove 86 miles and stopped for the night. Mr. Cole drove 81 miles along the same road and stopped for the night. How far apart did the families spend the night?

16. Kenneth had saved 88 cents to spend at the circus. He went to the circus and spent only 81 cents. How many cents did he have left?

17. There were 46 children in the third grade and 45 in the fourth grade. How many more were there in the third grade than in the fourth?

18. There were 28 blackbirds sitting in a tree and 24 of them were singing. How many blackbirds were *not* singing?

19. There were 59 people in a car before the train reached the station. After the train left the station there were 54. How many people got out at the station?

20. There were 29 people at a picnic. When it was over, 27 of them went home and the rest stayed to put things in order. How many stayed?

21. There were 16 cows in the pasture. Harvey and his dog brought 13 of them up to the barnyard. How many cows were left in the pasture?

22. Fanny had 19 words to write. She had written 16 and then she said she was so tired she could not write any more. Mother said, "Oh, you are almost through now. You can finish in a moment." How many more words did Fanny have to write?

23. Mr. Bartlett had 19 bushels of potatoes. He sold 13 bushels and kept the rest for his family. How many bushels did he keep?

24. Mr. Foster asked the man at the gasoline station how far it was to the next town. "Thirty miles," said he. But Mr. Foster found it was 33 miles. How much farther away was the town than the gasoline man said it was?

25. There were 27 pupils in the fifth grade. After school 21 went home and the others stayed to water the school garden. How many pupils stayed?

26. Raymond had 38 stone blocks. He built a house that took 35. He used what were left for the chimney. How many blocks did he use for the chimney?

27. Gladys had saved 27 cents to buy a Christmas present for baby. She bought a picture book for 25 cents. How many cents did she have left?

28. There were 39 pupils in Miss Hill's class, but one stormy day only 32 were present. How many pupils were absent?

29. One day 37 robins met in the old apple tree and planned a trip to the Southland. They said they would start early next morning, but when starting time came only 33 of them were there. How many robins were missing?

30. Mrs. Harris did laundry work. She earned 39 dollars in September. In October she earned 35 dollars. How many more dollars did she earn in September than in October?

31. Uncle John got 46 quarts of milk one June morning. He sent 40 quarts of it to town. How many quarts were left for the family to use that day?

32. Mr. Gray brought 78 ears of green corn from the garden. Mrs. Gray sold 72 ears and cooked the rest for lunch. How many ears of corn did she and Mr. Gray have for lunch?

33. Mr. Fisher had 69 hens last summer. In the fall he sold 63 of them. How many hens did he keep?

34. Raisins were 16 cents a box on Friday, but on Saturday they were 14 cents. How many cents would you have saved if you had bought a box on Saturday?

35. Walter brought in 26 fresh eggs. Mrs. Mitchell came in and said they looked so good she would like to buy 24 of them. Walter sold her the 24. How many were left?

36. Mrs. Cackle took her 13 little chickens to the wheat field to look for seeds. She found a nice big wheat seed and when she called "Cluck, cluck, cluck!" 12 little chickens ran to her as fast as they could go. How many little chickens stayed behind?

37. Mr. McGregor had 25 fine heads of lettuce in his garden. One morning he found that he had only 23. How many heads were gone?

38. Mr. Peterson planted 75 apple trees in the fall. The next spring he found that only 72 of them were alive. How many had died in the winter?

39. Alice bought some ribbon for 21 cents. She gave the woman in the store a 25-cent piece. How many cents did she get in change?

40. Earl lived on a fruit farm. He put 57 nice ripe peaches into a basket and went to the station at train time. The people at the station and on the train bought 54 of them. How many peaches did he have left?

41. Mr. Dowling put 84 eggs in nests for the hens to hatch. They hatched 81 little chickens. How many of the eggs did not hatch?

42. Mrs. Brown bought 24 blue buttons. She used 20 of them for trimming her new dress. How many blue buttons were left?

43. Charles went to the store one evening to buy a dozen eggs. He had 21 cents. "Eggs have gone up," said the storekeeper. "They are 23 cents a dozen now. You may leave me the rest on your way to school in the morning." How many cents did Charles have to take in the morning?

44. Grandpa is 65 years old and grandma is 64. How many years older is grandpa than grandma?

45. Mr. Anderson and Mr. Black drove their automobiles for an hour. Mr. Anderson drove 34 miles and Mr. Black drove 33 miles. How many more miles did Mr. Anderson drive than did Mr. Black?

46. Harold and Ray picked 34 quarts of cherries. A man came along and said, "Boys, I should like to buy a bushel of those cherries." (That was 32 quarts.) So they sold 32 quarts. How many quarts of cherries had they left?

47. Sam wants to buy a bicycle that will cost 18 dollars. He has 14 dollars. How many more dollars must he save in order to pay for the bicycle?

48. Albert is 10 years old. He will go to high school when he is 14. In how many years will he go to high school?

49. One winter Mr. and Mrs. Jackson began to save as much money as they could for a vacation trip in the summer. Each one tried to save more than the other. When summer came Mr. Jackson had saved 45 dollars and Mrs. Jackson 42 dollars. How many more dollars did Mr. Jackson save?

50. Joe and Maria cut 39 beautiful roses one morning. They made them into bunches and took them out by the roadside. Before noon people in automobiles had bought 36 of the roses. Then Joe and Maria put the rest on the table in mother's room. How many roses did they have left to give mother?

51. Mrs. Rose had a piece of pink silk that was 18 inches long. She cut off a piece 1 foot long to use for a cap and gave the rest to Sadie for a lamp shade. How many inches long was Sadie's piece?

52. There were 15 red buttons in the show case. Helen bought a dozen of them. How many red buttons were left in the show case?

53. Ray wanted a stick 1 foot long to put into a kite. He found one that was 16 inches long. How many inches did he have to cut off?

54. May and Ralph were hunting for nuts. May said she had 14 and Ralph said he had a dozen. How many more nuts had May than Ralph?

The Progress Test

In the strictest sense the following test only partly covers the work of this chapter. As has already been pointed out, there are four hundred and five examples under Case I and an equal number under Case II. Even when the less important combinations are left out of account, the combinations of unquestioned importance are still too many to include in a test. We have therefore availed ourselves of the fact that these combinations occur in series and that the members of each series are closely related. To such an extent is this true that, in the case of any given series, knowledge of the basic combination, together with a few of the members of the series, will undoubtedly guarantee a knowledge of all the combinations in the series. Consider, for example, the $6-2$ series. Here the basic combination is $6-2$ and the series for Case I of higher-decade subtraction is $16-2$, $26-2$, $36-2$, etc. The series for Case II is $16-12$, $26-22$, etc.

We may therefore be content if in this test all the series are represented. This is only another way of saying that we

shall be content if all the basic combinations are represented, since there is a series for each such combination. In the following test every basic combination is represented once and a few of them twice. There are twenty examples of Case I and forty of Case II. Added to these are two problems in each case. The test is divided into two parts, each consisting of thirty examples and two problems. In analyzing the pupils' responses by means of the class record sheet you will need a sheet having thirty-two columns besides the column at the left for the pupils' names and the column at the right for totals. As before, cross-ruled paper with rulings one fifth of an inch apart will meet the needs. To save space the combinations should be written at the tops of the columns after the manner shown in connection with the class record sheet in Chapter XVII.

Test on Higher-Decade Subtraction

PART I

34 − 32 =	35 − 2 =	47 − 45 =
36 − 5 =	49 − 48 =	39 − 7 =
30 − 30 =	36 − 2 =	38 − 36 =
49 − 49 =	43 − 42 =	28 − 7 =
89 − 81 =	58 − 1 =	25 − 3 =
32 − 2 =	56 − 50 =	24 − 4 =
18 − 18 =	18 − 15 =	16 − 4 =
87 − 81 =	59 − 56 =	31 − 30 =
47 − 42 =	42 − 40 =	28 − 23 =
13 − 10 =	18 − 14 =	46 − 42 =

1. There are 18 houses on our street; 3 of them are made of brick. How many are not made of brick?

Ans. ____

2. The fruit man had some fine oranges this morning. Mrs. Smith bought 48 and Mrs. Ellis bought 42. How many more oranges were there in Mrs. Smith's basket than in Mrs. Ellis's?

Ans. ____

PART II

27 − 27 =	24 − 21 =	25 − 5 =
21 − 1 =	44 − 2 =	28 − 5 =
48 − 6 =	22 − 21 =	26 − 24 =
39 − 32 =	16 − 16 =	35 − 30 =
46 − 1 =	57 − 54 =	39 − 35 =
17 − 13 =	48 − 40 =	23 − 1 =
57 − 6 =	49 − 3 =	27 − 20 =
35 − 31 =	18 − 12 =	64 − 63 =
66 − 63 =	69 − 64 =	49 − 40 =
43 − 3 =	24 − 20 =	65 − 64 =

1. Madge and her little brother were cleaning the silver. They cleaned 28 spoons, but her little brother cleaned only 5. How many did Madge clean?

Ans. ____

2. Tony had 37 cents in the morning. At evening he had only 32 cents. He had spent the rest. How many cents had he spent?

Ans. ____

CHAPTER XIX

DRILLS AND GAMES. IV

Many games may be used in connection with column addition and higher-decade combinations. An appeal to the play instinct should continue throughout the whole primary-school period. Indeed, the game in which the element of contest is prominent has important educational uses throughout the entire elementary-school period.

Games previously Described

In previous chapters certain games have already been described. A number of these may be used to afford drill on higher-decade combinations and on easy column addition. Attention is called to the possibilities of this sort in the following games:

	PAGE
1. A bean-bag game	71
2. Crossing the stream	73
3. The ladder game	73
4. The fishing game	102
5. The flower game	103
6. Going to the lunch counter	104
7. A ball game	163
8. Bringing home the cows	164
9. Hopscotch	164
10. Relay race	165
11. The game of silence	166
12. The number match	166

Not all of these games are equally useful in their application to work beyond the basic combinations. Some of them

become more applicable after a slight modification. Such games as crossing the stream, the ladder game, the fishing game, the flower game, and bringing home the cows are, from one point of view, less useful than the other games just mentioned because they do not permit a large amount of drill. In other words, they are slow. Their value, however, lies in another direction. They are interesting, at least until their novelty has worn off, and this matter of interest is important. We have not appreciated as fully as we should the value of making the learning process pleasant. Enough evidence, however, is at hand to show that the emotional tone with which learning is invested is exceedingly important; and this importance is especially great in the case of young children.

The bean-bag game described on pages 71 and 72 is particularly useful with higher-decade addition and subtraction. It is suggested, however, that more squares than those represented on page 72 be provided. In these squares may be written higher-decade addition combinations or higher-decade subtraction combinations of either Case I or Case II. Even column addition as presented in Chapter XVI may be practiced by inserting in the squares appropriate material.

In the bean-bag game, as in some of the other games, drill in column addition may also be provided by the device of having the children work in teams of three. Each child may be permitted three throws of the bean bag; and corresponding to each throw is the opportunity to score one. His maximum score will therefore be three. The team-score will, of course, be the sum of the individual scores. If there are only three children on each team and the maximum score for each child is three, the sum can never exceed nine. The limitation here is that this scoring method will never furnish addends larger than three. Accordingly, it can be more successfully employed after more combinations have been learned and after work such as that of Chapter XXIV has been under-

taken. The full use, however, of summing scores in contests will not come until column addition has been extended to a larger number of addends.

The game of hopscotch described on page 164 is admirable for higher-decade addition and subtraction. The relay race described on page 165 may be easily adapted to higher-decade work, if the teacher first writes the combinations on the board rather than allowing the pupils to do so. If the pupils are permitted to do this, they may not keep within the limits of the higher-decade combinations already presented. The relay race when so modified is also well adapted for drill in column addition. The ball game (page 163) is especially well suited to higher-decade work, as is the number match (page 166).

Using the Number Chart

The number chart described in Chapter V lends itself especially well to drill on higher-decade addition and subtraction. For the former you may, for example, give the pupils a one-digit number with instructions to add it to each two-digit number on the chart to which you point. Be careful, however, not to introduce a combination requiring bridging the tens until after the work of Chapter XXI has been covered. Let us call the top row of the chart the zero row, because the unit figure is always zero; the next, the one row; the next, the two row; and so on. If you tell the children to "take" 7, you must select all the two-digit numbers from the zero, one, and two rows; otherwise the sum will go over into the next decade. If 6 is the constant number, the rows from zero to three become available. We may make the following rule about this: In higher-decade addition do not go beyond the row whose number added to the number selected makes a sum of nine. If 9 is selected, do not go beyond

the zero row. If 8 is selected, do not go beyond the one row. Finally, if 1 is selected, do not go beyond the eight row.

In Case I of higher-decade subtraction a constant number may likewise be given the pupils with instructions to subtract it from each two-digit number to which the teacher points. Use only the rows whose numbers are equal to or greater than the selected number. Thus, if you have indicated 1 as the number to be subtracted, you may use any row from one to nine. If 2 is to be subtracted, use any row from two to nine; if 3 is to be subtracted, any row from three to nine, and so forth.

In Case II of higher-decade subtraction all that is necessary is to stay in the same column for both minuend and subtrahend. Point to the minuend first and then higher up in the column point to the subtrahend. Give preference to subtrahends which are multiplication-table products.

The number chart lends itself to use in games. For example, without the use of any other device a contest may be planned in which the children choose sides or in which one row works against another. A pupil scores one for his side or row each time he gives a correct response. Moreover, in some games requiring the visual presentation of number combinations, the chart may be used as a substitute for flash cards. This is especially valuable in dealing with higher-decade combinations. They are so numerous that anything like an adequate provision of them in flash-card form involves considerable labor, and besides, the resulting material is bulky and clumsy to handle.

Additional Games

Puss-in-the-corner. This is a popular game. As usually played, it has to do with the basic combinations. All the pupils who are to play the game, except one, are arranged in a circle and given one-digit numbers in such a way that a given num-

ber always belongs to two children. For example, two children are given the number 6, two others the number 3, and so forth. The pupil who has no number is "it" and takes his place in the center. This latter pupil names a combination, for example, 9 less 3; whereupon the two children who have the number 6 attempt to exchange places before the pupil in the center can reach the place or "corner" of one of them. If the pupils having the number 6 successfully make the exchange, the pupil in the center continues to be "it" and calls another combination. As soon as one of the pupils on the circle is "left out" he becomes "it," and his number is taken by the pupil who captured his corner.

This game, like most of the games we have described in previous chapters, may be adapted to higher-decade drill. For example, the pupils may be given the same set of numbers as suggested above in connection with the use of the basic combinations. The pupil in the center of the circle may announce higher-decade subtraction combinations of Case II. For example, he may say "67 less 64"; whereupon numbers 3 would attempt to exchange places.

Or the understanding may be that higher-decade addition combinations are to be used. In this case the pupils having the units' figure of the sum may exchange corners. For example, the child in the center of the circle may announce 23 and 4. This would be the signal for those whose numbers are 7 to exchange places. Similarly, the higher-decade subtraction facts of Case I may be practiced by taking account of the unit figure of the answer; for example, 38 less 5.

Still another variation, applicable to this game as well as to many others, may direct attention not to the sum or the difference but to one of the other terms of a combination. Thus practice on combinations of the type $32 + 4$ may be secured by having the pupil in the center of the circle state his proposition in the form of the question "32 and how many

[what] are 36?" This will be the signal for the pupils whose numbers are 4 to change places.

Fox and geese. Let the geese stand in a circle and the fox stand in the center. The fox calls a combination (either basic or higher-decade) and asks a goose to give the answer. If the answer is correct the goose stays in the circle; if not, the goose is "caught" by the fox and goes to the center of the circle. The fox then gives additional combinations as before. A goose who has joined the fox in the center of the circle may "escape" and go back into the circle by giving a correct answer in place of a wrong one which the fox has accepted.[1] For variety the game may be played under the name of "Cat and mice."

Tit-tat-toe.[2] The teacher may draw the familiar diagram on the board, — two horizontals cut by two verticals with the resulting nine compartments. The game may be to have the children find out which row of three numbers — diagonal, horizontal, or vertical — makes the largest sum. This game lends itself especially well to column addition. Care must be taken at present not to have any of the rows total more than nine. The game may also be used for seat work.

Tennis toss. The teacher stands in the center of a circle with a tennis ball in her hand. She announces a combination, either basic or higher-decade, and tosses the ball to a pupil. He must give a correct answer before he catches the ball. The teacher should pause after announcing the combination before throwing the ball. A bean bag may be used in lieu of a tennis ball.

[1] Adapted from W. S. Guiler, "Teaching Arithmetic through Games and Other Pupil Activities," p. 28. Edwards Brothers, Ann Arbor, Michigan, 1924.

[2] Ibid., p. 15.

CHAPTER XX

ADDITION WITHOUT CARRYING AND SUBTRACTION WITHOUT BORROWING

There are two further applications of the basic number facts which have been taught up to this point. We introduce them here in order to secure the benefit of the early use of the number facts. Drill through use is better than drill for its own sake.

Addition

It is clear that if pupils know all the addition facts whose sums are less than 10, they can without much further difficulty add any numbers the sums of whose corresponding digits do not exceed 9. In other words, they can add without carrying. The new difficulty is merely a more sustained attention and the correct writing and placing of each digit of the answer. For example, a child who can handle all the combinations whose sums are less than 10 will have no more number facts to learn in order to add the two numbers given in the following example:

$$\begin{array}{r} 71436 \\ \underline{25361} \end{array}$$

Reading and writing numbers to 1000. Large numbers, however, have no meaning to pupils of this age. Nevertheless, most courses of study require the reading and writing of three-place numbers in the second grade. It will therefore be advisable at this time to extend the previous notation (units and tens) one step further to the left. As a preliminary to this, let the children count by 100's; that is, 100, 200, 300, and so on

to 1000. Let it be shown that 100 is ten 10's; that 200 is two 100's; that 300 is three 100's; and so on. Show that if we want to tell "how many," we often need larger numbers than 100. Show by counting that after we reach 100 we again begin at one, prefixing the words "one hundred"; for example, one hundred one, one hundred two, and so on. If the number is more than 200, we say two hundred one, two hundred two, and the like. Show how these numbers are written and teach the fact that the number which tells how many 100's there are is in hundreds' place. Have the children read and analyze three-place numbers and write them from dictation.

There is now no reason why children should not perform such additions as the following:

25	271	863	351
34	324	26	8

They have the requisite knowledge and, if they put it to use, they will make it more thoroughly their own. Moreover, the combining of such numbers is a common human need.

The first presentation. Consider the example $\begin{array}{r} 12 \\ 15 \\ \hline \end{array}$. This, or a similar example, may very properly be brought before the children as the result of an activity or as the representative of an interesting problem. For example, one side in a game may have "captured" 12 cards containing numerical combinations. The other side may have captured 15. No doubt the most interesting point is the amount by which one side beat the other, but a not unreasonable question may be "How many cards did both sides get?" In the flower game (page 103) it may be "How many flowers did they pick?" or in another game (page 164) it may be "How many cows did they bring home?" One way to find out how many cards were captured is to count them. If one side has 12 and the other side has 15, let the children put the two packs of cards

together and count them as one group. Let several children count them. It may easily be established that there are 27 cards. This number may be written on the board for reference.

Then let the teacher show a short way of getting the answer. Let her first write the example on the board,

$$\begin{array}{r} 12 \\ +\ 15 \\ \hline \end{array}$$

TEACHER. How many are two and five?

PUPIL. Seven.

TEACHER. I write seven under the two and five. [*Does so.*] One and one are how many?

PUPIL. Two.

TEACHER. I write two under the one and one. [*Does so.*] How many cards did you say both sides had captured? [*Pointing, if necessary, to the number written on the board*]

PUPIL. Twenty-seven.

TEACHER. [*Pointing to 27 in the example*] This is the answer we have just got. Read it.

PUPIL. Twenty-seven.

The teacher should make clear the fact that this way has brought us to the same result that the counting did.

The teacher will now recapitulate, the object being to set up the different steps as clearly and sharply as possible. The steps will be (1) add the units' figures and write the result under the units' figures; (2) add the tens' figures and write the result under the tens' figures; (3) the number thus made is the answer.

Try this same process with two or three more problems, involving such numbers as 14 and 23, 15 and 11, etc. Count out the material for each addend separately and then count the resulting group. Have the children add the units and write their sum. Have them add the tens and write their sum. Then let them observe that they have obtained the same answer that they did by counting.

Somewhat later when the children have had practice with two-place numbers, introduce three-place numbers. For example: "Mike has 127 marbles and Tom has 132. How many have they together?" To find this out we must add the numbers. If it seems desirable, we may play that the sticks we so often use for showing tens and hundreds are the marbles. The 127 marbles may be represented by a bundle of 100, two bundles of 10, and 7 loose sticks. The 132 marbles may be similarly represented. We may now put all these together, count the ones and find 9, the tens and find 5, and the hundreds and find 2. Then the teacher may ask "What number has two hundreds, five tens, and nine units, or ones?" The pupils should be able to give the number 259. Show that the short way to get this answer is to write the example this way, $\begin{array}{r} 127 \\ \underline{132} \end{array}$, and to think 7 and 2 are 9 (write 9), 2 and 3 are 5 (write 5), and 1 and 1 are 2 (write 2).

After practicing this kind of addition, introduce some problems and examples in which a two-place and a three-place number are added; also a one-place number and a three-place number; for instance, $\begin{array}{r} 236 \\ \underline{62} \end{array}$ and $\begin{array}{r} 462 \\ \underline{7} \end{array}$. The order of addition being immaterial, the smaller numbers may just as well precede the larger, for example, $\begin{array}{r} 62 \\ \underline{236} \end{array}$ and $\begin{array}{r} 7 \\ \underline{462} \end{array}$. The children should practice with such examples written both ways.

Subtraction

The argument for introducing at this point the subtraction of two-place and three-place numbers is analogous to that given above for addition. The pupils have the requisite knowledge; and this particular application of it is socially useful. Moreover, the application affords useful drill on the simple combinations. When a two-place number is subtracted from a

two-place number without borrowing, the numbers should not be in the same decade. (If they are, they belong to Case II of higher-decade subtraction, which has already been treated in Chapter XVIII.)

The first presentation. Consider the example $\begin{array}{r} 25 \\ -\,13 \end{array}$. This can easily come about through making change from a quarter of a dollar, or it may come about through the playing of a game in which one pupil or side scores 25 and the other 13. If the example arises so that the actual objects with which it is concerned may be counted, this opportunity should be utilized. Suppose the objects to be cent pieces, or pennies. Have the pupils count the 25 with which the example starts. Have them "take away" 13 by counting. Then let them count what they "have left," finding that this is 12 cents.

TEACHER. [*After writing* $\begin{array}{r} 25 \\ -\,13 \end{array}$ *on the board*] Five less three are how many?

PUPIL. Two.

TEACHER. [*After writing* 2 *under the* 5 *and the* 3] Two less one are how many?

PUPIL. One.

The teacher writes 1 under the 2 and the 1 and has the resulting number read as "twelve," pointing out that this is the same as the number of cents found by counting after having taken away 13 cents from 25 cents.

Trying the same method with other pairs of numbers such as 23 less 12 and 38 less 24, the pupils may readily be brought to see that this method *always gives the right answer*. The method may be formulated in substance as follows: (1) subtract the units and put the result under units; (2) do the same with the tens; (3) the number you get is the answer.

In all these subtractions it is recommended that the *same form of thought and expression be used as has been used in saying*

the basic combinations. Thus in the example $\begin{array}{r} 25 \\ -\ 13 \end{array}$ have the pupils say "5 less 3 are 2" and "2 less 1 are 1," *not* "3 from 5 are 2" and "1 from 2 is 1." There is no reason why the combinations should be said one way when they are being *learned* and another way when they are being *applied.* The children must feel that these are the same number facts with which they have long been dealing. The additional difficulty is the new setting. It is introducing a gratuitous difficulty to begin at this very moment to give these old friends new names. Note that if this recommendation is followed the movement of eye and hand is continuously downward. That is, in $\begin{array}{r} 25 \\ -\ 13 \end{array}$ it is from 5 to 3 to 2, and from 2 to 1 to 1. If the answer is to be written, the hand is in position to do so as fast as each subtraction is made.

At a later date take up subtraction involving three-place numbers. For example: "Our school gave an entertainment and raised 247 dollars. We spent 125 dollars for pictures and put the rest in the bank. How many dollars did we put in the bank?" Show that to find this out we must subtract 125 dollars from 247 dollars. We may, as in adding three-place numbers (see page 248), let sticks represent the numbers. We may take away (to spend for pictures) 5 single sticks (dollars), 2 ten bundles, and 1 hundred bundle, showing that 1 hundred bundle, 2 ten bundles, and 2 sticks are left. This may be shown to be 122 sticks or dollars. The problem may be set down in this form: $\begin{array}{r} 247 \\ -\ 125 \end{array}$. We may then think "7 less 5 are 2 [writing 2]; 4 less 2 are 2 [writing 2]; and 2 less 1 are 1 [writing 1]."

Introduce, as soon as convenient, the subtraction of a two-place number and a one-place number from a three-place number; for instance, $\begin{array}{r} 284 \\ -\ 61 \end{array}$ and $\begin{array}{r} 547 \\ -\ 5 \end{array}$.

Remember that zero is a real difficulty in subtraction. We have already pointed out that the zero number facts are hard even when considered separately. When these facts occur in examples, they are even more difficult. See that you provide practice in these matters. In subtraction without borrowing zero cannot occur in the minuend. It can, however, occur in the subtrahend and in the answer. In three-place numbers there are six ways in which zero may appear in the answer; namely, in units', tens', or hundreds' place only, and in units' and tens', units' and hundreds', or tens' and hundreds' places. The fact that zero is not written when it belongs in hundreds' place must be carefully noted and provided for. There are three ways in which zero may appear in the subtrahend; that is, in units' place, in tens' place, or in both. Thus there are nine types of these examples. Then since *each* of the six ways in which zero may occur in the answer may exist in the same example with each of the three ways in which it may occur in the subtrahend, we have 18 more kinds of zero arrangements. This makes 27 all together. Table VI on the following page gives an illustration of each of these 27 arrangements. The experienced teacher will recognize many of them as particularly troublesome. The first 27 examples in Part III of the Progress Test at the end of this chapter are the same as the examples in Table VI.

Illustrative Material

We present below both abstract and concrete material on addition without carrying and subtraction without borrowing.

Addition: abstract examples. Three hundred examples are given. As in all the examples we have furnished, an accurate score of the number of occurrences of each basic combination has been made. There is some reason to believe that the appearance of a combination in units' place is different from

the appearance of the same combination in tens' place; in other words, that the difficulty of the combinations is influenced by their position. In the first two hundred examples (that is, in those limited to two-place numbers) all the fifty-five addition combinations thus far taught appear in units' position either three or four times; and all these combinations except the ones involving zero appear in the tens' position either five or six times. In general the combinations which appear most frequently are the harder ones. In the last one hundred examples each of the "easy" basic combinations occurs five times.

TABLE VI. THE WAYS IN WHICH ZERO MAY OCCUR IN THE SUBTRACTION OF THREE-PLACE NUMBERS WITHOUT BORROWING

ZEROS IN ANSWER	ZEROS IN SUBTRAHEND			
	None	In Units' Place	In Tens' Place	In Units' and Tens' Places
None	——	872 − 510	319 − 207	681 − 400
In units' place	358 − 138	960 − 420	952 − 302	750 − 600
In tens' place	268 − 162	837 − 630	704 − 201	504 − 100
In hundreds' place	345 − 324	162 − 150	697 − 604	438 − 400
In units' and tens' places	759 − 559	970 − 270	507 − 207	800 − 300
In units' and hundreds' places	861 − 831	590 − 560	835 − 805	970 − 900
In tens' and hundreds' places	746 − 741	639 − 630	407 − 403	206 − 200

a

23	19	26	34	35
12	10	23	15	23
27	11	33	45	54
32	47	51	30	33
42	35	14	53	23
47	32	60	45	43
13	43	25	54	22
86	54	54	22	62
54	28	65	68	72
14	71	21	10	15

b

76	87	61	16	32
21	10	32	70	64
13	36	12	41	41
50	42	33	21	13
12	21	32	22	21
26	16	20	31	45
30	40	50	41	60
36	34	22	58	25
50	20	24	21	70
38	70	51	24	13

c

41	10	35	30	10
12	16	54	42	57
20	31	11	43	15
41	10	47	41	60
53	22	65	64	16
43	67	32	10	72
37	17	43	74	65
61	82	54	24	23
54	86	43	42	12
12	13	30	22	31

d

13	32	12	53	19
56	46	15	32	80
76	48	25	27	12
21	40	61	50	73
34	58	16	51	32
51	41	40	21	10
10	31	60	41	20
23	34	18	23	14
31	20	24	23	44
62	27	73	35	15

e

51	80	11	70	60
28	15	30	12	30
30	67	45	52	64
21	12	14	14	25
45	65	17	43	23
43	34	62	54	76
13	47	22	33	34
25	31	47	32	53
36	11	51	20	29
43	75	36	38	50

f

21	56	57	74	43
67	12	41	24	23
12	75	14	38	10
84	12	62	20	57
80	17	62	21	32
16	40	33	25	16
26	11	10	15	35
10	23	35	10	61
52	46	51	42	68
35	31	28	52	21

g

24	40	20	31	20
70	21	34	56	43
32	43	14	32	61
31	10	71	40	14
40	61	23	15	51
40	31	51	43	40
73	12	37	34	23
26	37	62	25	64
53	73	14	16	19
12	15	63	83	50

h

30	82	15	16	20
18	16	24	12	12
21	32	20	41	60
27	65	76	58	37
34	51	72	58	45
54	36	24	41	32
38	54	67	43	17
40	22	21	23	80
76	22	46	35	25
10	53	41	21	60

i

140	542	452	444	413
138	300	111	305	211
517	130	641	401	434
432	108	242	571	413
100	564	573	202	103
630	101	306	782	795
120	176	740	314	362
732	212	251	604	514
344	625	825	120	250
334	152	144	305	400

j

262	516	260	311	560
537	271	223	168	405
144	201	628	701	685
812	630	311	145	100
100	230	102	156	490
167	166	423	330	104
147	331	132	334	680
221	254	620	440	207
151	110	307	232	142
547	424	350	336	222

k

800	129	624	223	305
150	530	203	434	331
510	201	253	160	103
206	206	122	18	710
509	282	452	305	769
170	715	42	323	30
271	278	953	380	113
108	620	31	109	652
700	600	130	311	225
236	399	802	283	570

l

876	3	37	782	256
10	244	850	4	500
271	32	731	40	697
5	460	64	314	300
471	700	300	100	200
220	6	410	312	751
710	853	4	43	753
44	5	284	950	100
762	580	50	440	214
6	117	928	509	600

Addition: concrete problems. The following thirty-five problems afford practice well distributed over the addition combinations which the children have learned.

1. Frances and Fred sat in the automobile while their father and mother went into the stores. They counted all the people who walked past them. They counted 62 grown-up people and 34 children. How many people did they count in all?

2. Then they stopped in front of the post office. Frances counted the people who went into the post office. There were 12. Fred counted those who came out. There were 15. How many people did they both count?

3. Herman paid 75 cents to go to the circus and 20 cents for lemonade and balloons. How many cents did Herman spend at the circus?

4. Frank's suit cost 22 dollars and his overcoat cost 17 dollars. How many dollars did his suit and coat cost?

5. Tom planted 22 hills of corn and Tim planted 22 hills of corn, too. How many hills of corn did Tom and Tim both plant?

6. Tom and Tim saved money for mother's birthday present. Tom saved 33 cents and Tim saved 46 cents. How many cents did they both save?

7. Tony and Joe went to the animal show. Tony counted the animals that could climb. There were 11 of them. Joe counted those that could not. There were 45 of them. How many animals did both boys count?

8. The Davis family drove 50 miles and then stopped and cooked dinner beside the lake. In the afternoon they drove 48 miles. How many miles did they drive that day?

9. In a park in New York City lived a very large elephant named Gunda. High up in Gunda's room was a wooden box which had a hole in the cover. This was Gunda's bank. Above the bank was a little handle which rang a bell. Many people gave Gunda pennies. He would take each penny in his trunk, drop it into his bank, and turn the handle which rang the bell. One morning Gunda put 37 pennies into his bank, and in the afternoon he put in 52. How many pennies did Gunda get that day?

10. Marie and Isabelle went to a park. They paid 40 cents to go swimming in the pool and 35 cents for sandwiches and milk. How many cents did they spend in the park?

11. In a pet shop there were 35 plain goldfish and 14 spotted goldfish. How many fish were there all together?

12. Uncle Jim took Ned to a park in a large city. Ned took a ride on the camel. That cost 15 cents. He said he would like to ride on the elephant, too. So he had a ride on the elephant. That cost 20 cents. How much did Ned's rides cost?

13. Emily and Eva want to go to the country to board for two weeks next summer. To go there and come back will cost 11 dollars. The board will cost 24 dollars. What will the vacation trip for both of them cost?

14. Wallace sat down by the roadside one afternoon. He counted the automobiles as they passed. There were 63 going one way and 21 going the other way. How many automobiles did he count?

15. Donald practiced his violin lesson 25 minutes in the morning and 32 minutes in the afternoon. How many minutes did he practice in all?

16. Wallace has a new book that tells about trips in a flying machine. Last night he read 18 pages and today he read 81 pages. How many pages has he read?

17. Mrs. Foster is a dressmaker. It took her 24 hours to make Mrs. Shelton's silk dress and 12 hours to make Peggy Shelton's white dress. How many hours did Mrs. Foster work for Mrs. Shelton and Peggy?

18. One spring morning Albert saw a large flock of wild geese flying toward the north. There were 42 of them. Toward evening he saw a small flock. There were 16 of these. How many wild geese did Albert see that day?

19. Florence had a trip to the seashore one summer. She liked to play in the sand and watch the people bathe in the ocean. One morning she saw 16 people in the water and in the afternoon she saw 33. How many people did Florence see bathing that day?

20. Last summer Albert earned 35 dollars by working in a store and 21 dollars by taking care of lawns. How many dollars did he earn in all?

21. In our schoolroom closet there are 53 reading books and 32 number books. How many books are there in the closet?

22. The first and second grades had a Flag Day drill. In it there were 14 first-grade children and 11 second-grade children. How many children were there in the drill?

23. Mr. and Mrs. Squirrel put away 44 nuts for winter use. The Squirrel children put away 25. How many nuts did they all put away?

24. Mr. and Mrs. Frog started a singing school down in Meadow Brook. There were 14 froggies in Mr. Frog's class and 13 in Mrs. Frog's. How many froggies were there in both classes?

25. Julia had been reading for 40 minutes. Mother said, "Julia, you may read just 10 minutes more." How many minutes did Julia read?

26. Philip had read to page 114 in a storybook. One day he read 54 pages more. How many pages had he then read?

27. Mr. Brown drove his car 213 miles on Friday and 184 miles on Saturday. How many miles did he drive in both days together?

28. In the afternoon 246 people came to our school play and 532 came in the evening. How many people came to our school play?

29. Robert rode 876 miles on the train and 23 miles in an automobile. How many miles did he ride in all?

30. A baseball team played 124 games in one year and 125 th next year. How many games did they play in both years?

31. Mr. Roberts has 455 books at home and 234 at his office. How many books has he all together?

32. Paul lives 430 yards north of the schoolhouse and Howard lives 165 yards south of the schoolhouse. How many yards away from each other do Paul and Howard live?

33. Mr. Havens has a chicken farm. He had 853 chickens and bought 125 more. How many chickens did he then have?

34. Charles and Frank are brothers. Each of them delivers papers in the evening. Charles delivers 122 and Frank delivers 76. How many papers do both together deliver?

35. The second grade sold 236 tickets for the school entertainment and the third grade sold 431. How many tickets did the second and third grades sell?

Subtraction: abstract examples. Here are two hundred two-place examples which afford drill on all the fifty-five easy subtraction combinations. These examples are followed by one hundred others which involve three-place numbers. All the possible zero difficulties as set forth in Table VI are exemplified.

a

76	25	63	89	60
− 24	− 13	− 41	− 35	− 20
93	75	86	55	67
− 82	− 41	− 45	− 42	− 35
63	34	56	39	98
− 10	− 12	− 23	− 28	− 45
87	67	96	79	48
− 56	− 54	− 72	− 66	− 36
49	82	78	87	99
− 17	− 61	− 53	− 23	− 54

b

47	75	64	98	89
− 27	− 64	− 34	− 64	− 73
78	97	59	98	76
− 37	− 32	− 12	− 28	− 11
91	89	58	74	38
− 71	− 10	− 32	− 50	− 21
99	84	97	54	86
− 19	− 43	− 51	− 21	− 36
75	68	73	69	82
− 45	− 50	− 23	− 41	− 52

c

67	96	37	86	25
− 20	− 40	− 16	− 24	− 13
93	89	52	63	41
− 81	− 65	− 40	− 12	− 30
45	76	95	57	83
− 11	− 35	− 62	− 35	− 70
94	46	59	98	85
− 12	− 23	− 18	− 35	− 10
97	76	39	68	62
− 24	− 12	− 26	− 36	− 51

d

52	67	79	70	59
− 32	− 43	− 43	− 60	− 24
89	86	78	94	95
− 47	− 36	− 50	− 51	− 75
47	98	58	98	79
− 27	− 47	− 48	− 34	− 32
98	95	89	94	63
− 82	− 64	− 21	− 23	− 23
89	78	89	94	87
− 79	− 23	− 50	− 14	− 62

e

46	54	35	87	76
− 31	− 10	− 23	− 41	− 14
61	67	96	68	79
− 51	− 14	− 70	− 31	− 56
62	25	78	71	75
− 40	− 15	− 46	− 60	− 21
35	87	46	87	49
− 10	− 17	− 25	− 30	− 18
84	93	76	93	85
− 64	− 60	− 32	− 41	− 72

f

69	82	93	93	87
− 25	− 51	− 82	− 53	− 26
54	96	58	50	97
− 32	− 13	− 25	− 40	− 35
94	68	76	48	89
− 21	− 12	− 56	− 10	− 69
79	52	39	47	98
− 41	− 12	− 10	− 23	− 68
76	79	28	48	89
− 10	− 24	− 13	− 34	− 17

g

67	65	38	69	87
− 50	− 34	− 27	− 43	− 42
75	49	98	54	87
− 60	− 12	− 71	− 13	− 31
96	81	72	87	94
− 41	− 71	− 30	− 57	− 80
61	59	58	97	35
− 20	− 48	− 26	− 54	− 12
54	46	89	96	65
− 34	− 32	− 26	− 24	− 13

h

89	76	98	27	79
− 64	− 15	− 34	− 15	− 27
98	76	98	79	87
− 15	− 53	− 62	− 40	− 16
54	97	98	49	98
− 32	− 83	− 43	− 35	− 57
89	67	95	89	73
− 53	− 32	− 74	− 72	− 62
84	69	78	67	89
− 43	− 48	− 36	− 54	− 36

i

985	947	826	923	405
−872	−320	−305	−512	−401
963	579	641	887	780
−943	−421	−200	−123	−740
834	206	462	740	998
−804	−200	−332	−310	−866
680	987	321	734	515
−600	−750	−101	−601	−513
976	650	845	679	999
−753	−100	−101	−670	−432

j

575	732	738	978	918
−500	− 10	−131	−652	−318
796	547	698	306	987
−125	−448	− 83	−204	−863
460	984	906	476	609
−260	−510	−100	− 21	−509
955	773	246	700	415
−932	−750	− 33	−400	−100
996	979	499	854	868
− 52	−764	−404	−513	−744

k

876	646	884	837	769
−765	−600	−521	−237	−222
401	592	780	556	689
−300	−101	−480	−235	−634
384	300	587	505	786
−262	−200	−335	−105	−101
895	906	875	285	657
−512	−201	−764	−255	−344
768	990	987	124	357
−740	−970	−643	−104	−302

l

650	497	345	878	953
−600	− 82	−142	−876	−741
510	844	538	749	925
−300	−121	−530	− 3	−220
976	208	375	748	107
−443	−204	−234	−308	−100
962	971	379	839	867
−311	−560	− 5	−706	−431
894	296	769	246	970
− 62	−100	−129	− 34	−120

Subtraction: concrete problems. These thirty-eight problems offer distributed drill on the easier subtraction facts.

1. Elizabeth and Margaret sat in the automobile waiting for mother, who was buying shoes. Elizabeth counted all the people who walked past on the near side of the street. There were 36. Margaret counted those who passed on the other side. There were 25. How many more people did Elizabeth count than Margaret?

2. Then they counted all the automobiles that passed; 28 went down the street and 17 went up. How many more automobiles went down the street than went up?

3. In the flower-picking game the girls picked 67 flowers and the boys 52 flowers. How many more flowers did the girls pick than the boys?

4. Clifford had 37 cents one week, but on Saturday afternoon he went to a picture show. It cost him 15 cents. How many cents did he have left?

5. Jack and Fred said that they were going to count stars one evening and in the morning they would see who had counted more. Next morning Jack called, "Hello, Fred. How many stars did you count?" "Fifty-eight," said Fred. "Oh, you beat me," said Jack, "I counted only 45." How many more stars did Fred count than Jack?

6. Jane had some orange paper and some black paper for Halloween. She had enough orange paper. She needed a sheet of black paper 72 inches long. She found one 60 inches long. How many more inches of black paper did she need?

7. Wallace bought groceries that cost 63 cents. He gave the grocer 75 cents. How many cents in change did the grocer give him?

8. Eva was in the country visiting her cousin. Father came to take her home in his automobile. "Well, Eva," he said, "we shall have to drive 95 miles today." When they stopped for lunch he said, "We have driven 54 miles." Eva said, "I know how many miles we have to drive this afternoon." How many miles did she say?

9. Stephen took 72 morning papers to sell before school. Finally he said to himself, "It is almost school time. I must take these papers back now." He counted those that were left. There were 10. How many papers had he sold?

10. Nancy had a party. Her mother had bought 42 tiny nut cakes, but only 31 of them were used. How many nut cakes were left?

11. Mr. McGregor picked 37 ears of sweet corn one morning. He kept 13 ears for dinner and sold the rest. How many ears of corn did he sell?

12. There were 23 chairs in a row on one side of the room. The children took away 12. How many chairs were left in the row?

13. Mr. Morgan and his boys picked 67 quarts of berries. Mrs. Morgan used 32 quarts of them to put in jars for the winter, and the boys sold the rest. How many quarts of berries did the boys sell?

14. Jack went to school with 38 cents in his pocket. When he came home he said, "Mother, I am hungry. I did not buy a very large lunch today." "How much did you spend for lunch?" asked mother. "I forget," said Jack, "but here are 24 cents that are left." How many cents did Jack spend?

15. There were 89 spools of cotton in a little store. Mrs. Foster stopped and bought 12 spools. How many spools were left in the little store?

16. There were 79 children in a children's hospital. On May Day 24 of them were taken out to have lunch under the trees. Each of the others was given a little basket of wild flowers. How many of the children had baskets of wild flowers?

17. Grace and Irene have scrapbooks. There are 39 pictures in Grace's book and 27 in Irene's. How many more pictures are there in Grace's book than in Irene's?

18. Mrs. Howard ordered 36 buns for a school picnic. The baker sent her only 24 and John had to hurry off to the baker's shop to get the rest. How many buns did he get?

19. Jim wanted to buy a knife that would cost 75 cents. He had 42 cents. How many more cents must he save in order to buy the knife?

20. The children gathered autumn leaves. They had 48 red ones and 24 yellow ones. How many more red leaves did they have than yellow leaves?

21. Peter and Ray lived in the country. One summer afternoon they walked to town to do some errands. Peter said, "Let's get a pint of ice cream and sit under this tree and eat it on the way home." "Yes," said Ray, "but that would cost 28 cents and I have only 13 cents." "I wonder if I can pay the rest," said Peter as he put both hands into his pockets. How many cents did Peter need?

22. Jack had 89 postage stamps. He sold 66 and put the rest in a stamp book. How many did he put in the stamp book?

23. Eggs were 64 cents a dozen in the winter. In the spring they were 21. How many more cents did they cost in winter than in spring?

24. The children of the second grade have to sell 58 tickets for the school concert. They have sold 12. How many more tickets must they sell?

25. Jean's reader has 96 pages. Her picture book has only 31 pages. How many more pages has her reader than her picture book?

26. One summer Hilda picked 378 quarts of strawberries and Irene picked 353 quarts. How many more quarts did Hilda pick than Irene?

27. In the morning Walter had 175 ears of corn on his roadside stand. At night he had 31 ears left. How many had he sold?

28. One of Lucy's readers has 186 pages and the other has 142 pages. How many more pages has the first reader than the second?

29. Farmer Jones sold 947 bushels of potatoes this year and 824 bushels last year. How many more bushels did he sell this year than last?

30. In the bean-bag game the boys made 358 points and the girls 325. How many more points did the boys make than the girls?

31. The Whittier School has 995 pupils. The Longfellow School has 633. How many more pupils has the Whittier School than the Longfellow School?

32. Mr. Gordon bought a new car for 996 dollars. He got 245 dollars for his old car. How much more did he have to pay?

33. Mr. Scott and his family are driving to New York in their car. It is 869 miles to New York. The first day they drive 245 miles. How many miles are they from New York then?

34. Philip had 279 marbles. He traded 65 of them for a knife. How many marbles did he have left?

35. The Garfield School had a big doll sale. The mothers dressed 274 dolls and gave them to the school. At the sale 223 dolls were sold and the rest were given to the children's home. How many were given to the children's home?

36. Tom's pig weighed 229 pounds and William's weighed 207 pounds. How many more pounds did Tom's pig weigh than William's?

37. Farmer Rogers raised 857 bushels of oats. He sold 650 bushels and kept the rest to feed to his horses. How many bushels did he keep?

38. Our school library has 886 books. The children are reading 183 of these books. The rest are on the library shelves. How many are on the shelves?

Progress Tests

All the basic combinations thus far taught are represented in the progress tests which follow. The harder ones appear twice. The class record sheet should show precisely what, if any, combinations each pupil has failed to learn.

Part I. Addition without Carrying

34	62	50	80	755
21	36	28	16	201
24	12	530	50	23
60	67	260	42	31
40	356	30	75	46
57	412	44	414	30
219	12	248	31	50
50	71	340	658	35
212	34	10	410	627
732	32	23	19	340
361	106	33	205	742
620	461	50	513	250

1. The girls of our class dressed 32 dolls for Christmas, and the girls in Miss White's class dressed 46. How many did the girls in both classes dress?

Ans. ____

2. Yesterday Tom saw 27 birds flying south for the winter. Today he saw 32 more. How many did he see on both days?

Ans. ____

3. Emma bought a collar for $1.50 and a pair of stockings for $1.35. How much did she pay for both?

Ans. ____

Part II. Subtraction of Two-Place Numbers from Two-Place Numbers without Borrowing

75	34	66	89	74
− 40	− 21	− 30	− 12	− 20
99	87	79	65	79
− 25	− 20	− 36	− 41	− 13
24	50	41	65	785
− 14	− 30	− 31	− 22	− 35
95	86	78	68	58
− 13	− 56	− 54	− 57	− 40
97	79	89	98	62
− 77	− 69	− 60	− 88	− 12
43	93	81	83	52
− 23	− 41	− 40	− 70	− 20

1. Farmer John has 46 cows. Last year he had only 24. How many more cows has Farmer John this year than he had last year?

Ans. ____

2. In the bean-bag game our side got 27 and the other side got 16. How much more did our side get than the other side?

Ans. ____

3. Anna wanted a candle-holder that cost 89 cents. She had only 75 cents. How many more cents did she need?

Ans. ____

Part III. Subtraction without Borrowing, Three-Place Numbers

358	872	639	835	800
− 138	− 510	− 630	− 805	− 300
268	960	319	407	970
− 162	− 420	− 207	− 403	− 900
345	837	952	681	206
− 324	− 630	− 302	− 400	− 200
759	162	704	750	948
− 559	− 150	− 201	− 600	− 137
861	970	697	504	789
− 831	− 270	− 604	− 100	− 48
746	590	507	438	598
− 741	− 560	− 207	− 400	− 51
473	327	987	998	849
− 142	− 4	− 232	− 743	− 516

1. There are 867 children in the Main Street School; 421 of them are boys. How many are girls?

Ans. ____

2. Ida's book has 258 pages. She has read 125 pages. How many pages does she still have to read?

Ans. ____

CHAPTER XXI

THE HARD COMBINATIONS

As has already been pointed out, there are one hundred basic combinations in addition and one hundred in subtraction. For teaching purposes we have divided the addition combinations into an *easy* and a *hard* group. The easy combinations are the fifty-five whose sums are less than 10. The remaining forty-five, whose sums are 10 or more, are the hard combinations. Similarly, in subtraction we have recognized the fifty-five combinations whose minuends are less than 10 as the easy group and the remaining forty-five, whose minuends are 10 or more, as the hard group.

Thus far provision has been made for teaching and drill on the easy combinations of each operation. In this chapter we shall consider the hard combinations, namely, the ninety whose sums and minuends exceed 9. Again, we shall arrange the material in teaching units, each represented by its key combination. Each of these teaching units consists of four combinations, or facts, unless it is a double, in which case it consists of two facts. Thus the teaching unit whose key combination is $2 + 9$ contains the combinations $2 + 9$, $9 + 2$, $11 - 2$, and $11 - 9$. The teaching unit whose key combination is $6 + 6$ contains only the combinations $6 + 6$ and $12 - 6$, at least so far as addition and subtraction are concerned. As a matter of fact, however, we are joining to each double three easily inferred facts, one each in multiplication, division, and partition. For example, with $6 + 6$ are to be taught 2 6's are 12, 6's in 12 are 2, and $\frac{1}{2}$ of 12 is 6. Accordingly, with each double there are five number facts to be taught, although

only two of them are in addition and subtraction. The ninety hard combinations in addition and subtraction are organized into twenty-five teaching units. Twenty of these include four number facts each, and five of them, being doubles, include but two facts each. The twenty-five teaching units are arranged in three groups: nine in the first, nine in the second, and seven in the third.

The Order of Teaching

We may apply to the arrangement of these teaching units the same principles which were stated in connection with the similar organization of the easy combinations; namely, that the order of difficulty should be regarded in the order of teaching, and that combinations having the same terms should be kept apart.

Table VII shows the order of presentation according to these principles. We have divided the teaching units into three groups. The first column indicates these groups. The second column shows the order of presentation, the third column the key combination for each teaching unit, and the fourth column the difficulty of the number facts in each teaching unit. This last is the average difficulty of the four — or two, in the case of doubles — combinations belonging to each teaching unit. This index is obtained by pooling the results of eight studies made by five different investigators. It will be observed that the teaching order is not always the precise order of difficulty. In no case, however, do the ranks on these bases differ by more than 3; $4 + 7$ is twentieth in the order of teaching and nineteenth in the order of difficulty. The fact that there is any difference at all in these orders arises from the attempt to arrange the teaching units so that no two combinations with the same term should be consecutive.

TABLE VII. ORDER OF PRESENTING THE HARD COMBINATIONS

TEACHING GROUP	ORDER OF PRESENTATION	KEY COMBINATION FOR EACH TEACHING UNIT	DIFFICULTY INDEX
I	1	5 + 5	30.5
	2	6 + 6	38.0
	3	1 + 9	39.4
	4	7 + 7	44.7
	5	4 + 6	45.7
	6	8 + 8	45.5
	7	3 + 7	51.5
	8	9 + 9	49.4
	9	2 + 8	51.1
	Group average		44.9
II	10	5 + 6	52.2
	11	4 + 8	53.7
	12	2 + 9	57.7
	13	7 + 8	61.0
	14	3 + 9	60.1
	15	6 + 7	62.2
	16	8 + 9	65.5
	17	5 + 7	61.9
	18	3 + 8	66.1
	Group average		60.0
III	19	6 + 9	68.8
	20	4 + 7	67.5
	21	5 + 9	71.3
	22	6 + 8	69.7
	23	4 + 9	69.2
	24	5 + 8	72.3
	25	7 + 9	71.5
	Group average		69.7

TABLE VIII. THE DIFFICULTY OF THE COMBINATIONS WHOSE SUMS AND MINUENDS ARE OVER NINE (After Clapp)

(This is not the order in which the combinations are to be taught.)

COMBINATION	PER CENT OF ERROR	COMBINATION	PER CENT OF ERROR	COMBINATION	PER CENT OF ERROR
5 + 5	1.0	5 + 6	8.7	11 − 8	16.3
8 + 8	2.2	9 + 4	9.0	11 − 5	16.8
9 + 2	2.4	9 + 5	9.0	13 − 6	17.2
9 + 9	2.6	6 + 7	9.0	15 − 8	17.2
7 + 7	2.6	4 + 9	9.1	11 − 7	17.6
2 + 8	3.1	10 − 4	9.1	11 − 9	17.8
2 + 9	3.3	7 + 5	9.5	18 − 9	18.9
6 + 6	3.5	4 + 7	10.0	16 − 8	19.0
9 + 1	3.7	8 + 6	10.8	11 − 6	19.5
6 + 4	3.8	8 + 9	10.8	11 − 4	19.7
8 + 3	3.9	10 − 6	10.9	12 − 5	19.8
8 + 2	3.9	5 + 9	10.9	12 − 7	19.8
1 + 9	4.6	10 − 5	11.2	14 − 8	20.1
6 + 5	5.5	9 + 6	11.5	15 − 6	20.4
4 + 8	5.8	8 + 7	11.6	12 − 3	20.9
8 + 4	5.8	7 + 8	11.7	14 − 6	21.0
7 + 3	5.9	5 + 7	11.8	11 − 3	21.0
3 + 9	6.4	12 − 9	12.1	13 − 8	21.6
3 + 8	6.5	6 + 9	13.1	13 − 7	21.8
9 + 3	7.0	6 + 8	13.8	16 − 7	22.4
3 + 7	7.6	9 + 7	14.2	13 − 5	23.1
9 + 8	7.7	12 − 8	14.3	13 − 9	23.5
10 − 2	7.8	5 + 8	14.9	15 − 7	24.3
10 − 1	8.0	8 + 5	15.0	17 − 8	24.4
11 − 2	8.0	7 + 9	15.0	15 − 9	25.8
7 + 4	8.0	10 − 8	15.2	17 − 9	26.3
7 + 6	8.1	12 − 4	15.3	14 − 5	26.7
10 − 7	8.1	14 − 7	15.4	16 − 9	26.8
12 − 6	8.3	10 − 9	15.6	13 − 4	26.9
4 + 6	8.6	10 − 3	15.8	14 − 9	27.9

The Difficulty of the Combinations

In order that you may the better regulate drill and distribute emphasis, Table VIII is given (see page 272). It represents the difficulty of all the hard combinations (forty-five in addition and forty-five in subtraction). This table may be compared with Table II, a similar table for the easy combinations, on page 25. You will no doubt note that many combinations in Table VIII are easier than some of the combinations in Table II. If, however, we confine our comparison to the addition combinations of the two tables, we shall find that the overlapping of difficulties is not very extensive. The same is true if we compare the subtraction facts of one table with the subtraction facts of the other. What gives Table II so many difficult combinations is the presence of the subtraction facts. Similarly, what gives Table VIII so many easy combinations is the presence of the addition facts. In a general way it remains true that for each operation the combinations given in Table VIII justify our calling them the hard combinations.

Suggestions as to Teaching

In teaching the harder combinations bear in mind the principles and methods set forth in earlier chapters. A few brief reminders may not be amiss.

Continue to teach the corresponding addition and subtraction facts together. Each will facilitate the learning of the other. In taking up a teaching unit, teach first the key combination, or direct addition fact. Whether to teach the reverse addition fact next or the direct subtraction fact is immaterial. It would seem that by this time the *teaching* of the three additional facts after the key combinations have been taught would be a very brief process. With the objective material already set up for $2 + 9$, the presentation of $9 + 2$, $11 - 2$, and $11 - 9$ will be easily and quickly accomplished.

This is one reason for organizing the combinations into teaching units so that those which bear upon and support one another may be presented together.

In the presentation of the key combination any suitable method which avoids obtaining the answer by counting may be adopted. Two have already been suggested. According to the first, the total number of objects is shown and apprehended; this total is divided into two groups according to the two terms of the combination to be taught; and a statement of the combination is secured, written on the board, and repeated both by the teacher and by the pupils. The combination is then posted in the place reserved for the permanent display of number facts and is applied in concrete problems (see Chapter III; see also the outline of this method on pages 76 to 77).

According to the second method, the teacher first selects as many objects as the total of the combination indicates, 12 objects if the combination is $5 + 7$. Part of these, corresponding to the first term of the combination, she then presents, making sure that the pupils apprehend this number. The objects are then concealed, and the additional objects representing the second term of the number fact are shown and apprehended. These are also concealed along with the first group. All the objects are then brought to view and their number ascertained. This leads to a statement of the number fact which has been concretely presented. It is written on the board and repeated frequently both by the teacher and the pupils. It is then posted and applied in verbal problems. A fuller description of this method in its application not only to addition facts but also to subtraction facts is given on pages 77 to 79.

Have children frequently "tell the whole story" when a single fact of a teaching group is asked for (see page 41). Be sure to adopt in much of your teaching and drill work a

varied response. Let it be frequently your practice to ask such questions as "4 and how many are 10? How many and 9 are 11? 12 less how many are 8?" and "How many less 5 are 6?" (See page 42.)

Finally, let the accuracy ideal prevail. Do not crowd the children to produce rapid work. They should not dawdle, but they should at no time get the notion that by doing many examples they may offer an excuse for inaccuracy. Teach the children to admit that they do not know or that they are in doubt when these admissions are justified. Teach them to use the posted combinations freely and never under any circumstances to "take a chance" in responding. Teach them that in written tests if they do not know the answer they should write nothing. Make frequent and resourceful use of concrete problems. Collect situations for use the next time you have to teach a class of this grade. Get the pupils interested in making up problems; and if they find this difficult give them models to imitate and sufficient data to bring the task within their powers. Keep a record of any interesting situations used by the pupils in making problems.

While the teaching of these combinations is in progress, it is of the highest importance that drill on the easier combinations should be continued. Our analysis of the total number of combinations into easy and hard ones is really an artificial division. It would play serious havoc with the learning of the pupils if it led you to abandon the easy facts when you began teaching the hard ones.

As has already been indicated (see page 170), the *hard* combinations may well be begun while column addition and higher-decade work based on the *easy* combinations is in progress. In this way much of the work called for in Chapters XVI, XVII, and XVIII may appropriately be done after the teaching of the hard combinations, set forth in the present chapter, has commenced.

Column Addition and Higher-Decade Work based on the Hard Combinations

The pupils have already become familiar with the ideas of column addition and of higher-decade addition and subtraction. These ideas should be applied as fast as the hard combinations are taught. For example, in Group I (see Table VII) all the facts whose sums and minuends are 10 are included. Accordingly, as soon as this group of facts has been taught, you may introduce column addition of three one-place numbers whose sums are 10, thus anticipating in part the work of Chapter XXIV. Such columns might be the following:

$$\begin{array}{ccc} 5 & 1 & 3 \\ 3 & 6 & 4 \\ \underline{2} & \underline{3} & \underline{3} \end{array}$$

One of the ideas which we have continually tried to bring into play is illustrated by this last suggestion. It is the idea of *putting a fact to use as soon as it is learned.* Too often the child is expected to hold the number combinations in suspension until some remote time when he will have use for them. Under such circumstances he has little or no present interest in them and, although he appears to learn them, they are continually dropping out of his mind, as things will when they are not tied to other things. There are many ways in which the fundamental number facts may be "tied to other things" and thus made meaningful. One way is to put them to use in new and hitherto untried operations. Another is to practice them from a new point of view. Both these ways are illustrated in the application of number facts to column addition as fast as they are learned. As soon as you teach $5 + 5$ you may, if you wish, introduce these six new columns of three addends:

$$\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 & 0 \\ \underline{5} & \underline{5} & \underline{5} & \underline{5} & \underline{5} & \underline{5} \end{array}$$

Moreover, as each of the new combinations is taught, as called for in this chapter, it may be drilled not only as a basic fact but also in its higher-decade applications. Thus with $2+8$ you may introduce $12+8$ and $22+8$. With $8+2$ you may teach $18+2$ and $28+2$. These higher-decade addition facts will be useful later in column addition. Similarly, with $10-2$ one may introduce the corresponding higher-decade subtraction combinations of Case I and Case II (pp. 217 and 220). The combinations are $20-2$ and $30-2$ (Case I) together with $20-12$ and $30-22$ (Case II). Associated with $10-8$ are $20-8$ and $30-8$ (Case I) together with $20-18$ and $30-28$ (Case II). Of these, $20-18$ and $30-28$ are especially important because they occur in short division.

Thus it may be seen that we have an extension of the idea of "telling the whole story" (see Chapter III). We have already pointed out that when $5+6$ is called for, the pupils may sometimes be expected to respond with the statement not only of this fact but also of the three related basic facts. We may now have a different story told. When $5+6$ is called for, the pupil may sometimes be expected to say "Five and six are eleven; fifteen and six are twenty-one; twenty-five and six are thirty-one." When $11-5$ is called for, the story may be indicated as $11-5=6$; $21-5=16$; $31-5=26$; or (Case II) it may be $11-5=6$; $21-15=6$; $31-25=6$. Observe that the sums and minuends in all these cases are less than 40. This is more or less arbitrary; but the relative importance of the smaller higher-decade combinations is unquestioned. Moreover, it is desirable to keep the related work from becoming too extensive. When the pupil sees how the basic facts apply in the 'teens and twenties, he is likely to have little difficulty in applying them in still higher decades.

Multiplication, division, and partition. As has already been indicated, with the doubles ($5+5$, $6+6$, etc.), it is recommended that the corresponding multiplication, division, and

partition facts be taught. It is suggested that the sign of multiplication ($\times$) be introduced at this point. Review, if necessary, the use of the sign of equality ($=$), and show that it may be read "are" or "is." Then in connection with teaching the multiplication of 5 by 2 introduce the form $2 \times 5 = 10$. Have this form read "two fives are ten," *not* "two times five are ten." Similarly, $2 \times 6 = 12$ will be read "two sixes are twelve."

In presenting the division facts introduce the form $5\overline{)10}\,2$. This form is to be read just as the division facts have hitherto been read, namely, "5's in 10 are 2." For the partition facts use the form $\frac{1}{2}$ of $10 = 5$. Have this form read "one half of ten is five."

The following facts are to be taught and posted, written and practiced, in connection with the doubles occurring in the first teaching group:

With $5 + 5$,	With $6 + 6$,	With $7 + 7$,
$2 \times 5 = 10$	$2 \times 6 = 12$	$2 \times 7 = 14$
$5\overline{)10}\,2$	$6\overline{)12}\,2$	$7\overline{)14}\,2$
$\frac{1}{2}$ of $10 = 5$	$\frac{1}{2}$ of $12 = 6$	$\frac{1}{2}$ of $14 = 7$

With $8 + 8$,	With $9 + 9$,
$2 \times 8 = 16$	$2 \times 9 = 18$
$8\overline{)16}\,2$	$9\overline{)18}\,2$
$\frac{1}{2}$ of $16 = 8$	$\frac{1}{2}$ of $18 = 9$

Progress Test on Group I, Table VII

The progress test presented herewith includes all the combinations of Group I. There are forty-one of them, five for each key combination that is a double and four for each of the other key combinations. Make the usual record sheet.

Test on the Hard Combinations of Group I

2	12	8	9	6	14	18
8	− 6	8	1	4	− 7	− 9
10	3	1	10	8	5	10
− 2	7	9	− 4	2	5	− 5
10	7	4	10	9	10	6
− 1	3	6	− 9	9	− 7	6
	16	7	10	10	10	
	− 8	7	− 3	− 6	− 8	

2×5	2×9	$\frac{1}{2}$ of 14
$6\overline{)12}$	2×7	$9\overline{)18}$
$\frac{1}{2}$ of 16	$5\overline{)10}$	2×6
$7\overline{)14}$	$\frac{1}{2}$ of 12	$\frac{1}{2}$ of 10
2×8	$\frac{1}{2}$ of 18	$8\overline{)16}$

1. Jean had 10 candles on her birthday cake. She let Baby Jack blow out 3 of them. How many candles were left for Jean to blow out?

Ans. ____

2. Ruth had 4 oranges for her party. She bought 6 more. How many oranges did she then have?

Ans. ____

The Combinations of Group II, Table VII

This group consists of nine teaching units involving a total of thirty-six different number facts. When Group II has been taught, all the number facts involving sums or minuends of 12 have been covered. This fact releases for use in problems a

great many useful applications of *foot* and *dozen*. See that these applications are made in your problems. Again it is suggested that drill in the use of these combinations be carried into column addition and into higher-decade addition and subtraction. Any column may now be used whose sum is 11 or 12, provided it does not involve $4+7$. This combination belongs in the next group. The progress test given in connection with this group contains all the new basic facts.

Progress Test on the Hard Combinations of Group II

5	15	6	8	11	12
6	− 7	7	4	− 2	− 3
5	8	6	7	17	9
7	3	5	6	− 9	2
3	11	9	4	13	8
8	− 9	8	8	− 6	9
12	7	12	2	11	13
− 8	8	− 9	9	− 5	− 7
11	9	11	12	3	12
− 3	3	− 6	− 7	9	− 4
12	15	7	11	8	17
− 5	− 8	5	− 8	7	− 8

1. Harry bought a dozen eggs. When he got home, he found 3 of them were broken. How many were not broken?

Ans. ____

2. Alice had a piece of ribbon one foot long. She cut off 4 inches. How many inches were left?

Ans. ____

The Combinations of Group III, Table VII

These are the hardest facts with which the children will have to deal in addition and subtraction. There are seven teaching units in this group and a total of twenty-eight number facts. These facts should be presented with unusual care, and each should receive plenty of drill and application in problems before the succeeding one is taught. They may be applied in column addition and in higher-decade work.

Progress Test on the Hard Combinations of Group III

6 9	11 − 7	6 8	9 5	4 7
13 − 8	8 6	9 6	13 − 9	7 9
14 − 8	15 − 6	11 − 4	5 9	16 − 9
13 − 5	7 4	8 5	14 − 9	9 4
5 8	14 − 5	4 9	15 − 9	14 − 6
	13 − 4	9 7	16 − 7	

1. Mabel wanted 15 cents for the school bank. She had only 9 cents. Her father gave her the rest. How many cents did her father give her?

Ans. ____

2. Robert sold 8 papers and Henry sold 5. How many papers did they both sell?

Ans. ____

Problems on Group I

Below are problems applying the number facts of Group I. In following sections there are problems which exemplify the facts of Group II and Group III. As to the use of these problems, the teacher's attention is called to what we have said in Chapters III and VII in connection with the problems given in those chapters.

Five and five

1. Lucy spent 5 cents for a ride on the merry-go-round and 5 cents for an ice-cream cone. How many cents did she spend for both?

2. Harriet brought her teacher 5 pink roses and 5 red ones. How many roses did she bring her teacher?

3. William rode 5 miles yesterday. Henry rode 10 miles. How many more miles did Henry ride than William?

4. Susie had written 10 words on her paper. Lillian had written 5 on hers. How many more words were there on Susie's paper than on Lillian's?

5. Ethel bought a 5-cent cake of chocolate. She gave the storekeeper a dime. How many cents should he give her in change?

6. There are 5 school days in a week. How many school days are there in 2 weeks?

7. If a yard of narrow ribbon cost 5 cents, how many cents will 2 yards cost?

8. Mrs. Green gave Tom 10 pennies to get changed into nickels. How many nickels did he get?

9. Sam feeds 5 apples a day to his rabbits. Now he has only 10 apples left. How many more days will the apples last?

10. One Saturday morning 2 boys bought some kite string for 10 cents. Each boy paid $\frac{1}{2}$ the cost. How many cents did each boy pay?

11. Hal said, "Let's have a snowball match." There were 10 boys and they took sides, $\frac{1}{2}$ of them on each side. How many boys were on each side?

Six and six

1. Esther bought half a dozen large oranges and half a dozen small ones. How many oranges did she buy?

2. Clara did 6 examples Monday and 6 Tuesday. How many examples did she do on both days?

3. Jane bought half a dozen white buttons and half a dozen brown ones. How many buttons did she buy?

4. Mrs. Smith had 1 dozen eggs. She kept 6 and sold the rest. How many did she sell?

5. Mrs. Brown ordered 12 handkerchiefs from the store. When they came there were only half a dozen. How many handkerchiefs were missing?

6. Mrs. Harper bought 2 packages of matches. Jane said, "Oh, mother, how many boxes of matches did you buy?" And Mrs. Harper said, "There are 6 boxes in each package. You tell *me* how many boxes I bought." What number do you suppose Jane said?

7. There are 6 working days in a week. How many working days are there in 2 weeks?

8. Nora washed some blankets Monday. She sent Polly to bring some clothespins, and Polly brought 12. Nora said, "I shall need 6 for each of these large blankets." How many blankets could she hang with the 12 pins?

9. Henry had a dozen tomato plants. If he set them 6 in a row, how many rows could he make?

10. Ray bought half a dozen oranges. How many oranges did he buy?

11. How many inches long is a ruler that is half a foot long?

One and nine

1. There was just 1 man (the driver) in a bus. The bus stopped and 9 men got in. How many men were there in the bus then?

2. First 1 little rabbit found a hole in the pen and went through it. Then 9 more little rabbits found it and *they* went through. How many little rabbits went through the hole?

3. On our baseball team there were 9 little boys besides 1 little boy to play for anyone who stayed away. How many boys were there on the team all together?

4. Last summer Harold earned 9 dollars by helping the grocer. His father paid him 1 dollar for watering the garden. How many dollars did Harold earn last summer?

5. Bessie had a dime, but she had to spend 1 cent for a pen. How many cents did Bessie have left?

6. Mother told Louis to go to the garden and pull up 10 beets. When he had pulled 1, how many more beets did he have to pull?

7. Harry had 10 newspapers. He sold 9. How many had he left?

8. Albert picked 10 quarts of cherries and Ted picked 9. How many more quarts did Albert pick than Ted?

Seven and seven

1. There are 7 girls and 7 boys playing a game. How many children are in the game?

2. Mother used 7 pounds of sugar for making grape jelly and 7 pounds for apple jelly. How many pounds of sugar did she use in the jelly?

3. At recess 14 boys had a snowball battle. There were 7 on one side. How many boys were there on the other side?

4. Tom and Ted were hunting for nuts. Tom found 14 and Ted found 7. How many more nuts did Tom find than Ted?

5. Lena asked her father how long it would be before she went to see her cousin. Father said, "Just 2 weeks." Lena knew that there are 7 days in a week. So she knew how many days it would be before she went away. How many?

6. Jane went to the store to buy 2 pounds of rice. It was 7 cents a pound. How many cents did she pay for the rice?

7. Mr. Carter bought a set of 14 books, and 7 of them filled a shelf in his bookcase. How many shelves did his set of books fill?

8. If 1 bunch of silk thread costs 7 cents, how many bunches can you buy for 14 cents?

9. Charles and Walter planned to walk to a place 14 miles away. They were going to use 2 days for the journey, and walk half the distance each day. How many miles did they plan to walk each day?

10. The butcher said that the ham hanging up weighed 14 pounds. Mrs. Horn said she would like to have $\frac{1}{2}$ of it. How many pounds of ham did Mrs. Horn want?

Four and six

1. Last summer 4 children were living on a farm, and 6 little cousins came from the city and lived with them. How many children lived on the farm last summer?

2. Wallace is 4 years old. Henry is 6 years older than Wallace. How many years old is Henry?

3. John planted 6 rows of corn. Philip planted 4 rows. How many rows did they both plant?

4. There were 6 cows in the pasture and Mr. White put in 4 more. How many cows were in the pasture then?

5. Harold had saved 10 dollars, but he spent 4 dollars for Christmas presents. How many dollars had he left?

6. James is 10 years old. His little brother is 4 years old. James is how many years older than his little brother?

7. Frederick and William hoed 10 rows of beans. Frederick hoed 4 rows. How many rows did William hoe?

8. Mrs. White bought 10 towels; 6 of them had red borders and the others had blue borders. How many blue-bordered towels did Mrs. White buy?

9. There were 10 blackbirds on the wheat stack; 6 flew away. How many blackbirds remained on the stack?

10. This morning Henry found a nest with 10 eggs in it. He sold half a dozen of the eggs to Mrs. Brown. How many eggs did he have left?

Eight and eight

1. Allen lived 8 blocks from his school. How many blocks did he walk in going to school and back?

2. A baker boy sold 8 loaves of white bread and 8 loaves of brown bread. How many loaves of bread did he sell?

3. Albert and Charles stood facing each other 16 feet apart. Charles walked 8 feet towards Albert. How far apart were they then?

4. There were 16 cookies in the jar. Mother went to get some and there were only 8. How many cookies had been taken?

5. Billy said he had 8 chestnuts in each pocket. He had 2 pockets. How many chestnuts did Billy have?

6. Grace made two badges. Each one took 8 inches of ribbon. How many inches did both badges take?

7. Jennie had 16 cents. She wanted to buy something to give to the Children's Home. If tops were 8 cents each, how many could she buy?

8. It took Mr. Peterson 16 hours to paint the barn. He worked 8 hours a day. How many days did it take him?

9. Harold came into the house and said, "Oh, Hattie, I have found some big blackberries. I'll give you half of them." When they counted the blackberries they found that there were 16. How many did Hattie get?

10. There were 16 children at a party. One table was set in the dining room and another just like it on the porch. How many children sat at each table?

Three and seven

1. Raymond picked 3 quarts of berries one day and 7 quarts the next. How many quarts did he pick in the two days?

2. One day 3 girls went out in one boat and 7 in another. How many girls were there in both boats?

3. In a week there are 7 days. In a week and 3 days, how many days are there?

4. Mr. White planted 7 rows of cabbages and Donald planted 3 rows. How many rows did they both plant?

5. Tom took 10 kernels of corn in his hand to feed his pet chicken Dick. Dick was busily eating when along came little dog Jack and frightened Dick away. In Tom's hand were 3 kernels of corn. How many kernels had he given his pet chicken?

6. Polly took a dime to the store and spent 3 cents for a yeast cake. How much change did she get?

7. There were 10 haycocks in the hayfield. Mr. Hill carted away 7 of them. How many haycocks were left in the field?

8. Herbert used to walk to school in 7 minutes. Now his little sister goes with him and it takes him 10 minutes. How many more minutes does it take now than it used to take?

Nine and nine

1. The baker made 9 apple pies and 9 custard pies. How many pies did he make in all?

2. Yesterday 9 girls and 9 boys went on the lake for a boat ride. How many children went on the boat ride?

3. Baby Ruth is 18 months old. Baby Dick is 9 months old. How many months older is Ruth than Dick?

4. A broom seller had 18 brooms in the morning. At noon he had only 9. How many brooms had he sold?

5. Mrs. Gray was a dressmaker. She bought 2 papers of pins at 9 cents each. How many cents did they cost?

6. Herbert's brother earns 9 dollars a week. Can he earn enough in 2 weeks to buy a raincoat that costs 18 dollars?

7. The coal man's big truck will bring 9 tons of coal at a time. How many trips will it have to make to bring 18 tons?

8. It takes 9 boys to make a baseball team. In the fifth grade there are just 18 boys. How many teams can be made in the fifth grade?

9. Harry and Bill were going fishing. They had to buy hooks and lines. They said each would pay half. If the hooks and lines cost 18 cents, how much would each have to pay?

10. Mrs. Hudson bought 18 milk tickets for 2 dollars. How many could she have bought for 1 dollar?

Two and eight

1. There are 2 spools of black thread and 8 spools of white thread in the work basket. How many spools of thread are there in the basket?

2. At Mr. Jones's house the milkman left 2 bottles of cream and 8 bottles of milk. How many bottles did he leave?

3. All the girls in camp were busy; 8 of them were putting the tents in order and 2 were washing dishes. How many girls were in camp?

4. There were 8 ladies at our Mothers' Meeting. Then 2 more ladies came in. How many ladies were then at the meeting?

5. Paul caught 10 fish, but he let 2 of them go. How many did he keep?

6. Esther had a fruit and vegetable stand by the roadside. She had 10 boxes of plums and sold 2 boxes. How many boxes did she still have?

7. Mother gave Clifford a dime and told him to go to the store and buy her an 8-cent loaf of bread. She said that he might keep the change. How many cents were left for Clifford?

8. There were 10 boys playing on a porch, and 8 ran away to hide. How many boys were left on the porch?

9. There were 10 new books on the shelf; 2 of them were green and the rest were brown. How many brown books were there?

Problems on Group II

Five and six

1. Ella paid 5 cents for a skein of silk and 6 cents for a spool of cotton. How many cents did she pay for both?

2. John earned 5 cents washing the porches and 6 cents cleaning the walks. How many cents did he earn?

3. Jack's mother made 6 glasses of jelly. The next day she made 5 glasses more. How many glasses of jelly did she make?

4. Tom worked for the baker. He sold 6 apple pies and 5 pumpkin pies. How many pies did Tom sell?

5. Mr. Hulse bought 11 gallons of gasoline. He took a long drive and used up 5 gallons. How many gallons had he left?

6. Mr. Smith's cow gave 11 quarts of milk. Mr. Smith had to keep 5 quarts for his family. How many quarts could he sell?

7. Farmer Brown had 11 bushels of apples. He sold 6 bushels. How many bushels did he keep?

8. Jack had 11 raisins. He gave 6 to his little brother. How many did he have for himself?

Four and eight

1. The grocer put 4 bananas into a sack. Then he put in 8 more. How many bananas did he put into the sack?

2. Ella sold pop-corn balls at a fair. She sold 4 pink ones and 8 white ones. How many pop-corn balls did she sell?

3. Dick and Tom were going to have a lemonade stand. Dick bought 8 lemons and Tom bought 4. How many lemons did they have?

4. There are 8 rows of beans and 4 rows of corn in Walter's garden. How many rows of beans and corn are there together?

5. There were 12 red checkers on the board and 4 were taken off. How many red checkers remained on the board?

6. There were a dozen eggs in a box. Mother took out 4 for breakfast. How many eggs were left?

7. Jeannette is trying to sell 12 tickets. She has sold 8 of them. How many tickets does she still have to sell?

8. There are 12 months in a year. When 8 months are passed, how many months are left in the year?

9. Walter has saved 8 eggs. When he has saved a dozen he will sell them. How many more eggs must he save before he sells them?

Two and nine

1. There were 2 children skating on the pond. Then 9 more came to skate. How many children were then skating on the pond?

2. George rode 2 miles on his bicycle and 9 miles on the train. How many miles did he ride?

3. The tail of Jack's kite was 9 feet long. If he added 2 feet to it, how many feet long was it then?

4. Mr. Davis sold 9 bushels of apples at the market and 2 bushels to Mrs. Gray. How many bushels of apples did he sell?

5. Philip picked 11 quarts of berries and sold 2 quarts. How many quarts did he have left?

6. John put 11 grasshoppers in a box. Soon he found there were only 2. How many grasshoppers had hopped out?

7. One Monday Mother Robin brought her little ones 11 cherries. Tuesday she brought only 9. How many more cherries did she bring on Monday than on Tuesday?

8. The teacher told Helen to read 11 pages. She has read 9 of them. How many more pages has she to read?

Seven and eight

1. Biddy took her baby chickens out for a walk; 7 walked on one side of her and 8 on the other. How many baby chickens were walking with Biddy?

2. There were 7 yellow tulips and 8 red ones in the garden. How many tulips were there in the garden?

3. Mr. McGregor was in the field dropping corn. He looked behind him and saw that there were 8 blackbirds and 7 crows picking up the corn. How many birds were picking up the corn?

4. Eight boys and 7 girls were playing picture show. How many children were at the show?

5. Fifteen children were making a school garden. The teacher called 7 of them in. How many children stayed out working in the garden?

6. Recess was 15 minutes long. Florence used 7 minutes of it to finish writing her words. How many minutes did she have left for play?

7. Mother had 15 apples. She saved 8 for the children and put the rest into pies. How many apples were put into the pies?

8. Mabel has 15 "jacks." Elsie has 8. How many more "jacks" has Mabel than Elsie?

Three and nine

1. Albert sold 3 bunches of beets and 9 bunches of carrots. How many bunches of vegetables did he sell?

2. Amy had a fruit stand by the roadside. On Saturday afternoon she sold 3 baskets of pears and 9 baskets of peaches. How many baskets of fruit did she sell that afternoon?

3. Play it is 9 o'clock now. In 3 hours more the circus parade will pass. At what time will the parade pass?

4. The circus train has 9 cars for animals, tents, and other things, and 3 cars for people. How many cars are there in the circus train?

5. There were a dozen oranges in a paper sack and we took out 3. How many oranges were left in the sack?

6. There were 12 spoons on the table and Baby took off 3. How many spoons were left on the table?

7. There were a dozen buttons on a card and Mary took off 9. How many buttons were left on the card?

8. In a house I know there are 12 windows upstairs and 9 downstairs. How many more windows are there upstairs than downstairs?

9. Mrs. Gray bought a dozen eggs and boiled 9 for a picnic lunch. How many eggs did she have left?

10. Mr. White wants to buy a dozen chickens. Harry will sell him 9. How many more chickens will Mr. White have to buy?

Six and seven

1. Harold has 6 brown rabbits and 7 white rabbits. How many rabbits has Harold?

2. Some houses are called apartment houses. One apartment house had 6 rooms on the first floor and 7 rooms on the second floor. How many rooms were there on both floors?

3. Howard wants to buy a bicycle. He has 7 dollars, but he says that he will have to save 6 dollars more before he can buy it. How much will the bicycle cost?

4. Seven girls and 6 boys went out to the grove for a picnic. How many children went to the picnic?

5. There were 13 sparrows on the back porch. Harry opened the door and 6 of them flew away. How many sparrows were left on the porch?

6. Margaret took 13 cents to school for lunch money. She spent 6 cents for sandwiches and the rest for fruit. How many cents did the fruit cost?

7. A farmer had 13 loads of hay. He sold 7 loads. How many loads did he keep?

8. It was June 14 — Flag Day. The storekeeper put 13 small flags in his window. In a little while there were only 7 left. How many flags had he sold?

Eight and nine

1. Stephen has 8 wooden soldiers and 9 tin soldiers. How many soldiers has he?

2. There are 8 buttons on Martha's best dress and 9 on her school dress. How many buttons are there on both dresses?

3. Some Sunday schools have a birthday box. Jane put 9 pennies into the box and Eva put 8. How many pennies did they both put into the birthday box?

4. Mr. Jones made 9 gallons of maple sirup one day, and the next day he made 8. How many gallons did he make in both days?

5. Mr. McGregor had 8 cabbage plants in his garden. He planted 9 more. How many cabbage plants did he then have?

6. There were 17 candies in a box. The children took out 8. How many candies were left in the box?

7. There were 17 children in a Sunday-school class. One Sunday it snowed and 8 children stayed home. How many children were in the class that day?

8. There were 17 chairs in the reading corner of our room. Now there are only 9. How many chairs have been taken away?

9. On Sunday 17 automobiles passed our house and 9 passed on Monday. How many more automobiles passed on Sunday than on Monday?

Five and seven

1. Ernest sold 5 papers before supper and 7 after supper. How many papers did he sell that evening?

2. In a field there were 5 little pumpkins and 7 big ones. How many pumpkins were there in the field?

3. Albert gathered 7 ears of yellow corn and 5 ears of white corn. How many ears of corn did he gather?

4. There are 7 red beads and 5 blue beads on a string. How many beads are there on the string?

5. A carpenter had a board that was 12 feet long. He sawed off 5 feet. How many feet were left in the board?

6. In an orchard there are 12 apple trees and 5 pear trees. There are how many more apple trees than pear trees?

7. Mother bought a piece of muslin 12 yards long. She used 7 yards of it. How many yards were left in the piece?

8. Ernest received 12 Christmas cards this year. Last year he received only 7. How many more cards did he receive this year than last?

9. Mr. McGregor set out a dozen celery plants. The next morning there were only 7 of them left. How many had Peter Rabbit eaten?

Three and eight

1. Susie bought 3 yards of wide lace and 8 yards of narrow lace. How many yards of lace did she buy?

2. There were 3 boys and 8 girls playing a game. How many children were playing?

3. There were 8 apples in a basket and Lulu put in 3 more. How many apples were then in the basket?

4. Joe's watch cost 8 dollars and his chain cost 3 dollars. How many dollars did the watch and chain cost?

5. If little Bo-Peep had 11 sheep and lost 3 of them, how many sheep did she still have?

6. There were 11 ducks in a pond and 3 of them went up on the shore. How many ducks remained in the pond?

7. Speckle has 11 baby chicks and Biddy has 8. How many more chicks has Speckle than Biddy?

8. Eleven children stayed after school to practice a play. Then 8 of them went home and the others stayed to help Miss Brown. How many children stayed to help?

Problems on Group III

Six and nine

1. It took Frances 6 minutes to get her books and put on her wraps and 9 minutes to walk to school. Books, wraps, and walk — how long did they take?

2. Susie and Nellie sold lemonade at a picnic. Susie gave half a dozen lemons and Nellie gave 9. How many lemons did they both give?

3. There were 9 little dogs and 6 big dogs acting in a circus. How many dogs were in the circus?

4. Mrs. Gray sold 9 quarts of milk each morning and 6 quarts each evening. How many quarts did she sell each day?

5. There were 15 children in the reading class. They had only 9 reading books. How many more books did they need?

6. A vegetable man had 15 bushels of potatoes in his wagon and sold 9 bushels. How many bushels were left in the wagon?

7. There were 15 boys in a line playing soldier; 6 were called away. How many boys were left in the line?

8. John needs 15 feet of string. He has a piece that is 6 feet long. How many more feet does he need?

Four and seven

1. There are 4 people who live at Margery's house, and 7 people came to a dinner party. How many people were at Margery's house when they had the party?

2. William carried home 4 pounds of meat and 7 pounds of sugar. How many pounds did William carry?

3. Miss Howell's class was planning for a drill. Miss Howell bought 7 yards of red bunting and 4 yards of blue. How many yards of bunting did she buy?

4. In the drill there will be 7 girls and 4 boys. How many children will there be in the drill?

5. There were 11 books on the bookshelf. Jane took down 4. How many books were left on the shelf?

6. Paul had 11 marbles and gave 4 to Harry. How many marbles did Paul still have?

7. Ruth is 11 years old. Emma is 7. How many years older than Emma is Ruth?

8. A grocer had 11 packages to send out. The boy could carry only 7. How many packages were left at the store?

Five and nine

1. What would be the cost of a box of matches at 5 cents and a package of oatmeal at 9 cents?

2. Allen earned a nickel by raking the lawn and 9 cents by picking strawberries. How many cents did he earn by his work?

3. In going to grandma's Ella rode 9 miles on the train and 5 miles in an automobile. How many miles did she ride in going to grandma's?

4. Goldilocks picked 9 daisies and 5 buttercups. How many flowers did she pick?

5. Arthur earned 14 cents and from it he took a nickel to Sunday school. How many cents did he have left?

6. Catherine wants a fur piece that will cost 14 dollars. She can pay 5 dollars and her mother will pay the rest. How much will her mother have to pay?

7. Alice read 14 pages. Mary read 9. How many more pages did Alice read than Mary?

8. There were 14 white chickens and 9 black ones in the chicken yard. How many more white than black chickens were there?

Six and eight

1. Last summer Lucy spent 6 days at the seashore. Next summer she will spend 8 days more than she did last summer. How many days will she spend at the seashore next summer?

2. Tom and Ted were playing dominoes. Tom won 6 games and Ted won 8. How many games did they play?

3. One day Mr. Squirrel stored away 8 nuts for the winter and Mrs. Squirrel stored 6. How many nuts did they both store that day?

4. Sarah sold 8 tickets. Alfred sold 6 more than Sarah. How many tickets did Alfred sell?

5. Once 14 animals held a meeting to see what should be done about Brother Fox, and 8 of them said, "Let him stay with us." The others said, "Send him away." How many animals wanted to send him away?

6. Walter promised to pick 14 quarts of strawberries one day. By noon he had picked 8 quarts. How many quarts did he have to pick in the afternoon?

7. There were 14 books on the teacher's desk. She lent 6 of them to some of the children to take home. How many books were left on the teacher's desk?

8. Mr. McGregor set out 14 tiny cabbage plants one day. The next day there were only 8 left. How many plants had been taken?

Four and nine

1. On father's desk there are 4 picture post cards and 9 plain post cards. How many post cards are there on the desk?

2. Ella spent 4 cents for pencils and 9 cents for blank books. How many cents did she spend for pencils and blank books?

3. Daisy spent 9 cents for a package of flower seeds and 4 cents for a package of radish seeds. How many cents did she spend for both packages?

4. One day 2 boys set out on a long walk. They had walked 9 miles and were very tired. Just then they saw on a sign an arrow pointing straight ahead, and the words "4 miles to Restville." And so they walked 4 miles more. How far did the boys walk that day?

5. Father said he had to write 13 letters. After he had written 4 of them, how many letters did he still have to write?

6. Miss Clark had a piece of red-and-white ribbon that was 13 inches long. She cut off 4 inches to use in making a monitor's badge. How many inches of ribbon were left?

7. Ethel picked 13 roses. She gave 9 to her mother and the rest to her teacher. How many roses did she give to her teacher?

8. John wants to buy a coat that costs 13 dollars. He has 9 dollars. How many more dollars does he need?

Five and eight

1. Brother Rabbit had an animal meeting; 5 animals came and a little later 8 more came. How many animals came to the meeting?

2. Harry has 5 large marbles and 8 small ones. How many marbles has he all together?

3. Red Riding Hood carried her grandmother 8 red apples and 5 yellow ones. How many apples did she carry in all?

4. Robert has 8 cents. His mother gives him a nickel. How many cents will he have?

5. There are 13 rooms in the boarding-house and 5 rooms in the cottage. How many more rooms are there in the boarding-house than in the cottage?

6. There were 13 sheep and 5 lambs in a pasture. How many more sheep than lambs were there?

7. There were 13 children dancing on the lawn; 8 of them stopped to rest. How many children were still dancing?

8. There were 13 little girls at a lawn party. Every little girl wore a pink or a white dress; 8 of them wore white dresses. How many wore pink dresses?

Seven and nine

1. Louis's shoes cost 7 dollars and his coat cost 9 dollars. How many dollars did his shoes and coat cost?

2. There are 7 rabbits in one pen and 9 in another. How many rabbits are there in both pens?

3. There are 9 ducks in one flock and 7 in another. How many ducks are there all together?

4. There were 9 cars in one train and 7 in another. How many cars were there in both trains?

5. Alice saw 16 robins and 7 wrens. How many more robins than wrens did she see?

6. John said, "I have seen 16 robins this spring." Wallace said, "I have seen only 7." How many more robins has John seen than Wallace?

7. A freight train carried 16 cars; 9 of them were box cars, and the others were coal cars. How many of them were coal cars?

8. Mr. Hawkins's car will go 16 miles and Mr. Miller's 9 miles on a gallon of gasoline. How many more miles will Mr. Hawkins's car go on a gallon of gasoline than Mr. Miller's?

CHAPTER XXII

HIGHER-DECADE ADDITION WITH BRIDGING

In Chapter XVII it was pointed out that life situations constantly call for the addition of a one-place number to a two-place number, that is, to a higher-decade number. Problems often present themselves to children as well as adults in that form. Moreover, column addition and carrying in the process of multiplication require this kind of addition.

It was likewise suggested in Chapter XVII that the application of the basic combinations to higher-decade addition afforded an opportunity to put them to work on material of high social value. The examples and problems in that chapter, however, were limited to instances in which the sum was in the same decade as the two-place number, such as $16 + 3$, $24 + 2$, and $32 + 6$. In other words, we were there concerned with higher-decade addition *without bridging.*

Now that the basic combinations whose sums exceed 9 have been taught, we are prepared to extend higher-decade addition to instances where the sum is in the decade next higher than that of the two-place addend. For example, since the pupils have learned $6 + 7 = 13$, they may well apply this fact to $16 + 7$ or to $36 + 7$. This chapter will be devoted to such applications; and since in each case the pupil must carry his thinking over into the next higher decade, or "bridge the tens," we call this *higher-decade addition with bridging.*

A moment may be spent on the question, How is a child to know whether a given combination "bridges the tens"? In other words, When does he go into the next higher decade? Let the pupil be guided by the units' figure; that is, in $16 + 3$

by the 6 and the 3, in $18 + 5$ by the 8 and the 5. If these units' figures when added give 10 or more, that is, if they are what we are calling a "hard" combination, then the answer is one decade higher than the two-place number. If the units' figures when added are less than 10, an "easy" combination, then the answer is in the same decade as the two-place number. In $16 + 3$, since $6 + 3$ is less than 10, the answer, like 16, is in the 'teens. In $18 + 5$, since $8 + 5$ is 10 or more, the answer is in the decade next higher than the 'teens; that is, in the twenties.

Analysis of Higher-Decade Combinations Involving Bridging

Corresponding to the four hundred and five combinations without bridging (see page 194), there are three hundred and sixty which involve bridging. Of these three hundred and sixty, there are ninety whose sums are less than 40. Since most columns which we have to add are short, we may very well limit our work at this juncture to those whose sums are less than 40. This means that, for the present, as far as use in column addition is concerned, we need not consider higher-decade combinations whose sums exceed 39. It is worth pointing out that even when longer columns have to be added, requiring combinations whose sums are 40 or more, the lower combinations are *passed through* on the way to the higher ones. We cannot get to the forties without passing through the 'teens, twenties, and thirties. Thus the smaller combinations are used even in adding long columns. In addition to the ninety higher-decade combinations whose sums are less than 40, there are thirty-five more which are used in carrying in multiplication. These one hundred and twenty-five are especially useful, and they should be carefully provided for.

In order to identify the one hundred and twenty-five most useful higher-decade combinations involving bridging, we have first to consider the ninety whose sums are less than 40. There will be no difficulty here. To the number 10 no one-place number can be added so as to cause bridging. To 11 only the number 9 may be added. Therefore $11 + 9$ is the first higher-decade combination with bridging. To 12 we may add 8 and 9. Thus two more combinations arise. To 13 we may add 7, 8, and 9; to 14 we may add 6, 7, 8, and 9; and so on up to 19, to which we may add 1, 2, 3, 4, 5, 6, 7, 8, and 9. We thus find that there are forty-five of these combinations in which the two-place number is in the 'teens. By the same process we find that there are forty-five in which the two-place number is in the twenties. This makes the total of ninety to which reference has already been made. It is clear that the two-place addend cannot be in the thirties because bridging the tens would then mean that the sum would be 40 or more.

It remains to identify the thirty-five higher-decade combinations involving bridging whose sums are 40 or more, but which have high social utility because they are used in multiplication. It is suggested that at this point the teacher re-read pages 194 to 198 inclusive. From what is there stated we may say that each of the thirty-five combinations which we are now considering is characterized by the following facts: (1) the two-place addend is a product in the multiplication table, like 36, 49, or 56; (2) the one-place addend is less than the larger factor in this product, that is, less than 9, 7, or 8 in the two-place products just mentioned; (3) the sum of these two addends is 40 or more; and (4) the sum is in the decade next above that of the two-place addend. Table IX gives the thirty-five combinations which meet these conditions.

TABLE IX. HIGHER-DECADE COMBINATIONS USED IN MULTIPLICATION; SUMS 40 OR MORE, BRIDGING INVOLVED

35 + 5	36 + 4	45 + 5
6	5	6
	6	7
	7	8
	8	
48 + 2	49 + 1	54 + 6
3	2	7
4	3	8
5	4	
6	5	
7	6	
56 + 4	63 + 7	64 + 6
5	8	7
6		
7		72 + 8

Thus we have seen that of the total of seven hundred and sixty-five possible higher-decade combinations arising, as indicated on page 194, there are three hundred and sixty which involve bridging the tens; that of these three hundred and sixty there are one hundred and twenty-five which are especially useful; and that the remaining two hundred and thirty-five are less useful. It is not intended, however, that the two hundred and thirty-five which we have called "less useful" should be omitted from the drill. Such combinations as $57 + 4$, $48 + 9$, and $64 + 8$ are often needed, even though they do not occur in adding short columns or in multiplication. In the counting and measuring activities of children, 44 is scarcely less likely to occur than 45, and additions to the former may be as common as additions to the latter. In *problem work*, therefore, any higher-decade additions may be needed.

Method of Presentation

The method suggested in Chapter XVII may be used. The following procedure has also been found to be effective.

Review tens and units. Have pupils tell how many tens and units there are in a given number and identify numbers whose tens and units are given. Put these combinations on the board:

$$6 + 4 = 10 \qquad 26 + 4 = 30$$
$$16 + 4 = 20 \qquad 36 + 4 = 40$$

Have the pupils verify these combinations. Count 16 objects; conceal them; count 4 objects of the same kind, and conceal them with the 16; bring all to view and count the resulting 20. Write this answer in its proper place. Proceed in the same way with the other two higher-decade combinations. Then have the pupils read the combinations singly and in concert. Tell them that this is in the six-and-four family (pointing to the first example). In the second, third, and fourth examples show that the units' figure of the answer is a zero the same as it is in $6 + 4 = 10$. Show that in $6 + 4 = 10$ the 6 has no ten and the answer has one ten, one more ten than the first number. Show that the same is true with the other examples; that is, the tens' figure is one more than the tens' figure of the first number. Show, in summary, that all the six-and-four family behave like $6 + 4 = 10$. The units' figure of the answer is always a zero, and the tens' figure is one more than the tens' figure of the first number.

"What would $46 + 4$ be? $56 + 4$? Say the six-and-four family to $56 + 4 = 60$."

Put another family on the board and go through the same procedure. This may be:

$$4 + 7 = 11 \qquad 24 + 7 = 31$$
$$14 + 7 = 21 \qquad 34 + 7 = 41$$

Bring out the fact that $4 + 7 = 11$ tells the whole story. It tells that the units' figure of the answer is one and that the tens' figure is one more than the tens' figure written beside the 4.

Let the teacher now drill the pupils on telling the family to which given higher-decade combinations belong. For example: "We have seen that $16 + 4 = 20$ belongs to the six-and-four family and that $24 + 7 = 31$ belongs to the four-and-seven family. To what family does $13 + 9 = 22$ belong? $15 + 8 = 23$? $27 + 3 = 30$? $26 + 8 = 34$?" Give the pupils higher-decade combinations *without* the answers, and have them tell the corresponding *complete* basic facts. Then let them infer the answer to the higher-decade combinations. For example, if they say that the incomplete combination, $14 + 9$, belongs to the four-and-nine family, and if they know that 4 and 9 are 13, they should be able to give $14 + 9 = 23$. Similarly, if they recognize $6 + 7 = 13$ in $16 + 7$, they should be able to give $16 + 7 = 23$.

Considerable drill on work of this sort should be given, at first going back to the basic combination each time. The number chart (see page 62) may then be used frequently. Introduce some combinations in which no bridging of the tens is required, such as $26 + 2$ and $15 + 3$. For the most part, though not entirely, use the numbers in the second, third, and fourth columns of the chart and the multiplication-table products in the columns farther to the right. It will be a good plan to underscore or put a ring around these products to guide you in your drill. Remember that the one-place numbers you especially wish to add to these products are in each case less than the larger factor in the product. To 45 you may add any number from 1 to 8. If you add 9, you are going beyond the carrying that will occur in multiplication. There is, of course, no harm in doing this, but the chief emphasis should be on the combinations that will be most often used. You will observe that when 1, 2, 3, or 4 are added to 45 there is no bridging. These combinations have been introduced in

Chapter XVII. As has been indicated, they should be drilled at this juncture not only to keep the pupils proficient in them but also to break up the idea that all the combinations are now going to result in an answer in the next higher decade.

When higher-decade combinations are written, they should be in horizontal form, generally with the two-place number first.

Practice Material, Higher-Decade Addition with Bridging

Below are given a number of sets of examples in higher-decade addition involving bridging the tens. Section I, consisting of ten sets each of forty-five examples, is limited to combinations whose sums are less than 40. As has already been indicated, there are ninety of these combinations. Each is practiced in Section I exactly five times. None of them occurs twice in the same set.

Section II comprises examples whose sums are 40 or more and in which the two-place number is a product in the multiplication table. As we have already shown, there are thirty-five combinations of this kind. In Section II each of these occurs either five or six times. The arrangement in Section II is likewise such that a given combination does not appear twice in the same set.

Section I. Sums less than 40

a

19 + 5	15 + 5	17 + 5	3 + 29	23 + 7
15 + 9	17 + 3	29 + 7	13 + 8	26 + 9
22 + 8	14 + 8	29 + 9	19 + 6	23 + 9
7 + 13	17 + 4	19 + 8	9 + 17	29 + 1
26 + 7	16 + 8	22 + 9	26 + 5	9 + 11
14 + 7	8 + 23	4 + 19	24 + 9	29 + 5
25 + 7	29 + 4	15 + 6	25 + 5	21 + 9
8 + 29	15 + 7	19 + 2	28 + 8	6 + 29
14 + 9	19 + 7	17 + 8	26 + 4	27 + 9

b

7 + 17	16 + 9	6 + 17	9 + 13	25 + 9
24 + 8	18 + 9	25 + 6	26 + 8	3 + 19
28 + 7	25 + 8	9 + 19	26 + 6	1 + 19
28 + 9	12 + 8	15 + 8	2 + 29	12 + 9
16 + 4	4 + 18	14 + 6	28 + 2	27 + 3
18 + 7	27 + 4	28 + 5	24 + 6	16 + 6
27 + 6	18 + 8	16 + 7	18 + 6	16 + 5
27 + 7	28 + 4	24 + 7	18 + 2	28 + 6
27 + 5	18 + 3	27 + 8	18 + 5	28 + 3

c

15 + 7	4 + 19	17 + 9	26 + 9	19 + 5
15 + 5	22 + 8	29 + 1	24 + 9	19 + 2
17 + 5	14 + 8	26 + 7	5 + 29	28 + 8
3 + 29	29 + 9	16 + 8	25 + 7	29 + 6
7 + 23	19 + 6	22 + 9	29 + 4	14 + 9
15 + 9	23 + 9	26 + 5	15 + 6	7 + 19
17 + 3	7 + 13	9 + 11	25 + 5	17 + 8
29 + 7	13 + 8	19 + 8	23 + 8	29 + 8
27 + 9	4 + 17	14 + 7	21 + 9	26 + 4

d

28 + 4	28 + 5	2 + 29	3 + 19	7 + 17
16 + 9	28 + 7	12 + 9	24 + 6	24 + 7
18 + 2	16 + 6	16 + 4	25 + 8	6 + 17
9 + 13	9 + 19	18 + 4	27 + 6	28 + 6
27 + 5	18 + 8	14 + 6	26 + 6	25 + 9
24 + 8	1 + 19	28 + 2	16 + 7	18 + 3
27 + 8	18 + 6	27 + 3	28 + 9	18 + 9
25 + 6	12 + 8	7 + 18	16 + 5	18 + 5
28 + 3	27 + 7	27 + 4	15 + 8	26 + 8

e

15 + 5	19 + 5	17 + 5	26 + 9	22 + 8
14 + 8	9 + 17	29 + 1	26 + 7	19 + 4
24 + 9	29 + 5	15 + 7	19 + 2	28 + 8
3 + 29	23 + 7	15 + 9	9 + 29	19 + 6
23 + 9	16 + 8	22 + 9	26 + 5	25 + 7
29 + 4	15 + 6	29 + 6	14 + 9	7 + 19
17 + 3	29 + 7	8 + 13	13 + 7	17 + 4
8 + 19	11 + 9	14 + 7	8 + 23	25 + 5
21 + 9	8 + 29	17 + 8	26 + 4	27 + 9

f

9 + 16	7 + 17	6 + 17	3 + 19	28 + 7
25 + 8	2 + 29	12 + 9	16 + 4	28 + 5
24 + 6	16 + 6	28 + 4	24 + 7	18 + 2
9 + 13	25 + 9	24 + 8	9 + 19	26 + 6
1 + 19	18 + 4	14 + 6	28 + 2	27 + 6
18 + 8	16 + 7	28 + 6	27 + 5	18 + 3
18 + 9	25 + 6	26 + 8	28 + 9	12 + 8
15 + 8	27 + 3	18 + 7	27 + 4	18 + 6
16 + 5	27 + 7	27 + 8	18 + 5	28 + 3

g

5 + 19	26 + 9	13 + 8	19 + 8	15 + 5
22 + 8	29 + 7	17 + 4	17 + 5	14 + 8
17 + 3	13 + 7	3 + 29	23 + 7	6 + 19
9 + 29	23 + 9	15 + 9	17 + 9	19 + 4
23 + 8	29 + 8	14 + 7	21 + 9	29 + 1
24 + 9	11 + 9	25 + 5	26 + 7	5 + 29
16 + 8	25 + 7	22 + 9	29 + 4	26 + 5
15 + 6	15 + 7	27 + 9	2 + 19	26 + 4
28 + 8	8 + 17	6 + 29	19 + 7	14 + 9

h

3 + 19	7 + 17	26 + 8	15 + 8	16 + 9
28 + 7	25 + 6	12 + 8	6 + 17	25 + 8
18 + 9	28 + 9	9 + 13	3 + 19	24 + 8
1 + 19	25 + 9	26 + 6	2 + 29	28 + 5
27 + 4	27 + 7	12 + 9	24 + 6	18 + 7
16 + 5	16 + 4	16 + 6	27 + 3	18 + 6
18 + 4	6 + 27	28 + 2	16 + 7	14 + 6
18 + 8	28 + 4	18 + 5	28 + 3	24 + 7
18 + 2	28 + 6	27 + 8	18 + 3	27 + 5

i

19 + 5	17 + 5	23 + 7	17 + 3	13 + 8
22 + 8	29 + 9	23 + 9	4 + 17	17 + 9
26 + 7	22 + 9	9 + 11	23 + 8	24 + 9
25 + 7	15 + 6	21 + 9	15 + 7	28 + 8
14 + 9	8 + 17	27 + 9	15 + 5	3 + 29
15 + 9	29 + 7	26 + 9	14 + 8	19 + 6
13 + 7	8 + 19	1 + 29	16 + 8	26 + 5
14 + 7	19 + 4	29 + 5	29 + 4	25 + 5
8 + 29	19 + 2	29 + 6	7 + 19	26 + 4

j

7 + 17	6 + 17	25 + 9	18 + 9	26 + 8
28 + 7	9 + 19	1 + 19	12 + 8	2 + 29
16 + 4	14 + 6	27 + 3	27 + 4	24 + 6
27 + 6	16 + 7	16 + 5	28 + 4	18 + 2
27 + 5	27 + 8	28 + 3	16 + 9	9 + 13
24 + 8	25 + 6	3 + 19	25 + 8	26 + 6
28 + 9	15 + 8	12 + 9	4 + 18	28 + 2
18 + 7	28 + 5	16 + 6	18 + 8	18 + 6
27 + 7	24 + 7	28 + 6	18 + 3	18 + 5

Section II. Sums 40 or more. Combinations used in multiplication

a

36 + 4	45 + 6	48 + 2	63 + 7	35 + 5
48 + 4	36 + 6	49 + 1	49 + 3	64 + 6
35 + 6	45 + 5	36 + 7	72 + 8	49 + 2
48 + 5	45 + 8	36 + 5	49 + 6	54 + 7
56 + 4	49 + 5	48 + 3	56 + 5	64 + 7

b

48 + 7	56 + 6	63 + 8	54 + 6	49 + 4
56 + 7	48 + 6	36 + 8	45 + 7	54 + 8
36 + 4	63 + 7	45 + 6	35 + 5	48 + 2
48 + 4	64 + 6	49 + 3	36 + 6	49 + 1
45 + 5	35 + 6	72 + 8	49 + 2	36 + 7

c

36 + 5	45 + 8	48 + 5	49 + 6	54 + 7
64 + 7	56 + 5	48 + 3	49 + 5	56 + 4
54 + 6	48 + 7	63 + 8	49 + 4	56 + 6
45 + 7	54 + 8	56 + 7	48 + 6	36 + 8
63 + 7	36 + 4	35 + 5	48 + 2	45 + 6

d

48 + 4	64 + 6	49 + 1	36 + 6	49 + 3
49 + 2	72 + 8	36 + 7	45 + 5	35 + 6
45 + 8	54 + 7	36 + 5	48 + 5	49 + 6
56 + 4	48 + 3	49 + 5	64 + 7	56 + 5
63 + 8	54 + 6	49 + 4	56 + 6	48 + 7

e

48 + 6	36 + 8	45 + 7	54 + 8	56 + 7
48 + 2	35 + 5	45 + 6	36 + 4	63 + 7
36 + 6	49 + 1	49 + 3	64 + 6	48 + 4
72 + 8	35 + 6	49 + 2	45 + 5	36 + 7
45 + 8	36 + 5	49 + 6	54 + 7	48 + 5

f

56 + 5	64 + 7	48 + 3	56 + 4	49 + 5
56 + 6	49 + 4	63 + 8	48 + 7	54 + 6
48 + 6	54 + 8	45 + 7	36 + 8	56 + 7
36 + 4	35 + 5	45 + 6	63 + 7	48 + 2
64 + 6	48 + 4	49 + 3	36 + 6	49 + 1

g

35 + 6	45 + 5	36 + 7	72 + 8	49 + 2
54 + 7	49 + 6	36 + 5	45 + 8	48 + 5
56 + 4	49 + 5	48 + 3	56 + 5	64 + 7
49 + 4	54 + 6	63 + 8	56 + 6	48 + 7
48 + 6	45 + 7	36 + 8	56 + 7	54 + 8

h

36 + 4	48 + 4	35 + 6	48 + 5	56 + 4
45 + 6	36 + 6	45 + 5	45 + 8	49 + 5
48 + 2	49 + 1	36 + 7	36 + 5	48 + 3
63 + 7	49 + 3	72 + 8	49 + 6	56 + 5
35 + 5	64 + 6	49 + 2	54 + 7	64 + 7

Illustrative Problems

In the following problems all the hard basic combinations are illustrated in their applications to higher-decade addition. Most of them appear twice. Although in these problems preference has been given to the one hundred and twenty-five most useful higher-decade combinations, we have not hesitated to introduce others, for example, $75 + 9$ in the seventh problem and $85 + 6$ in the eighth. The point is that apart from column addition and use in multiplication, higher-decade combinations outside the one hundred and twenty-five designated as "most useful" occur with considerable frequency, especially in problem situations.

1. The children were playing circus. Philip and Ralph took the tickets. Philip took 17 and Ralph 8. How many tickets did they both take?

2. Ben and Uncle Will went to town to sell watermelons. They had sold 49 and thought they could not sell any more. Just then a man came along in an automobile. He stopped and said he wanted to get some good melons for his boarders. He took all they had. That was 7. How many melons did Ben and Uncle Will sell in all?

3. It took Howard 27 minutes to ride home from the post office and 8 minutes to put away the saddle and take care of the pony. How many minutes did it take in all?

4. George fed his pet chicken 28 kernels of corn and his squirrel 8 kernels. How many kernels did he feed to both?

5. At the school picnic 15 boys and 9 girls went swimming in the lake. How many children went swimming in the lake?

6. There were 29 reading books and 6 music books on the schoolroom table. How many books were there on the table?

7. Anna's pony cost 75 dollars and her saddle 9 dollars. How many dollars did the pony and saddle cost?

8. Dora went to her grandpa's farm. She traveled 6 miles in an automobile and 85 miles on the train. How many miles did she travel?

9. One day 16 children worked in the school garden in the morning and 7 worked there in the afternoon. How many children worked in the garden that day?

10. Harry went to a picture show. His ticket cost him 15 cents. He paid 6 cents for car fare. How many cents did he pay for the trip to the show?

11. Inez weighed 77 pounds. She went to the country for a vacation. When she came home, her father weighed her and told her she had gained 5 pounds. How many pounds did she weigh then?

12. Father paid 57 dollars for his overcoat and 5 dollars for his hat. How many dollars did his coat and hat cost?

13. The milkmen were trying to see who could sell the most butter. One morning Mrs. Gray's milkman told her he had sold 67 pounds. Mrs. Gray bought 6 pounds. How many pounds of butter had he then sold?

14. Esther made a window curtain. The window was 48 inches long. She added 4 inches for the hems. How long was the cloth for the curtain?

15. Wallace spent 18 cents for lunch at school and 4 cents for marbles. How many cents did he spend for both?

16. There were 49 little ducks and 5 large ducks in the pond. How many ducks were there in the pond all together?

17. Tom worked for the baker one Saturday. He sold 7 loaves of brown bread and 44 loaves of white bread. How many loaves did he sell in all?

18. Uncle Will picked 32 quarts of cherries and Ben picked 9 quarts. How many quarts of cherries did they both pick?

19. Catherine and Flora were playing croquet. Catherine won 12 games of croquet and Flora won 9. How many games did they play?

20. Ida and Jessie sold pop-corn balls at a fair. Ida sold 13 and Jessie sold 9. How many pop-corn balls did they both sell?

21. There were 13 little white chickens and 7 little black chickens out in the grass. How many little chickens were there out in the grass?

22. Tom drove mother to town to do some shopping. She spent 58 cents for her lunch and 6 cents for car fare back home. How much did she spend for both?

23. David planted 18 cabbages and Nan planted 6. How many cabbages did they both plant?

24. Susie bought 18 spools of white cotton and 9 spools of black. How many spools of cotton did she buy?

25. There were 18 goldfish in a big bowl. A man put in 7 more. How many goldfish were there in the bowl then?

26. One day 27 school children and 3 teachers from the same school went to see a children's play. How many people went from that school?

27. Carl paid 38 cents for coffee and 3 cents for a yeast cake. How many cents did he pay for both?

28. It took Joe 18 minutes to walk to school and 3 minutes to take his little brother to his room. How many minutes did it take him all together?

29. Anna was up on the stepladder picking peaches. She had counted 19 as she put them in her basket. Her tall brother reached up and picked 3 peaches and put them in her basket. How many peaches were there in the basket?

30. Tim sold 9 morning papers and 11 evening papers. How many papers did he sell?

31. There was a parade in the city; 16 policemen rode by the school on horseback, and then 8 more rode by. How many policemen rode by?

32. Thursday afternoon the second-grade children counted the words they had put in their spelling books. There were 16. On Friday they put in 4. How many words did they have that week?

33. It took Louis 42 minutes to do his arithmetic lesson and 8 minutes to do his spelling lesson. How many minutes did it take to do both?

34. There were 16 lambs in a field and Will put in 9 more. How many lambs were there in the field?

35. Bessie and her father took a walk. They saw 16 robins and 6 doves. How many birds did they see?

36. It was a cool autumn afternoon. "Children," said father, "there will be frost tonight. Run down to the garden and gather the pumpkins. I will bring them in with the wheelbarrow." So Patsy and Molly ran away and soon had a pile of yellow pumpkins by the side of the garden. "There," said Molly, "that's soon done, and we have 39 fine pumpkins. Hurrah for Halloween and Thanksgiving." Then father came. "Well, that is good," he said. "I will look around a bit. These pumpkins are so fine, we mustn't waste any." Then he looked all about and came out of the garden with 4 more. How many pumpkins did they all find?

37. A school paid 89 dollars for a phonograph and 8 dollars for some records. How many dollars did both cost?

38. The bus stopped in front of the school; 17 children and 4 grown people got out. How many people got out at the school?

39. Some boys had a circus. Before it began 16 children went in. After a while 5 more children went in. How many children went to see the circus?

40. Henry worked in an ice-cream store one Saturday. He sold 9 cones of chocolate ice cream and 33 of vanilla. How many cones did he sell?

41. One day while Tom was working for the baker he sold 13 custard pies and 8 pumpkin pies. How many pies did he sell?

42. John came into the kitchen. He saw a pile of warm cookies on the table. "O Jenny," he said, "how many cookies are there in that pile?" "Twenty-four," said Jenny. "Is that all we are going to have?" asked John. "No. There are 8 more in the oven," said Jenny. How many cookies were there in all?

43. Before school Mary practiced her music lesson 45 minutes and studied her spelling lesson 8 minutes. How many minutes did Mary spend with her lessons?

44. Some children were playing dog show; 15 children went to see the show and 7 went to take care of the dogs. How many children were at the show?

45. The children wanted to paint some eggs for Easter. The eggs cost 18 cents and the paints cost 5 cents. How many cents did the eggs and paints cost?

46. Ralph had 46 cents in his toy bank and his father put in 5 cents more. How many cents did he then have in his bank?

47. There were 24 chickens in the chicken yard and 6 more went in. How many chickens were then in the yard?

48. John and Henry each had a dog. One day they weighed their dogs. John's weighed 19 pounds, and Henry said that his weighed 2 pounds more. How many pounds did Henry's dog weigh?

49. Ethel had 76 cents in her missionary box. Her brother put in 8 cents more. How many cents were then in the box?

50. Marie earned 25 cents by taking care of the neighbor's baby, and 8 cents by picking berries for mother. How many cents did she earn?

51. Walter found 36 eggs in the henhouse and 9 under the haystack. How many eggs did he then have?

52. There were 19 women at a Mothers' Meeting in our room. Then 4 more women came. How many women were at the meeting?

53. Tony sold 34 tickets for the school entertainment. His little sister sold 9. How many tickets did they both sell?

54. The dressmaker bought 6 black buttons and 36 white ones. How many buttons did she buy in all?

55. David and Nan had a garden. One morning David sold 19 bunches of radishes and Nan sold 8 bunches. How many bunches did they both sell?

56. Alfred rode 11 miles on his bicycle one day and 9 miles the next. How many miles did he ride on both days?

57. Mr. Stevens set out 17 tomato plants and Harry set out 7. How many tomato plants did they both set out?

58. Philip sold electric-light bulbs. Saturday morning he made 67 cents. In the afternoon he made only 9 cents. How many cents did he make on Saturday?

59. Jane paid 68 cents for her reading book and 5 cents for a spelling tablet. How many cents did she pay for both?

60. On Mothers' Day, William gave his mother 8 white roses and 12 pink ones. How many roses did William give his mother?

The Progress Test

In the two parts of the following test are represented all the basic combinations on which higher-decade addition with bridging depends. Sums do not exceed 39 except in the case of the larger combinations used in multiplication.

Test on Higher-Decade Addition with Bridging

PART I

49 + 2 =	9 + 23 =	8 + 29 =	9 + 15 =
18 + 6 =	29 + 1 =	49 + 5 =	28 + 8 =
27 + 4 =	17 + 8 =	37 + 7 =	27 + 6 =
35 + 5 =	72 + 8 =	16 + 7 =	27 + 3 =
18 + 2 =	45 + 7 =	63 + 8 =	14 + 6 =
54 + 8 =	27 + 5 =	28 + 4 =	21 + 9 =

1. John paid 25 cents for oranges and 8 cents for bread. How many cents did he spend?

Ans. ____

2. Henry did 15 examples at home and 7 more at school. How many examples did he do?

Ans. ____

PART II

28 + 2 =	24 + 7 =	16 + 4 =	49 + 4 =
17 + 9 =	45 + 8 =	29 + 7 =	63 + 7 =
49 + 3 =	26 + 9 =	6 + 19 =	26 + 8 =
24 + 6 =	15 + 6 =	56 + 7 =	14 + 9 =
9 + 12 =	9 + 28 =	56 + 5 =	18 + 5 =
48 + 7 =	16 + 6 =	48 + 3 =	9 + 29 =

1. There were 12 boys and 9 girls playing in the yard. How many children were playing in the yard?

Ans. ____

2. How many cents are a 25-cent piece and 7 pennies?

Ans. ____

CHAPTER XXIII

HIGHER-DECADE SUBTRACTION WITH BRIDGING

A general discussion of the subject of higher-decade subtraction has already been given in Chapter XVIII. In that chapter the examples and problems were such as required no bridging of the tens. Two cases were distinguished: Case I in which the *subtrahend* is a one-place number, such as $27 - 4 = 23$; and Case II in which the *remainder* is a one-place number, such as $19 - 15 = 4$. It was shown that there were four hundred and five possible combinations, not involving bridging, under each of these cases, but that some of them were used much more frequently than others. It was pointed out that in general those involving small numbers are the most important, and that in Case II the combinations used in short division have a special value.

In this chapter the rest of the higher-decade subtraction combinations will be considered, namely, the three hundred and sixty in Case I and the three hundred and sixty in Case II which involve bridging. The meaning of these cases is the same as before. For example, $36 - 8 = 28$ is an instance under Case I; $32 - 25 = 7$ is an instance under Case II. All that was said in Chapter XVIII as to the importance of the higher-decade subtraction combinations in life, as to the desirability of training pupils to make single rather than piecemeal reactions to them, and finally as to the opportunity which they offer of using the basic combinations has the same application in this chapter. The chief difference is that the combinations treated in this chapter are more difficult than those treated in Chapter XVIII. They correspond to the basic combinations

having minuends of 10 or more. Thus the basic combination $10-4$ corresponds, with respect to Case I, to the series of higher-decade combinations $20-4, 30-4, 40-4, \cdots, 90-4$. There are evidently eight of these combinations, one for each decade from the twenties to the nineties inclusive. Similarly, the basic combination $10-4$ corresponds, with respect to Case II, to the series of higher-decade combinations $20-14, 30-24, 40-34, \cdots, 90-84$. There are likewise eight of these combinations. In short, corresponding to *every* "hard" basic subtraction fact there are eight higher-decade combinations of Case I and eight of Case II. Since there are forty-five of these basic facts, this gives us three hundred and sixty combinations in each case.

This brings us to a summary of the whole matter of higher-decade subtraction, both that which involves bridging and that which does not. Table X gives the facts.

TABLE X. SUMMARY OF HIGHER-DECADE SUBTRACTION COMBINATIONS

	Case I		Case II	
	Number of Combinations	Illustration	Number of Combinations	Illustration
Without bridging .	405	$38-5$	405	$38-35$
With bridging . .	360	$31-4$	360	$31-24$

TEACHING HIGHER-DECADE SUBTRACTION INVOLVING BRIDGING

Case I. The object in this teaching is to enable the child to infer the right answer from his knowledge of the corresponding basic combinations. Let the teacher write on the board the following series of facts, using the horizontal form as indicated:

$$11-2=9 \qquad 21-2=19 \qquad 31-2=29 \qquad 41-2=39$$

The pupils may read this series several times. It should then be pointed out that in each answer the units' figure is a 9 and that the tens' figure is one less than the tens' figure in the

first (the two-place) number. Other series may be written on the board and the corresponding facts may be pointed out; namely, that the units' figures in the answers are the same as in the basic combination and that the tens' figure is "one less than you start with." [1] Do not have the pupils "borrow." No *process* of subtraction is to be thought of. In finding 31 less 2 the answer, 29, is to be inferred directly from the basic combination 11 less 2. Most of the drill should be in the twenties, thirties, and forties. Use the number chart freely (see page 62).

Case II. Using the horizontal form of expression, put on the board a series such as the following:

$11 - 8 = 3 \quad 21 - 18 = 3 \quad 31 - 28 = 3 \quad 41 - 38 = 3$

The pupils already know the first fact and they will readily see that the answers to the succeeding facts are the same. They may easily be shown that the units' figure in the minuend is the same as the units' figure in the minuend of the basic combination with which they are already familiar. A similar observation may be made with reference to the units' figure of the subtrahend. If desirable, the series may be called the "eleven-less-eight family." Put other series on the board and point out the corresponding facts, or similarities with the basic combination. Be sure the right basic combination is selected. Nowhere in this work is a step-by-step process of subtraction to be used. The answer to $41 - 38$ is 3 because $11 - 8$ is 3. Do not have the children "borrow." They should respond to these combinations just as they do to basic combinations.[2]

Analysis of Case II

As has already been shown (Chapter XVIII, p. 220), there are four hundred and five higher-decade combinations of Case II, forty-five for each of the decades from the 'teens to the nineties inclusive. The smaller these combinations are

[1] Additional suggestions as to method are given in Chapter XVIII, p. 218.

[2] For additional suggestions as to teaching Case II see Chapter XVIII, p. 222.

the more important they are likely to be, that is, the more frequently they are likely to occur in the ordinary affairs of life. But the higher-decade combinations of Case II which are of greatest importance are those which are used in short division. We have already pointed out in Chapter XVIII that there are one hundred and fifteen which do not involve bridging the tens (see page 222 and Table V). There are sixty more which do involve bridging the tens. They are listed in Table XI.

TABLE XI. THE SIXTY HIGHER-DECADE SUBTRACTION FACTS, USED IN SHORT DIVISION, WHICH INVOLVE BRIDGING THE TENS

20–14	20–16	20–18	30–24
	21	21	31
	22	22	
	23	23	
		24	
		25	
		26	
30–27	30–28	40–35	40–36
31	31	41	41
32	32		42
33	33		43
34	34		44
35			
50–45	50–48	50–49	60–54
51	51	51	61
52	52	52	62
53	53	53	
	54	54	
	55	55	
60–56	70–63	70–64	80–72
61	71	71	
62			
63			

At this point the reader would do well to consult what we have said on pages 220 to 222 inclusive. Remember that the higher-decade combinations used in short division have as their subtrahends the products in the multiplication table and as their minuends the numbers which exceed these products by amounts less than the larger factor in the product. With reference to the teaching of such facts Osburn has this to say.[1]

> In subtraction two sorts of situations need careful attention. They are the one hundred combinations as ordinarily given in the subtraction table and the one hundred and seventy-five subtraction combinations which are needed in short division. To neglect either of these groups is to insure poor results in accuracy. The first group is usually taught carefully, but the second is universally neglected even in our *best* textbooks. When children meet the situation $8\overline{)712}$ they must subtract 64 (held in mind) from 71. But no previous practice has been given in this sort of thing. The only clue that even the bright child has is his knowledge that 4 from 11 leaves 7. Now, the question is what does 64 from 71 leave — a question which is considerably different. Is it any wonder that children make mistakes? To attempt to teach the mastery of short division under such conditions is rank stupidity. Let's give the child a chance. *Teach him the one hundred and seventy-five combinations.*

These one hundred and seventy-five combinations needed in short division consist of the one hundred and fifteen, treated in Chapter XVIII, which do not involve bridging and the sixty, presented in this chapter, which do. The teacher should now give special drill on these sixty subtractions, using for the purpose the number chart, flash cards, the games which have already been described, or any other device which seems appropriate. The one hundred and fifteen facts of similar usefulness given in Table V may well be combined with these sixty and the entire one hundred and seventy-five regarded as

[1] W. J. Osburn, Corrective Arithmetic, pp. 18–19. Houghton Mifflin Company, Boston, 1924.

one unit of drill work. It must be remembered, however, that the sixty which involve bridging are considerably harder and will need much more attention.

Practice Material, Case I

The following material under Case I consists of two hundred abstract examples and sixty-two verbal problems. Since there are three hundred and sixty higher-decade subtraction facts of Case I involving bridging, it is clear that this material does not exemplify all the combinations. The smaller combinations, however, are fully treated and, as has been pointed out, these are especially important. This is true not only because they occur more frequently in the numerical work of children and adults, but also because when once the transfer has been made from basic combinations to a few combinations in the lower decades the further transfer to still higher decades is greatly facilitated. Of the two hundred examples given, one hundred and fifty-seven have their minuends below 50 and their answers below 40.

Since higher-decade combinations afford practice on basic combinations, an attempt has been made in the material given to represent the basic combinations with the same or with nearly the same frequency. In the two hundred examples each basic combination is practiced from three to five times. Those that are practiced as many as five times are the hardest ones.

The sixty-two verbal problems show the application of Case I of higher-decade subtraction. Our reason for giving particular attention to this case is that natural situations call for its use. As in problems 1, 36, and 40, we often have tasks to perform which may be expressed numerically. We do a part of them and we are at the same time keenly interested in how much remains to be done. Then, of course, there are

the inevitable money problems involving the questions of change and cost (problems 4, 20, and 25). The kind of subtraction called for in Case I is likewise frequently required when we wish to find how much more or less one number is than another, provided one of them is considerably larger than the other. Life is full of such situations. For example, two boys attend the same class, one from a distance and the other from near by (problem 54). Two children count the automobiles, one on a congested street and the other on a quiet street (problem 56). A girl counts robins and bluebirds in the early spring (problem 58). Edith and Mabel play tit-tat-toe (problem 60) with the game running heavily against Edith. Sammy Squirrel encounters an alarming shortage of acorns compared with hickory nuts (problem 57).

There is also the situation in which a certain number of things is reduced by a relatively small proportion of the whole. This generally involves some form of finding how much is left (problems 10, 39, 42, 43, 46, 48, 53, and 59). For example, in problem 59 there are twenty-three children skating on the pond. Seven of them take off their skates, say good-by, and go home. How many are left on the pond? The situation is not wildly exciting, but it is not unnatural. It may even be made highly interesting. It may be dramatized with all the vividness of detail which dramatization makes possible. We have taken occasion several times in the foregoing pages to point out the desirability of "acting out" problems. Many a difficult situation is cleared up and many an uninteresting problem is made attractive by this type of self-activity. Sometimes the interest is not in how much is left but in how much was used, lost, taken away, or sold (problems 26, 32, and 41).

In the verbal problems given, 54 to 62 inclusive are so worded that the subtrahend comes first. Our problems should not continue indefinitely to express the minuend first.

Abstract examples

a

23 – 5	32 – 6	20 – 2	34 – 5	43 – 8
22 – 7	20 – 5	24 – 6	53 – 9	32 – 3
30 – 4	26 – 7	35 – 8	45 – 9	24 – 7
25 – 6	22 – 5	33 – 7	40 – 7	21 – 2
34 – 8	50 – 8	41 – 6	38 – 9	27 – 8

b

27 – 9	26 – 8	51 – 3	62 – 8	71 – 7
60 – 6	23 – 6	21 – 8	34 – 9	41 – 5
53 – 4	35 – 7	70 – 3	81 – 9	32 – 4
36 – 9	21 – 4	72 – 9	25 – 8	36 – 7
32 – 5	52 – 3	90 – 9	34 – 8	42 – 6

c

27 – 9	24 – 5	80 – 1	41 – 6	38 – 9
23 – 8	20 – 4	42 – 7	34 – 6	23 – 5
45 – 9	63 – 7	25 – 6	31 – 2	24 – 7
56 – 8	20 – 8	33 – 9	21 – 7	72 – 8
30 – 7	21 – 4	27 – 8	33 – 6	40 – 5

d

22 – 4	41 – 5	50 – 2	36 – 9	25 – 7
33 – 4	54 – 9	21 – 8	60 – 9	41 – 9
34 – 6	24 – 5	32 – 7	23 – 5	35 – 8
26 – 7	28 – 9	70 – 6	22 – 6	31 – 3
22 – 5	34 – 8	37 – 9	33 – 8	21 – 6

e

22 – 9	56 – 8	80 – 3	22 – 3	35 – 6
90 – 1	33 – 9	24 – 7	20 – 7	31 – 2
37 – 8	25 – 9	21 – 7	72 – 8	20 – 5
63 – 7	22 – 4	41 – 8	33 – 6	35 – 7
24 – 9	41 – 5	36 – 9	21 – 4	30 – 4

f

72 − 9	24 − 6	23 − 4	21 − 9	33 − 5
32 − 7	40 − 8	22 − 4	25 − 8	37 − 9
25 − 6	24 − 5	32 − 3	53 − 8	63 − 9
24 − 7	50 − 6	31 − 6	42 − 5	45 − 9
51 − 2	33 − 6	26 − 8	60 − 9	31 − 7

g

38 − 9	21 − 3	27 − 8	42 − 6	33 − 7
34 − 8	70 − 3	45 − 7	21 − 8	64 − 9
72 − 8	53 − 4	80 − 2	26 − 9	31 − 4
31 − 5	26 − 7	81 − 9	90 − 1	72 − 9
25 − 9	24 − 7	23 − 5	30 − 7	35 − 6

h

61 − 3	40 − 5	26 − 8	24 − 5	27 − 9
30 − 4	45 − 6	35 − 8	93 − 9	42 − 7
50 − 2	42 − 6	35 − 7	23 − 8	42 − 5
20 − 6	42 − 4	31 − 6	28 − 9	21 − 7
42 − 6	63 − 7	40 − 8	26 − 9	27 − 8

Concrete problems

1. We have a new set of reading books in our class. There are 33 of them. Yesterday the teacher put paper covers on 9 of them. How many more have to be covered?

2. Mother asked 23 children to come to Bessie's surprise party; 8 came from out of town, so they stayed all night. The rest lived in town. How many lived in the town?

3. In the month of May there were 23 school days; 6 of them were rainy days and the rest were bright and sunshiny. How many school days were bright and sunshiny?

4. William bought some candy that cost 8 cents. He gave a quarter to the man who sold it to him. How much change did the man give him?

5. Julia cut out 20 paper dolls. When she looked them over she found that there were 3 she did not like. So she threw them away. How many did she have left to play with?

6. There were 21 ducks in the water. All but 2 of them went on shore to get something to eat. How many went on shore to get something to eat?

7. Jack brought in 22 ears of corn from the garden. Mother cooked 9 for dinner. How many were left?

8. "Little Boy Blue, come blow your horn. The 21 sheep you were told to watch are all out in the meadow except 5. Do you know, you sleepyhead, how many sheep are in the meadow? How many?"

9. Ray had 47 dollars in the bank. He drew out 9 dollars so that he could buy a set of tools. How many dollars did he have left in the bank?

10. The postman had 31 letters for the people on our street. He left 6 at the first house. How many more letters did he have for the people on our street?

11. Frank drove to town to sell some strawberries. He had 32 quarts to sell. Mrs. Jones took 7 quarts, and Mrs. Buckly said she would take all the rest. How many quarts did Mrs. Buckly take?

12. A man came and took a picture of our class. We all went outside near the big front door. There were 37 of us. The man had 8 children sit down in front. The rest were standing up. How many were standing up?

13. We have a fine new schoolhouse. Our teacher says there are 21 rooms in it; but she says that 4 of them are not used. How many rooms are being used?

14. We had a visitor in our class this morning and 23 little girls went up to the front of the room to sing a song; but 4 of them forgot the words of the song. How many little girls knew the words?

15. The king was in the counting house, counting out his money. "One, two, three, four," he said, until he had counted out 24 bright, yellow, golden pieces of money. Then the king put 7 of the golden pieces in his pocket so that he could have them with him when he wanted to count them. The rest he locked up in his strong box. How many pieces of money did he lock up in his strong box?

16. A crate full of berries weighs 53 pounds. The crate weighs 6 pounds. How much do the berries weigh?

17. There are 33 books on the teacher's desk; 8 of them are green. How many are not green?

18. Our teacher told us to cut a strip of paper 24 inches long. She then told us to cut off 8 inches and to measure the piece we had left. How many inches long was it?

19. One day last week Mrs. Black bought a 25-pound bag of sugar. Since then she has taken 6 pounds out of the bag. How many pounds are left in it?

20. If you had a quarter and spent 6 cents for gumdrops should you have enough left to buy a spelling tablet that costs 20 cents?

21. Last month had 31 days in it; 8 days were Saturdays and Sundays. The rest were school days. How many days did the children go to school?

22. Alice and her father had to drive 30 miles. They drove 8 miles and then stopped to mend a tire. How many more miles did they have to drive?

23. There were 32 children in Miss Norton's class. On a very snowy day 5 of them stayed at home. How many pupils were in school that day?

24. Susie threw the bean bag 22 times. She missed 6 times. How many times did the bean bag go in the right place?

25. Sam's mother gave him 75 cents this morning and sent him to the store to buy some groceries. He brought back 9 cents in change. "How much did the groceries cost?" asked his mother. Sam knew the answer. What was it?

26. Henry took 34 tickets to sell. They were tickets to the school picnic. He brought back 7. How many had he sold?

27. In our cloakroom there are 44 hooks. On all but 9 of them hats are hanging. How many of the hooks have hats hanging on them?

28. Tom went to the store and bought 24 eggs. Mother used 6 of them for dinner. How many did she have left?

29. Charles and Edward went fishing. They fished all the afternoon. Then they counted the fish they had caught. Charles had 27, and when he told this to Edward, Edward said, "You beat me by 9 fish." How many fish had Edward caught?

30. There are 26 letters in the alphabet. Tom has learned to write 8 of them. How many more does he have to learn?

31. In our class there are 34 children. Only 6 of them are boys. How many are girls?

32. Maud's mother bought 30 yards of silk. She used some for making curtains and found she had 6 yards left. How many yards did she use for curtains?

33. Elsie's mother gave her 20 cents to spend for lunch in the school lunchroom. First she bought a bowl of soup for 5 cents. How many cents did she have left for the rest of her lunch?

34. Edna invited 20 children to her birthday party, and they all came but Jimmy. He was sick and had to stay at home. How many came to the party?

35. William and Ted had to plant 21 hills of corn. William planted 9. How many did Ted plant?

36. "Susie," said mother, "your new dress is all done except those red buttons we are going to put on it." "How many are there?" asked Susie. "There were 34, but I have sewed on 8," said mother. "I will sew on the rest after school," said Susie. And so she sewed and sewed, and got them all on. How many buttons did Susie sew on?

37. John earned 35 cents but had to spend 6 cents for car fare. He put the rest in his bank. How many cents did he put in his bank?

38. Betty had saved 96 cents for Christmas money. She spent 8 cents for candles. How many cents did she have left?

39. Paul picked 32 quarts of cherries. His mother used 8 quarts and he sold the rest. How many quarts did he sell?

40. Grace had 20 examples to do for home work. She did 9 at night and left the rest until next morning. How many examples did she have to do in the morning?

41. Ben took 35 papers to sell before supper time. At supper time he had 7 left. How many papers had he sold?

42. Mr. Allen was given a vacation of 30 days. He has taken 7 days. How many days of vacation has he still left?

43. There were 24 dishes in Fanny's tea set but 5 of them got broken. How many dishes were left in the tea set?

44. Emma made 24 valentines. She sent away 9 and kept the rest to give her little school friends. How many valentines did she have for her school friends?

45. Joseph bought 36 oranges. The family ate 7 of them for breakfast. How many oranges were left?

46. Robert had a board 30 inches long. He cut off 4 inches. How long was the board then?

47. One morning Mr. Cooper, the grocer, had 60 oranges in a box. That night he wanted to know how many he had sold. There were only 2 left in the box. How many had he sold?

48. There were 33 words on the blackboard and the teacher rubbed out 7. How many words were there then on the blackboard?

49. There were 21 mice living in a barn and 3 ran away. How many mice were left in the barn?

50. There were 45 people on the train and 8 got off. How many people were left on the train?

51. There were 23 chickens in the chicken yard; someone left the gate open and 5 ran out. How many chickens were left in the yard?

52. Once 22 children planned to go to a picnic, but the 3 Thomas children could not go. How many children went to the picnic?

53. There were 22 boys out on the playground; 4 ran into the schoolhouse. How many were left on the playground?

54. Charles does not live very far from school. This morning it took him 9 minutes to walk to school. Robert lives much farther away and it took him 28 minutes. How many minutes longer did it take Robert than Charles?

55. Seven of the 21 children who were riding in a bus got out at the first stop. How many children were left in the bus?

56. Henry and Charles counted the number of automobiles they met on their way to school. Charles counted 9 and Henry counted 23. How many more did Henry count than Charles?

57. Sammy Squirrel was looking for nuts to put away for the winter. He found lots of hickory nuts. But acorns were so hard to find that Sammy Squirrel felt very bad about it. He counted his acorns and found he had only 8. Then he counted his hickory nuts and found he had 47. How many more hickory nuts did he have than acorns?

58. Last spring Ruth tried to count the birds she saw. One day Mary asked her if she had seen as many bluebirds as robins. "Oh, no, there are lots more robins than bluebirds," said Ruth. "I have counted only 7 bluebirds but I have counted 26 robins." How many more robins than bluebirds had Ruth seen?

59. Seven of the 23 children who were skating on the pond went home. How many children were left on the pond?

60. Edith and Mabel played tit-tat-toe all the evening. Edith won 9 games and Mabel won 20. How many more games did Mabel win than Edith?

61. The children were playing a game called "I am thinking of a number." Robert said, "I am thinking of a number which is 9 less than 36." What number was Robert thinking of?

62. Harold went to the store and bought a pound of sugar. The man said the sugar was 9 cents. Harold gave him a quarter. How many cents did Harold get in change?

Practice Material, Case II

The material which follows, consisting of three hundred and fifty examples and sixty-eight verbal problems, affords drill and application in Case II of higher-decade subtraction. In the abstract work all the applicable forty-five basic combinations are frequently represented. Each one occurs either seven or eight times. Small numbers are frequently used. Of the three hundred and fifty examples, two hundred and twenty-two are in the twenties, thirties, and forties. In the verbal problems all the basic combinations are represented at least once. The hardest ones are represented twice. In the last fifteen problems the subtrahend is expressed first.

Abstract examples

a

34 − 28	24 − 16	45 − 36	52 − 48	71 − 64
27 − 18	55 − 49	42 − 35	71 − 63	90 − 81
25 − 18	32 − 24	40 − 32	36 − 27	63 − 56
81 − 72	54 − 49	42 − 36	33 − 27	72 − 63
21 − 16	33 − 25	54 − 45	73 − 64	51 − 45

b

20 − 13	32 − 27	53 − 48	30 − 26	31 − 28
56 − 48	40 − 38	51 − 49	35 − 27	20 − 19
34 − 27	53 − 49	70 − 64	56 − 49	41 − 37
47 − 39	80 − 75	58 − 49	90 − 87	22 − 19
73 − 64	45 − 36	54 − 49	33 − 27	81 − 72

c

22 − 16	24 − 15	55 − 48	72 − 63	31 − 25
90 − 81	36 − 27	55 − 49	54 − 48	52 − 45
21 − 14	40 − 32	63 − 56	37 − 28	24 − 16
33 − 25	22 − 14	71 − 63	52 − 48	41 − 36
20 − 17	33 − 28	57 − 49	34 − 27	33 − 29

d

35 − 27	60 − 54	56 − 48	41 − 37	40 − 35
21 − 18	46 − 39	60 − 53	32 − 27	58 − 49
60 − 56	52 − 49	63 − 54	41 − 32	30 − 28
31 − 29	23 − 17	50 − 49	52 − 48	65 − 56
55 − 49	90 − 81	24 − 18	42 − 35	64 − 56

e

25 − 18	36 − 27	62 − 56	41 − 35	43 − 36
27 − 18	44 − 35	61 − 54	42 − 33	20 − 12
22 − 14	34 − 29	41 − 36	23 − 19	71 − 63
0 − 37	57 − 49	33 − 28	34 − 27	25 − 17
52 − 49	70 − 63	56 − 48	40 − 35	58 − 49

f

21 – 18	20 – 16	56 – 49	30 – 24	31 – 27
43 – 35	21 – 12	32 – 27	41 – 39	50 – 48
92 – 85	24 – 18	33 – 27	65 – 56	55 – 49
32 – 28	44 – 36	33 – 24	20 – 11	22 – 16
40 – 39	57 – 48	35 – 27	54 – 45	72 – 63

g

23 – 18	36 – 27	51 – 45	20 – 12	54 – 49
32 – 24	71 – 63	30 – 27	43 – 36	35 – 28
27 – 19	20 – 15	21 – 16	36 – 28	52 – 49
21 – 14	34 – 27	53 – 49	30 – 23	58 – 49
32 – 27	60 – 59	31 – 27	20 – 14	51 – 48

h

23 – 14	40 – 36	51 – 49	27 – 18	25 – 16
50 – 48	32 – 25	44 – 36	56 – 49	35 – 27
72 – 63	33 – 27	24 – 18	35 – 29	20 – 11
54 – 45	23 – 15	21 – 12	32 – 28	22 – 16
57 – 49	23 – 16	36 – 27	55 – 48	61 – 54

i

41 – 36	24 – 17	40 – 32	54 – 49	40 – 36
36 – 28	41 – 35	22 – 14	28 – 19	42 – 37
71 – 63	70 – 64	53 – 49	31 – 28	40 – 35
22 – 19	33 – 28	30 – 27	56 – 49	20 – 13
41 – 37	72 – 63	51 – 49	45 – 36	24 – 18

j

20 – 19	45 – 39	33 – 27	44 – 36	42 – 35
33 – 24	21 – 12	90 – 81	43 – 35	61 – 54
62 – 56	35 – 27	50 – 48	57 – 49	43 – 36
24 – 15	36 – 27	57 – 48	22 – 18	30 – 27
20 – 14	25 – 18	53 – 49	40 – 32	34 – 27

k

21 − 19	32 − 27	53 − 48	71 − 63	60 − 54
61 − 56	26 − 18	34 − 29	40 − 35	56 − 49
21 − 18	40 − 33	31 − 27	58 − 49	60 − 56
52 − 49	41 − 35	20 − 18	72 − 63	40 − 39
45 − 36	44 − 38	35 − 27	23 − 19	43 − 36

l

32 − 24	43 − 24	44 − 36	36 − 27	57 − 48
55 − 49	20 − 17	61 − 54	52 − 45	34 − 27
55 − 48	30 − 22	37 − 29	52 − 48	23 − 17
90 − 81	72 − 56	44 − 35	63 − 54	52 − 49
23 − 18	41 − 35	30 − 23	36 − 29	21 − 13

m

41 − 32	44 − 39	22 − 17	40 − 34	86 − 78
41 − 36	48 − 39	50 − 48	31 − 27	40 − 35
21 − 19	60 − 56	51 − 48	50 − 49	24 − 18
23 − 16	72 − 63	25 − 16	35 − 27	53 − 49
30 − 23	40 − 35	33 − 27	62 − 56	56 − 49

n

80 − 72	44 − 36	25 − 19	42 − 35	21 − 13
90 − 81	22 − 19	43 − 38	24 − 17	41 − 35
62 − 54	43 − 35	71 − 64	27 − 18	36 − 27
63 − 54	20 − 17	45 − 38	57 − 49	22 − 18
40 − 35	86 − 78	24 − 19	41 − 36	32 − 27

Concrete problems

1. Charles gave the grocer 30 cents for groceries that cost 29 cents. How much change did Charles receive?

2. Fred had 52 cents in his bank and took out 45. How many cents did he have left in his bank?

3. There were 32 candy bags on the school Christmas tree. The children made 28 bags and the teacher made the rest. How many did the teacher make?

4. Polly and Jane had to clean the silver Saturday morning. Polly cleaned 30 pieces and Jane cleaned 27. How many more pieces did Polly clean than Jane?

5. Irving had 20 marbles. He gave 14 to his cousin. How many marbles did he have left?

6. One day 37 crickets sang a summer song, and 29 of them liked it so well that they sang it again. The rest went off to have a hopping race. How many crickets went to hop while the others sang a summer song?

7. There were 41 buttons in the button box. Peggy took out 36. How many buttons were left in the box?

8. The class has had 40 words in spelling and Marian has them all in her notebook. Pearl has been absent and has only 31 of the words in her notebook. How many will Pearl have to copy from Marian's book?

9. William did 23 examples. He did 14 of them at his seat and the rest at the board. How many did he do at the board?

10. Jane had 31 words to write. She wrote 22 of them in school and the rest at home. How many did she write at home?

11. Joe had saved 21 dollars. He used 16 dollars for a visit to the city. How many dollars did he have left?

12. Nellie has to read 46 pages of her book this week. She has read 39 of them. How many more pages has she to read?

13. Ella knew there were 23 peanuts in the dish on the porch table because she had counted them. She looked out of the window and saw a squirrel going up and down the porch steps. Then she went out and counted the peanuts. There were only 16 left. How many did the squirrel get?

14. The picnic boat sailed across the lake. There were 34 people in it; 25 of them got out at the first stop. How many people were left in the boat?

15. Helen and her mother went in a large steamboat across the ocean. The first evening there were 77 people in their dining room. The next morning there were 68. How many of the people did not come to breakfast?

16. Harry needed 35 cents for a hair-cut. His mother had only 27 cents in her pocketbook; so she gave him the 27 cents and sent him to his father for the rest. How many cents did Harry have to get from his father?

17. There were 51 chickens in the chicken yard. They found a hole in the fence and 49 of them went out. How many chickens were left in the yard?

18. At first Old King Cole had no fiddlers at all. He had only his pipe and his bowl. Then he called 52 of the best fiddlers of his kingdom to play before him. But he found fault with 49 of them. The rest he took to be his fiddlers. How many fiddlers did Old King Cole have?

19. Dick had 20 cents in his pocket. But there was a hole in his pocket and some of his money dropped through it. When Dick counted his money he found he had only 18 cents. How many cents had he lost?

20. There were 45 people at the railroad station, but 36 went away on the train. The others had come to say good-by to their friends. How many had come to say good-by to their friends?

21. Nancy had 55 cents. She spent 49 cents for a lace collar. How many cents did she have left?

22. Ralph was tossing rings. He threw 25 times and got 18 rings on the stick. How many times did he miss?

23. There were 41 children waiting to go home from the park; 35 of them got on the trolley car and the others waited for a bus. How many children waited for the bus?

24. Jim's mother gave him 60 cents to buy fruit with. She said he might have the change. The fruit cost 54 cents. How many cents were left for Jim?

25. Albert wanted to buy a knife that cost 70 cents. He had 63 cents. How many more cents did he need?

26. James is helping his teacher make a set of cardboard dominoes. There are 28 dominoes in a set. They have made 19. How many more dominoes do they have to make?

27. The baker took out 80 cookies in his wagon. He sold 72 of them. How many cookies did he have left?

28. There were 23 goldfish in the pet-shop window one morning. In the afternoon there were only 18 left. How many of the goldfish had been sold?

29. Miss Wood had the names of 54 children who wanted to go to the school picnic. When it was time to start only 49 were present. How many of the children were absent?

30. Mr. Morrison cut 52 roses and sold 48 of them. How many roses did he have left?

31. Last week eggs cost 42 cents a dozen in the city and 36 cents in the country. How many more cents did they cost in the city than in the country?

32. There were 43 peaches on Harold's little peach tree. A strong wind came and blew down 36. How many peaches were left on the tree?

33. There were 32 children in a May-Day dance and 24 in a flag drill. How many more were in the May-Day dance than in the flag drill?

34. In Miss Nelson's class there were 44 children; 36 of them sang in a chorus and the rest took part in a play. How many children were in the play?

35. There were 24 checkers on the checkerboard; 18 were taken off. How many checkers were left on the board?

36. There were 63 clothespins in the clothespin bag. Nora used 54 to pin up clothes and gave the rest to baby to play with. How many clothespins did baby have?

37. Walter had 41 eggs. He sold 36 and gave the rest to his mother to keep. How many eggs did he give his mother?

38. Paul had room for 22 rows of vegetables in his garden. He planted 13 rows with corn and the rest with beans. How many rows of beans did he plant?

39. Henry helped Uncle John pick pears. He counted what he picked. There were 53. A man came along and bought 48 of them. How many of Henry's pears were left?

40. Mr. Thomas raised 53 bushels of potatoes. He sold 49 bushels and kept the rest to use. How many bushels of potatoes did he keep?

41. There were 34 blackbirds sitting in a tree; 27 were singing, and the others were just looking for a place to sleep. How many were looking for a place to sleep?

42. Harry had 40 cents. He paid 35 cents for a set of dominoes. How many cents did he have left?

43. Frances and her father were driving in an automobile to her aunt's farm. From Frances's home to her aunt's was 53 miles. They drove 45 miles and then stopped to eat lunch. How many more miles did they have to drive?

44. Peter Rabbit had a party and 34 animal folks went. Sammy Jay had a party and 28 bird folks went. How many more were at Peter Rabbit's party than at Sammy Jay's?

45. Fred earned 55 cents one week, but he spent 48 cents for books. How many cents had he left?

46. Elsie and Ida played bounce the ball. Elsie caught it 54 times and Ida caught it 48 times. How many more times did Elsie catch it than Ida?

47. Rita had 36 birthday candies in a box. She passed them around at school. The children took 27 of them. How many were left?

48. In the barnyard there were 26 ducks and 18 geese. How many more ducks than geese were there?

49. Tom took 71 papers from the office to sell. He sold 64. How many papers did he have to carry back to the office?

50. The pupils of Miss Roe's class were asked to sell 40 tickets for a school entertainment. They sold 36. How many tickets were left?

51. On a bright May morning 55 birds met in the woods to sing a morning song. At evening 49 of them met in the same place to sing an evening song. How many of the birds stayed away at evening?

52. There were 32 quarts of strawberries in a crate. Mr. Reed bought 27 quarts of them and Mr. Hawkins bought the rest. How many did Mr. Hawkins buy?

53. Mary had 31 nuts in a paper bag. She passed them around to her friends. They took out 24. How many nuts did Mary have left in the bag?

54. This morning our spelling class lasted 18 minutes. Our reading class lasted 25 minutes. How many more minutes did our reading class last than our spelling class?

55. There are 28 days in February and 31 in March. How many more days are there in March than in February?

56. Bob and Dick helped their father plant potatoes. Dick planted 18 rows and Bob planted 21. How many more rows did Bob plant than Dick?

57. Henry and George tried a race in doing examples. George did 27 and Henry did 31. How many more examples did Henry do than George?

58. Our class is selling vegetables from the school garden. Yesterday the class got 63 cents and today 71 cents. How many more cents did it get today than yesterday?

59. Tim has 67 cents. He needs 75 cents to go to the circus. How many more cents must he save?

60. Ralph has saved 17 dollars. How much more must he save to buy a bicycle that costs 23 dollars?

61. Harry's lunch costs 26 cents. He gives 35 cents. How much change does he get?

62. In a game the losers scored only 27 and the winners 34. How much more did the winners score than the losers?

63. My desk is 32 inches wide and 40 inches long. How many inches longer is it than it is wide?

64. At home a little pail and shovel cost 15 cents. At the seashore they cost 20 cents. How many more cents do they cost at the seashore than at home?

65. Henry sold 27 morning papers and 33 evening papers. How many more papers did he sell in the evening than in the morning?

66. To go to town by trolley car costs 16 cents. To go by bus costs 20 cents. How many more cents does it cost to go by bus than by trolley car?

67. Frank bought groceries that cost 67 cents. He gave the grocer 75 cents. How many cents did he receive in change?

68. Mr. McGregor planted 16 rows of red beans and 22 rows of white beans. How many more rows of white than red beans did he plant?

The Progress Test

The test which follows will permit the teacher to check on the ability of the pupils to do the work of this chapter.

Test on Higher-Decade Subtraction with Bridging

PART I

20 − 3 =	53 − 49 =	32 − 7 =	62 − 56 =	33 − 28 =
32 − 3 =	20 − 16 =	70 − 64 =	22 − 16 =	24 − 18 =
23 − 16 =	20 − 18 =	26 − 18 =	31 − 28 =	61 − 56 =
22 − 18 =	32 − 28 =	21 − 9 =	52 − 45 =	44 − 36 =
52 − 49 =	34 − 6 =	34 − 5 =	27 − 9 =	62 − 54 =
63 − 56 =	71 − 64 =	22 − 4 =	33 − 4 =	80 − 72 =

1. Harry has 28 dominoes; 9 of them are in the box and the rest are on the table. How many are on the table?

Ans. ____

2. There are 33 children in our class; 28 of them went home as soon as school was out. The rest stayed to help the teacher. How many stayed?

Ans. ____

PART II

$71 - 63 =$	$30 - 27 =$	$20 - 1 =$	$55 - 49 =$	$54 - 49 =$
$34 - 28 =$	$53 - 48 =$	$27 - 8 =$	$31 - 27 =$	$22 - 5 =$
$55 - 48 =$	$53 - 45 =$	$30 - 9 =$	$26 - 7 =$	$50 - 45 =$
$36 - 8 =$	$31 - 2 =$	$33 - 27 =$	$28 - 19 =$	$25 - 6 =$
$24 - 7 =$	$41 - 36 =$	$23 - 9 =$	$41 - 35 =$	$36 - 9 =$
$35 - 27 =$	$31 - 24 =$	$20 - 14 =$	$21 - 18 =$	$25 - 18 =$

1. Ruth read 34 pages of her book and Helen read 27 pages of her book. How many more pages did Ruth read than Helen?

Ans. ____

2. Mrs. Jones bought 24 eggs. She let Mrs. White have 6 of them. How many eggs did she have left?

Ans. ____

CHAPTER XXIV

EXTENSION OF COLUMN ADDITION

An analysis of the skills required in column addition will be instructive; and it will offer suggestions for guidance in teaching. One skill, or perhaps we should say a group of one hundred skills, is the correct addition of the basic facts — the combinations of the one-digit numbers. These combinations, however, are not equally difficult; $2+2$ is not so hard as $9+7$. In general, as we have seen, the combinations whose sums are less than 10 are easier than those whose sums are 10 or more. The former we have therefore called the easy group. Any column addition involving nothing but these combinations will be easy, so far as the separate acts of addition are concerned.

Long columns, however, are harder to add than short columns. The more sustained effort of attention required by the long column is, in the first place, a very important element of difficulty. In the second place, the higher-decade combinations which are always in evidence in long columns constitute a difficulty. Among the higher-decade combinations, moreover, those that involve bridging are harder than those which do not. Finally, if the pupil must write the column from dictation or copy it, the longer the column is the greater the likelihood of error. As soon as numbers of more than one place are introduced, further difficulties appear — difficulties in copying the addends, in carrying, in writing the answer, and in a more prolonged sustaining of attention.

Accordingly, we may list the difficulties involved in column addition as follows:

1. Knowledge of the easy group of basic combinations.

2. Knowledge of the hard group of basic combinations.
3. Knowledge of the higher-decade combinations which do not involve bridging.
4. Knowledge of the higher-decade combinations which involve bridging.
5. Ability to handle columns of more than three addends.
6. Ability to handle addends of more than one place.
7. Carrying.
8. Writing the addends (if required).
9. Writing the answer.

From this it will readily be inferred that an approach to the teaching of column addition should be a gradual one. At first the easiest examples should be given, those which involve the fewest possible number of these difficulties. The new idea of adding columns should be unaccompanied by any but the simplest processes. In Chapter XVI columns of this nature were introduced, for example:

$$\begin{array}{r} 2 \\ 5 \\ \underline{2} \end{array}$$

We called them Type A examples (page 169). Only the easier group of basic combinations then played a part. No higher-decade combinations were required. The columns had but three addends; and this is the fewest number that gives the characteristic of column addition, the holding of one or more unseen sums in mind and combining them with seen numbers. And, finally, the addends were one-place numbers. This fact reduced to a minimum the difficulty of copying the example and writing the answer and also excluded the difficulty of carrying.

If the pupils have been well drilled on column addition of Type A as presented in Chapter XVI, they have learned the *form* of column addition. The teacher should now review examples of Type A carefully. Let the pupils do a number of

examples of three one-digit addends. Let them add downward in all cases. Give them a column like this:

$$\begin{array}{r} 2 \\ 5 \\ \underline{1} \end{array}$$

Show them that they have to *think* 7, because it is not written, or in other words that 2 and 5 are the same as 7. The combination $7+1$ is called *unseen* because part of it (the 7) is not visually presented.

All column addition, no matter how extensive, involves successive combinations of two numbers. Four kinds of such combinations are possible *and no more*. They are (1) easy basic combinations, (2) hard basic combinations, (3) higher-decade combinations without bridging, and (4) higher-decade combinations with bridging.

Column Addition of Type B

After reviewing Type A, introduce a column of three one-digit numbers involving *one* hard combination and *no* higher-decade combination. Such a column will have first at the top a seen easy combination and second an unseen hard combination, for example:

$$\begin{array}{r} 2 \\ 5 \\ \underline{8} \end{array}$$

Let us call this kind of column addition Type B. The unseen hard combination constitutes the new difficulty, yet the pupils should be able to surmount this difficulty. It is only an application of basic combinations which they have already learned, with this difference, however, that one term of the combination is in each case unseen.

Give considerable drill on examples of Type B. They can easily be improvised if one is careful to have the two top

figures always make a sum less than 10 and the bottom figure large enough to make the sum of the column a two-place number. For example:

3 5 6 14	2 0 9 11	1 7 4 12	2 2 8 12	0 1 9 10	5 4 8 17

Pupils should have drill in writing columns of this kind both from dictation and from copy. They should also write the answers. Often, however, the work may be done orally from columns written on the board, and again the teacher may dictate columns, and the pupils may add "in their heads."

Many children will be able to construct such examples themselves. This kind of work has decided value, especially for bright pupils.

Type C

We may next introduce the difficulty of higher-decade combinations which do not involve bridging. The columns will still have only three one-digit numbers. This we may call Type C. It will always begin at the top with a hard basic combination. The bottom figure will be small enough to leave the sum of the column less than 20. For example:

6 5 4 15	9 9 1 19	8 4 5 17	6 7 3 16	5 5 9 19

In the first of the above columns $6 + 5$ is a seen hard combination and $11 + 4$ is an unseen higher-decade combination without bridging.

The suggestions regarding written and oral work that were made for Type B apply to Type C as well.

Column Addition of Type D

The only difficulty which remains to be introduced in the column of three addends is that of higher-decade combinations with bridging. The column having this difficulty we may call Type D. It begins, like Type C, with a hard basic combination, but it follows this with a number which carries the total into the twenties. For example:

6 5 } 11	9 9 } 18	8 4 } 12	6 7 } 13	5 9 } 14
9	9	9	8	9
20	27	21	21	23

Observe that in the first example $6 + 5$ is a seen hard combination and that $11 + 9$ is an unseen higher-decade combination with bridging. The suggestions given above under Type B as to methods of drill again apply.

Summary of Column Addition of Three One-Place Addends

Every instance of column addition in which three one-place addends occur requires the correct application of two combinations and no more. One of these, the top one when addition is downward, is a seen combination. The other is unseen. That is, one item in the case of the second combination must be held in mind without being seen.

According to the nature of these two combinations we may arrange the gradation of our work. It is almost certain that Type A is easier than any of the other three types and that Type D is the hardest of the four. There is some doubt as to the order of gradation for Types B and C. At present we believe that Type C is harder than Type B. Table XII on page 346 puts the matter in tabular form.

TABLE XII. TYPES OF COLUMN ADDITION, THREE ONE-PLACE NUMBERS

TYPE	SEEN COMBINATION	UNSEEN COMBINATION
A	Easy basic	Easy basic
B	Easy basic	Hard basic
C	Hard basic	Higher-decade without bridging
D	Hard basic	Higher-decade with bridging

Type A has been introduced in Chapter XVI. Types B, C, and D may well follow in the order named. This exhausts the possibilities of columns of three one-place numbers. If the work has been thoroughly done, the pupil has had abundant opportunity to apply his knowledge of combinations. He will be ready to go into longer and wider columns without being troubled by difficulties which should be surmounted in a simpler setting. We believe this method of approach is an element in good teaching. Analyze first the difficulties, the skills required, in a given unit of work such as column addition, long division, or addition of fractions. Then, if possible, regulate the work so that these difficulties will be presented and surmounted *one at a time.*

PRACTICE MATERIAL FOR COLUMN ADDITION OF TYPES B, C, AND D

Type B. In the following one hundred and fifty examples each easy combination (seen) occurs three or four times.[1] The more difficult combinations are the ones which occur four times, although, of course, there are no *seen* combinations of the hard type, that is, none whose sum is more than 9. All the hard combinations (unseen) are repeated either three or four times. Those which appear four times are the more difficult.

[1] The combinations involving zero, however, occur but once or twice.

Type B. Single-column addition (Add down)

a

2	6	5	3	4	1	7	5	2	4
3	2	4	4	2	5	0	2	4	5
8	4	6	8	7	8	4	6	9	1
6	5	4	1	8	2	5	1	1	2
1	3	3	6	0	6	0	4	3	1
9	6	7	5	3	5	5	7	6	9
7	0	9	8	3	3	3	2	0	4
1	6	0	1	6	0	2	2	4	0
8	5	7	9	5	7	6	7	8	9
2	1	4	6	3	1	2	2	1	0
5	2	4	3	5	1	0	7	8	1
3	8	9	2	7	9	8	8	3	9
7	5	3	4	0	0	0	1	6	0
2	1	3	1	8	5	7	7	0	9
4	4	6	9	2	9	8	6	5	9

b

c

0	0	3	1	6	7	0	8	9	4
3	2	1	0	2	0	6	1	0	3
8	9	7	9	3	4	8	6	8	6
4	3	8	2	1	0	2	3	2	1
1	4	0	3	2	9	2	2	4	4
8	3	9	7	9	3	7	6	9	5
7	1	2	7	5	1	2	4	1	2
2	6	5	1	3	3	1	0	1	6
7	5	9	4	7	6	7	9	8	6
6	5	4	3	5	5	4	2	1	1
1	1	2	3	4	2	5	7	8	5
7	6	7	4	2	8	1	9	4	5
4	3	6	3	0	5	0	1	0	6
4	5	3	6	8	0	7	7	5	0
8	2	5	4	5	9	4	5	8	7

d

		e					*f*		
3	7	1	4	4	1	5	3	2	1
1	2	6	2	1	1	2	0	2	0
8	5	7	6	7	9	6	7	6	9
4	1	3	7	2	4	2	3	1	2
4	2	6	1	5	5	7	1	3	0
4	9	2	8	3	8	1	9	7	8
6	3	3	5	6	1	2	5	5	1
2	5	4	3	3	8	4	4	3	7
3	2	5	7	7	9	9	6	9	5
4	3	2	5	6	1	6	1	4	4
3	3	3	1	1	5	3	4	4	3
9	8	6	9	4	4	7	9	6	8
5	8	2	3	2	3	7	6	3	3
4	1	1	2	6	5	2	2	6	4
6	3	8	5	9	7	5	8	4	9

Type C. Two hundred examples of Type C are here given. The hard combinations (seen) occur either four or five times, the more difficult ones occurring five times. The higher-decade combinations (unseen) afford drill on the basic facts with the same frequencies and with the same slightly greater emphasis upon the most difficult facts.

As we have intimated, it is by no means certain that Type C is harder than Type B. We believe it is, although experimental data on the question are lacking. Both types involve a hard combination. Besides this, Type B has an easy and Type C a higher-decade combination. The latter is doubtless harder than the former. If, therefore, the two types are on a par because of the hard combination which is common to both, we may infer that Type C is harder than Type B. But Type B may acquire an added element of difficulty because of the fact that its hard combination is unseen.

Type C. Single-column addition (Add down)

a

8	6	3	1	7	9	8	9	5	7
5	7	9	9	5	3	8	9	5	4
3	4	7	6	4	2	1	1	7	2
9	6	3	8	5	6	9	6	5	4
4	8	8	9	6	6	7	4	7	9
5	4	7	0	4	3	2	5	1	2
7	5	6	2	4	5	8	8	2	6
8	9	9	9	6	8	6	3	8	5
3	2	4	6	9	1	3	5	4	8
9	8	7	8	9	7	9	9	8	9
8	7	6	4	5	3	2	6	2	1
2	1	6	6	3	3	1	2	1	2
3	7	4	4	7	9	6	3	9	4
7	7	7	8	9	8	7	8	5	7
8	5	3	5	3	1	3	6	4	3

b

c

7	6	6	3	9	5	8	7	6	4
5	9	8	7	4	8	8	4	6	9
1	0	2	6	5	2	0	8	3	1
8	4	8	5	8	9	7	9	6	9
9	8	4	9	5	7	3	3	4	9
2	7	5	5	4	2	5	2	4	0
3	7	8	5	2	8	5	9	9	6
9	8	7	6	9	6	5	2	8	7
4	1	3	7	4	3	3	2	1	3
7	7	4	1	7	8	4	3	9	9
9	7	6	9	6	2	7	8	5	6
3	1	8	9	6	2	6	3	4	2
9	5	2	6	8	9	5	4	7	8
6	7	8	5	3	1	9	6	9	9
4	6	7	1	5	1	2	9	1	2

d

e

6	4	2	3	7	9	7	6	4	5
9	8	9	7	7	3	4	4	9	8
4	1	7	6	5	3	8	5	1	2
8	6	1	2	7	7	9	8	9	8
4	8	9	8	8	3	1	6	2	2
5	1	8	7	3	2	1	3	2	4
8	3	5	7	6	3	6	9	4	7
7	9	7	6	5	8	7	8	6	9
1	6	4	5	4	6	3	1	9	2
8	5	7	8	5	9	4	5	6	9
3	6	5	5	5	6	7	9	9	5
1	5	2	6	3	2	3	4	1	2
9	9	6	8	9	8	3	7	8	9
9	7	6	8	4	9	7	6	5	9
1	3	7	1	4	2	8	4	6	1

f

7	6	8	6	4	9	5	9	8	4
8	6	3	5	8	3	8	1	3	9
3	1	5	7	2	3	1	2	4	2
6	2	1	5	2	7	9	6	3	9
8	9	9	7	8	9	8	9	9	9
1	4	7	5	6	2	1	2	1	1
9	8	7	5	9	9	7	8	9	8
7	4	7	6	4	7	8	8	6	7
1	6	5	1	5	3	4	1	3	1
7	3	8	5	7	8	9	8	6	6
5	9	7	5	3	9	5	5	8	7
4	7	4	3	4	2	3	6	2	5
8	9	6	7	8	7	5	9	7	9
8	2	4	4	2	7	9	4	6	2
3	2	5	8	1	5	1	4	3	8

g

7	6	8	6	4	9	5	9	8	4
8	6	3	5	8	3	8	1	3	9
3	1	5	7	2	3	1	2	4	2
6	2	1	5	2	7	9	6	3	9
8	9	9	7	8	9	8	9	9	9
1	4	7	5	6	2	1	2	1	1
9	8	7	5	9	9	7	8	9	8
7	4	7	6	4	7	8	8	6	7
1	6	5	1	5	3	4	1	3	1
7	3	8	5	7	8	9	8	6	6
5	9	7	5	3	9	5	5	8	7
4	7	4	3	4	2	3	6	2	5
8	9	6	7	8	7	5	9	7	9
8	2	4	4	2	7	9	4	6	2
3	2	5	8	1	5	1	4	3	8

h

9	5	9	8	4	9	5	9	8	4
3	8	1	3	9	3	8	1	3	9
3	1	2	4	2	3	1	2	4	2
7	9	6	3	9	6	2	1	5	2
9	8	9	9	9	8	4	7	5	6
2	1	2	1	1	1	3	5	6	8
9	7	8	9	8	9	7	8	9	8
7	8	8	6	7	4	6	3	5	7
3	4	1	3	1	2	8	4	1	3
8	9	8	6	6	5	9	8	7	4
9	5	5	8	7	6	4	2	9	1
2	3	6	2	5	7	3	5	8	6
7	5	9	7	9	8	6	4	2	5
9	4	4	6	2	1	5	9	3	7
1	1	4	3	8	4	2	6	1	3

Type D. The following two hundred and forty columns afford practice in seen hard combinations, but they do so somewhat irregularly. In order that the column may belong to Type D, the bottom number must carry the total into the twenties. If the sum of the first two numbers is only 10, no bridging to the twenties can occur in the next combination. Consequently the hard basic combinations which make 10 cannot be used. If the sum of the first two numbers is 11, the only one-place number which will carry the sum into the twenties is 9. Accordingly, only a few hard combinations which make 11 can be used. To a less degree there is likewise a limitation of practice on the hard combinations whose sums are 12. Obviously only 8 and 9 can be used in conjunction with these combinations to carry the sum into the twenties.

Similar limitations, although of an opposite nature, are placed upon the unseen combinations. We can have no instances where anything is added to 19, for the reason that no seen combination will yield 19. Similarly, in only a few cases can the unseen combination be 18 plus anything. There is only one seen combination, $9 + 9$, which will yield 18 as the first number of the unseen combination.

However, these two limitations in Type D, one on the seen combinations and the other on the unseen combinations, largely balance each other. The result is that the basic combinations are represented with fairly uniform frequency in these examples.

In spite of the fact that the number of basic combinations which may be utilized in Type D is relatively small, we are giving more examples of this type than of the other two types presented in this chapter. There are two reasons for this. First, Type C is hard and needs practice as a type. Second, the basic combinations represented in it need practice as such. Here is a chance to give proportional treatment to the hard combinations.

Type D. Single-column addition (Add down)

a

8	7	9	6	3
5	8	3	9	8
9	6	8	7	9
5	7	4	5	6
9	9	9	7	8
8	5	7	9	9
9	7	8	9	8
9	6	6	5	8
9	8	7	6	7
9	8	9	8	9
7	7	8	9	6
4	5	7	6	8

b

9	7	8	7	6
9	8	3	5	8
8	9	9	8	9
8	7	6	5	9
8	7	7	8	7
6	7	8	9	8
8	4	9	8	7
9	8	5	7	9
7	9	6	5	9
9	8	6	9	9
8	6	9	4	9
5	8	7	7	6

c

6	8	5	7	5
6	4	9	6	6
8	9	6	8	9
8	8	9	7	9
5	9	9	9	8
7	4	7	4	3
4	9	8	7	8
9	6	8	7	7
9	6	7	9	9
9	7	8	6	9
7	8	6	9	5
5	7	8	5	7

d

9	3	4	9	9
8	9	7	3	6
8	8	9	9	8
5	6	6	9	7
8	7	8	4	7
7	9	7	8	8
5	8	9	8	8
9	9	7	9	7
6	9	8	6	8
7	9	9	7	6
9	6	5	8	9
6	9	9	6	5

e

2	5	7	7	9
9	7	5	6	9
9	8	9	8	5
8	9	8	5	7
5	4	6	9	7
9	7	7	8	9
6	9	8	7	8
8	6	9	8	8
6	7	7	6	9
9	8	7	9	6
8	7	9	7	9
5	9	4	5	8

f

6	8	6	4	8
5	4	7	9	9
9	8	9	8	4
9	4	5	6	5
9	8	8	8	9
4	9	7	9	6
9	8	6	8	7
8	8	9	7	7
3	7	8	5	8
7	7	8	9	9
8	9	6	6	9
6	8	7	9	6

g

3	7	8	9	7
9	6	5	5	4
8	9	7	6	9
6	9	8	5	9
6	4	6	9	7
9	8	7	8	9
9	7	7	8	9
9	9	8	9	5
3	4	7	8	9
9	6	8	8	9
8	9	7	8	6
6	8	9	5	5

h

9	5	4	6	9
3	7	9	8	6
8	9	7	9	8
9	5	6	8	8
9	8	7	9	8
2	9	8	7	7
9	8	7	7	9
7	7	7	9	2
8	6	8	6	9
5	7	9	6	8
9	8	6	9	6
7	9	8	7	6

i

7	4	3	7	9
5	8	8	6	9
9	8	9	8	9
8	5	8	9	6
5	9	7	6	8
9	7	5	7	9
9	8	8	9	8
4	9	8	7	6
7	5	9	5	6
9	9	7	7	6
8	5	8	9	9
4	8	6	4	8

j

7	2	9	8	6
7	9	9	4	7
8	9	8	9	8
8	8	4	7	6
6	9	9	8	9
9	3	7	9	7
3	5	6	8	9
9	3	8	8	6
8	9	7	6	5
9	7	5	9	8
8	9	9	7	7
8	8	6	7	6

k

4	6	9	6	4
7	6	4	9	9
9	8	7	8	8
9	8	8	7	3
5	8	7	6	9
8	9	5	9	9
7	7	9	7	9
7	8	9	9	8
6	6	7	4	6
8	5	9	9	6
9	9	6	7	8
7	7	7	5	9

l

4	8	7	8	9
8	5	7	9	6
9	7	9	8	7
9	5	9	5	5
5	9	9	8	7
8	7	6	9	8
6	9	7	8	6
7	8	9	7	8
8	9	8	9	6
9	6	7	5	8
7	9	8	6	8
9	8	6	9	7

Illustrative Problems

We have classified the following problems according to the type of column addition which they exemplify. In doing so we have assumed that the numbers will be added in the order in which they appear in the problems. Consider, for example, the thirty-fifth problem: "At Mildred's party there were 9 boys, 9 girls, and 2 women. How many people were at the party?" Here it is assumed for purposes of classification that the numbers will be added in the order 9, 9, 2; and that if the process were written out in column form it would be

$$\begin{array}{r} 9 \\ 9 \\ \underline{2} \end{array}$$

This assumption must not be made to "go on all fours." It does not really matter what the order of addition is. The big thing is that the children should be interested in the problem, see its meaning, and solve it. Since they can solve these particular problems without paying any attention to the order of the numbers, to insist on a fixed order is to insist on an unessential. Most children, however, will add the numbers in the order in which they are expressed; if they write them, they will do so, for the most part, in the same order. Our classification is meant to do no more than recognize this fact. The last ten problems, however, are unclassified. They relate to a more or less familiar school project, that of a "Thanksgiving Table," in which contributions for needy families are collected.

This idea of using an interesting situation for a variety of numerical processes will be much more in evidence in the work of later grades. Up to the present we have been content to take a very simple, familiar, or fanciful situation and bring out one numerical fact about it. This in itself is sometimes rather artificial. Conditions with which both children and

adults are confronted are rich in numerical facts, relationships, and conditions. To select one of them is often to ignore what is obviously important and probably interesting. But hitherto we have been narrowly circumscribed. Only addition and subtraction have been available as operations and the numbers we have been able to use have been few and small. Problems involving more than one operation have been excluded. Not even the two familiar operations, addition and subtraction, have appeared in the same problem. A number of other limitations have been inevitable. But the situations of life are not thus limited. Other operations, more varied numbers, fractions as well as integers, and more operations are generally called for. As children advance in number work the "number project" will become more natural and more valuable.

Type B

1. Henry said he would make some kites to sell at the church fair. The day he began to make them he was pretty slow about it and made only 2. The next day he made 7, and the next 9. Then he thought he had made enough. How many had he made?

2. Mr. and Mrs. Rabbit and little Sonny Rabbit went out to find some lettuce leaves. Mr. Rabbit ate 5 leaves, Mrs. Rabbit ate 4, and Sonny Rabbit ate 3. How many lettuce leaves did the Rabbit family eat?

3. One day Mr. McGregor planted 3 rows of beets, 4 rows of carrots, and 4 rows of peas. How many rows of vegetables did he plant that day?

4. Some children of the first grade were asked to bring cans of fruit to give to poor families on Thanksgiving Day. They brought 7 cans of peaches, 2 of pears, and 6 of berries. How many cans of fruit did they bring all together?

5. Other children were asked to bring cans of vegetables. They brought 4 cans of peas, 5 of corn, and 5 of tomatoes. How many cans of vegetables did they bring?

6. The children of a school earned money for some school pictures. The first and second grades earned 4 dollars, the third grade earned 3, and the fourth grade earned 6. How many dollars did they all earn?

7. This morning the mailman left 6 letters, 3 magazines, and 4 post cards at Mr. Holt's office. How many things did he leave all together?

8. Donald is learning to swim. He swam 2 yards Monday, 4 yards Tuesday, and 6 yards Wednesday. How many yards did he swim in all?

9. Some children colored Easter eggs. They colored 5 of them blue, 3 red, and 4 yellow. How many eggs did they color?

Type C

10. One day some boys ate their lunch in the woods. While they were in the woods they saw 9 gray squirrels, 2 chipmunks, and 4 rabbits. How many animals did they see?

11. Most of the children in our class live a long way off and have to come every morning in the school busses; 7 come in one, 4 in another, and 5 in another. How many come in the school busses?

12. Harry, Charles, and William are brothers. They are saving money to buy their mother a Christmas present. Harry, who is the oldest, has saved 7 dollars; Charles has saved 3 dollars; and William has saved 2 dollars. How much have the boys saved for their mother's present?

13. "Helen," said mother one day, "you have left your books all over the house. Please go and pick them up and put them in your bookcase." Helen found 8 of her books in the living room, 3 in her bedroom, and 2 in the kitchen. How many of her books did Helen find?

14. John, Harry, and Dick went to look for wild blackberries. After a while they found a whole lot of them. John picked 5 quarts, Harry picked 7, and Dick picked 6. How many quarts of blackberries did they all pick together?

15. The expressman brought Mr. Jones a big package of groceries. There were some boxes of food in it. There were 8 boxes of oatmeal, 4 of raisins, and 5 of rice. How many boxes of food were there in the package?

16. There were some cakes of soap too. There were 9 cakes of laundry soap, 5 of toilet soap, and 3 of scouring soap. How many cakes of soap were there?

17. Sarah had learned to sew on buttons. One week she sewed on 8 for father, 3 for Fred, and 1 for Baby Lu. How many buttons did Sarah sew on that week?

18. Dandelions were coming. Ruth found 9, Harold found 3, and Jane found 7. How many dandelions did they all find?

19. Louis spent 5 cents for a ride on the merry-go-round, 8 cents for peanuts, and 4 cents for lollipops. How many cents did he spend all together?

20. Harvey paid 7 cents for soup, 4 cents for cocoa, and 6 cents for a sandwich. How many cents did he pay for his lunch?

21. Mr. and Mrs. Turner went out to buy Christmas presents for their children. Roller skates for Louise cost 3 dollars; an express wagon for Ted cost 7 dollars; and a doll for Polly cost 4 dollars. How many dollars did the Christmas presents cost?

22. In the barnyard there were 3 horses, 8 cows, and 1 dog. How many animals were there in the barnyard?

23. Some children were hunting Easter eggs. Susie found 5, Bess found 5, and Agnes found 8. How many eggs did they all find together?

24. The garden tools stood locked in a dark closet of the school basement. If they could have talked, they would have said they were tired of standing there and wanted to get out and work. One day there was a hurry of feet on the stairs. The closet door flew open; hands were stretched in; and out came 6 hoes, 5 rakes, and 3 spades. How many tools went to work in the garden?

25. Walter had saved enough money to buy his new spring clothes. His suit cost 9 dollars, his shoes 4 dollars, and his cap 1 dollar. How many dollars did they all cost?

26. There were 6 people in the Walsh family. Then they took 4 boarders and hired 2 people to work. How many people were then at the Walsh house?

27. Henry went to see his uncle. He walked 2 miles. Then he rode 9 miles on the train and 6 miles in an automobile. How many miles away did his uncle live?

28. Frances made curtains for her playhouse. She made 4 white curtains, 8 blue ones, and 2 green ones. How many curtains did she make?

29. Alfred was making a kite. His little neighbors wanted to help him. Alfred told them to go home and get some strips of cloth for the kite's tail. Tom got 7 feet, Don got 5, and Billy got 3. How many feet of cloth did they all get?

30. In the bean-bag game Harry's team had Mildred and Tom on it besides Harry himself. Mildred scored 5, Harry 9, and Tom 4. How many did Harry's team score?

31. On Edgar's team, John got 6, Edgar got 9, and Charles, who had broken his arm and had to throw with his left hand, got only 2. How many did Edgar's team get?

32. Maud and Emma were on William's team. Maud scored 4, William 7, and Emma 8. How many did William's team score?

33. Paul had saved 8 cents. His father gave him a nickel, and he found 3 cents. How many cents did he then have?

34. Frank and his father went to a hotel for dinner. In the dining room Frank saw 9 people sitting at a large table, 4 at a middle-sized table, and 2 at a small table. How many people did Frank see in the dining room?

Type D

35. At Mildred's party there were 9 boys, 9 girls, and 2 women. How many people were at the party?

36. The Tudor children were very fond of jam. So Mrs. Tudor made 9 jars of plum jam, 9 of blackberry, and 9 of raspberry. How many jars of jam did she make?

37. Mrs. Roberts filled 8 jars with strawberries, 6 with raspberries, and 9 with blackberries. How many jars did she fill with berries?

38. Mrs. Martin made pop-corn balls for the school picnic. She made 8 brown ones, 8 white ones, and 8 pink ones. How many pop-corn balls did she make?

39. Bobby and his friends went out on the grass with soapsuds and bubble pipes. Bobby blew 6 bubbles, Ted 9, and Tom 8. How many bubbles did they all blow?

40. Mrs. Taylor bought vegetables from the school garden. She paid 9 cents for a head of cabbage, 8 cents for some lettuce, and 5 cents for radishes. How many cents did she pay for the school vegetables?

41. Mary, Dick, and Kate had a lemonade stand under a big tree by the roadside. Mary sold 8 glasses, Dick sold 7, and Kate sold 9. How many glasses did they all sell?

42. The teacher asked some of the children to bring small flags to school to use on Flag Day. One brought 8, another 6, and another 7. How many flags did they bring?

43. In our fishing game, if Jack caught 9 fish, Helen 1, and Raymond 7, how many fish did they all catch?

44. Betsy, Biddy, and Speckle went out for a walk on Easter morning, with their new baby chicks. Betsy had 9, Biddy had 8, and Speckle had 9. How many baby chicks were out walking that Easter morning?

45. Rosa was a little flower girl in a large city. One day she sold 4 bunches of violets, 8 bunches of pinks, and 9 of roses. How many bunches of flowers did she sell in all?

46. James had some examples to do at home. He did 7 in the morning, 7 at noon, and 6 after school. How many examples did he do at home?

47. Herbert went to the store and bought a pound of sugar for 7 cents, a loaf of bread for 9 cents, and a cake of soap for 5 cents. How many cents did he pay for all?

48. Andrew has learned to write. He wrote 6 words in the morning, 7 in the noon hour, and 7 in the evening. How many words did he write that day?

49. Some children put wild flowers in a glass on the teacher's desk for a surprise. There were 8 daisies, 7 red clovers, and 6 wild roses. How many wild flowers were in the glass?

50. Florence and Kate were on a farm. They went to look for eggs. They found 3 goose's eggs, 8 duck's eggs, and 9 hen's eggs. How many eggs did they find in all?

51. Paul had 6 white rabbits, 7 brown rabbits, and 6 black ones. How many rabbits did he have all together?

52. Clara, Eleanor, and Lucy were looking for apples on the ground under the apple trees. Clara found 8, Eleanor found 7, and Lucy found 5. How many apples did they all find?

53. Polly, Edith, and Emma were dressing paper dolls for a fair. Polly dressed 9, Edith dressed 6, and Emma dressed 8. How many paper dolls did they all dress?

54. At grandma's birthday dinner party there were 7 women, 7 men, and 9 children. How many people were at the party?

55. Mr. and Mrs. Squirrel and their children sat down to a nut dinner. Mr. Squirrel ate 9 nuts, Mrs. Squirrel ate 6, and the children ate 9. How many nuts did the family eat for dinner?

56. Mrs. Atkins saw some pretty cloth in the store. She bought 8 yards for chair covers, 5 yards for a bedspread, and 9 yards for curtains. How many yards did she buy in all?

57. Mrs. Atkins also bought 6 large bath towels, 8 face towels, and 6 cup towels. How many towels did she buy?

58. Mrs. Davis made 5 jars of currant jelly, 6 of apple, and 9 of grape. How many jars of jelly did she make in all?

59. Some children planned to have a lawn party. Fred brought 8 Japanese lanterns, Bessie brought 7, and Louise brought 7. How many lanterns did they have in all?

60. Alice had some reading to do at home. She read 8 pages in the morning, 4 at noon, and 8 after school. How many pages did she read at home?

The Thanksgiving Table

61. Every day during the week of Thanksgiving the children in our school brought food to give to poor families. There was a Thanksgiving table in every room. One day the children in the second grade brought 8 turnips, 9 large potatoes, and 6 squashes. How many vegetables did they bring that day for their table?

62. A few brought dried fruit. They brought 4 pounds of prunes, 2 of figs, and 6 of peaches. How many pounds of dried fruit were there in all?

63. The day before Thanksgiving some of them brought bread. There were 6 loaves of plain white bread, 4 of raisin bread, and 7 of brown bread. How many loaves of bread were there in all?

64. Edna Warner's mother gave some buns. She sent 9 currant buns, 9 cinnamon buns, and 6 coconut buns. How many buns did she give?

65. On the Thanksgiving table there were also 4 pounds of coffee, 5 of sugar, and 7 of flour. How many pounds of these groceries were there?

66. Some of the children brought apples. There were 9 yellow apples, 7 green ones, and 9 red ones. How many apples did the children bring?

67. Horace's father was a farmer, so every morning Horace brought some fresh eggs for the Thanksgiving table. On Monday he brought 7, on Tuesday 6, and on Wednesday 5. How many eggs did Horace bring all together?

68. A few of the children brought meat. When the teacher weighed it, she found there were 6 pounds of bacon, 2 pounds of ham, and 8 pounds of sausages. How many pounds of meat were there?

69. Ethel's father keeps a grocery store. She brought 8 cans of corn, 2 of peas, and 9 of tomatoes. How many cans did she bring from her father's store?

70. Then 7 boys, 5 girls, and 3 teachers went out to carry the gifts to the people who needed them. How many people went to carry the gifts?

The Progress Test

The test here given contains in its two parts thirty-six examples and four problems. Of these forty exercises in column addition, eight are of Type B, seventeen of Type C,

and fifteen of Type D. Seventy of the one hundred basic combinations are represented either directly or by their corresponding higher-decade combinations. The thirty which are not represented are very easy, mostly involving 1 or 0. For suggestions on giving, recording, and following up this test the teacher is referred to pages 57 and 188.

Test on Column Addition, Types B, C, and D

PART I

6	9	6	4	3	8
2	3	5	9	7	9
5	2	6	4	4	3
8	8	3	2	6	8
2	6	9	5	9	8
9	7	8	8	9	7
3	6	6	9	5	2
5	6	8	8	4	9
7	4	8	4	7	8

1. A lady bought some cloth. There were 5 yards in one piece, 2 yards in another, and 6 yards in another. How many yards did she buy?

Ans. ____

2. One day Robert counted 6 birds in one tree, 8 in another, and 9 in another. How many birds did Robert count?

Ans. ____

PART II

8	5	4	6	7	6
3	7	5	4	7	3
4	7	4	7	3	2
7	7	5	7	7	3
5	9	8	9	6	8
6	6	6	8	3	9
1	9	9	9	9	6
7	6	1	9	5	9
9	6	8	4	6	5

1. On the teacher's desk there are 7 red pencils, 2 black pencils, and 8 brown ones. How many pencils are there on the teacher's desk in all?

Ans. ____

2. Tom is carrying 2 books to school. He has 8 books at home and 6 more at school. How many books has he all together?

Ans. ____

CHAPTER XXV

SUMMARY

It has been assumed that when children start to school they will be able to count to ten and to tell by inspection or by counting the number of objects in groups of two, three, or four. From this as a beginning, the following topics are included in the pre-third-grade work, according to the plan we have set forth:

1. Number concepts with special reference to those from 1 to 10.

2. Counting by 1's and by 10's to 100; by 2's to 20; by 3's to 30; and by 5's to 60.

3. Reading and writing numbers to 1000.

4. The one hundred basic addition facts.

5. The one hundred basic subtraction facts.

6. The multiplication, division, and partition facts associated with $1 + 1$, $2 + 2$, $\cdots$, $9 + 9$, etc.

7. The higher-decade addition facts, especially the 225 whose sums are less than forty and the additional 87 which are used in multiplication.

8. The higher-decade subtraction facts, Case I and Case II, with special emphasis on the 115 facts of Case II which are used in short division.

9. The technical and partly technical vocabulary appropriate to primary arithmetic together with an understanding of an appropriate general vocabulary such as is used in verbal problems.

10. Certain elementary facts of measurement and comparison involving the following units: inch, foot, yard, pint, quart, nickel, dime, quarter, half-dollar, and dollar.

11. The informal units, such as teaspoonful, tablespoonful, and cupful.

12. Ordinal numbers.

13. The signs of operation, + and −, and the sign of equality.

14. Telling time to the nearest five minutes; reading the calendar.

15. The addition of two numbers of not more than three digits without carrying.

16. The subtraction of two numbers of not more than three digits without borrowing.

17. Column addition, three one-place numbers.

18. The application of all the foregoing in the solution of verbal problems and in the formulation of verbal problems.

Review Problems

The following problems are intended to afford a partial review of the topics in the foregoing list.

1. Alice is 8 years old. Her brother has just gone away to stay 3 years. How many years old will Alice be when her brother comes back home again?

2. There are 4 cupfuls in a quart. How many quarts will 8 cupfuls make?

3. Howard walks 3 miles an hour. How many hours will it take him to walk 6 miles?

4. Mrs. White and Mrs. Brown each drove her automobile for an hour. Mrs. White drove 24 miles and Mrs. Brown 21. How much farther did Mrs. White drive than Mrs. Brown?

5. Jim had 20 popgun shots at a target. He hit it 12 times. How many times did he miss?

6. At Helen's party there were 8 girls, 6 boys, and 2 ladies. How many people were there at the party?

7. William hoed 25 rows of corn and his little brother hoed 8. How many more rows did William hoe than his little brother?

8. One summer Grandpa gave Paul 4 white ducks. The next spring some of these hatched 28 little ducks. Big ducks and little ducks! How many ducks in all?

9. Henry had 8 yellow ducklings and 11 little yellow chicks. How many little yellow fowls did he have?

10. There are 45 children in the first grade of a school and 33 in the second. How many more children are there in the first grade than in the second?

11. Flora bought a yard and 12 inches of pink satin. How many inches long was the piece of satin?

12. Wallace picked 16 quarts of cherries and Richard picked 9. How many more quarts did Wallace pick than Richard?

13. There were 35 sheep in a pasture and the farmer put in 4 more. How many sheep were there in the pasture?

14. Mrs. Holmes had 16 boarders who sat at the big table and 8 who sat at the small one. How many boarders sat at both of Mrs. Holmes's tables?

15. Peggy and her father were going to Aunt Belle's, a place 15 miles away. When they had gone 4 miles, Peggy said, "How many miles is it to Aunt Belle's now, Daddy?" What was her father's answer?

16. Earl had some pet hens that laid very fine eggs. A lady called one day and wanted to buy 24 eggs, but Earl had sold so many he had only 8 left. He said he would take her the rest next day. How many did he have to take her next day?

17. There are 19 cows in Mr. Smith's pasture and 16 in Mr. Poe's. How many more cows are there in Mr. Smith's pasture than in Mr. Poe's?

18. In Elizabeth's day-school class there are 22 girls. In her Sunday-school class there are 18. How many more girls are in the day-school class than in the Sunday-school class?

19. Ernest needed a tent pole 7 feet long. He had a pole which was just right except that it was too long. So he sawed off 7 feet. Then he measured the rest of the pole and found it was just 1 yard long. How many feet long was the pole at first?

20. Harold read 8 pages of his new book in the morning, 4 at the noon hour, and 9 in the evening. How many pages of the book did he read that day?

21. Wallace sold a head of cabbage for 12 cents and some beets for 15 cents. How many cents did he receive for both?

22. When Mrs. Hunter counts eggs, she takes 3 in each hand. How many does she take in both hands?

23. Last fall Mr. Lane had 68 chickens. He could keep only 25 through the winter. He sold the rest. How many chickens did he sell?

24. Mrs. Hall had 15 dollars to spend for fruit trees and shrubs. She bought trees for 9 dollars. How many dollars were left to buy the shrubs?

25. Polly measured her teacher's desk with a yardstick. She found the desk was 1 yard and 2 feet long. How many feet long was the desk?

26. Mr. Parks paid 4 dollars for a hat, 9 dollars for a pair of shoes, and 2 dollars for a tie. How many dollars did he pay for all?

27. Sadie had 15 shiny buttons on her new blue dress. One night she counted them and found she had only 12. How many buttons were lost?

28. William had saved 17 dollars. He paid 9 dollars for a tent. How many dollars had he then?

29. Miss Jones passed out sheets on which there were many examples. Then the children all worked until she told them to stop. After school Clarence told John that he had done 17 examples. John said, "I beat you. I did 22." How many more examples did John do than Clarence?

30. Marie sold flowers at a street corner in a large city. One morning she took out 36 bunches. At noon she had only 8 bunches left. How many bunches had she sold that morning?

31. Out of a crate holding 24 quarts of strawberries Mr. Schwartz sold 3 quarts. How many quarts were left in the crate?

32. Fourteen girls were to sing a song together. Mary counted them and found there were only 9. How many girls were absent?

33. Tommy Brown's big white hen had 13 soft baby chicks one morning. There were 2 eggs left in the nest. Tommy said, "Hurry, Snow White, and hatch those eggs. We shall have a fine fluffy family then." Soon 2 more little chicks tumbled out of the eggs. How many babies were in Snow White's family then?

34. Mr. Rogers bought a radio set for Bob at 54 dollars, and a tea set for Mary at 7 dollars. How many dollars did Mr. Rogers pay for the gifts?

35. Mr. Murray raised 98 bushels of oats. He kept 25 bushels and sold the rest. How many bushels did he sell?

36. Jean and Ruth each had 7 dolls. How many dolls did they both have?

37. There were 11 children at the board. Soon 6 had finished and passed to their seats. How many were still at the board?

38. Emma had to do 18 examples before noon. She had done 7 when the recess bell rang. How many examples did she have to do after recess?

39. Henry is a big boy. He earned 47 dollars last summer. Ray is his little brother. Ray earned 8 dollars. How many more dollars did Henry earn than Ray?

40. There were 27 people crossing the lake in a motor boat. The boat stopped to let 8 more people get on. How many people was the boat then carrying?

41. Nellie went to the orchard to find some apples for the picnic basket. She found 13 red ones and 8 yellow ones. How many apples did she find?

42. There were 13 boys in Miss Rice's class. When the hall monitor saw only 9 boys pass out, he knew that the rest had stayed to help Miss Rice. How many boys had stayed?

43. There were 18 boy scouts in the swimming pond. Then the master called out 10 of them. How many boys remained in the swimming pond?

44. Stanley and Charles go each summer to search for wild blackberries. Last summer they picked 29 quarts. This summer they picked 37. How many more quarts did they pick this summer than last?

45. The silk Mrs. Case bought for her dress was a yard wide. Mrs. Dixon bought some silk, too; but it was only 27 inches wide. How many inches wider was Mrs. Case's silk?

46. There were 7 men, 9 ladies, and 8 children in the trolley car going to the park. How many people were there in all?

47. Effie polished a dozen knives and a dozen forks. How many pieces of silver did she polish?

48. Jim bought 2 chocolate bars for 12 cents. How much were they apiece?

49. Nora went into the garden to hang out clothes and asked Polly to bring the clothespins. Polly brought 48 clothespins. Nora used 31 and told Polly to let Baby play with the rest. How many clothespins did Baby get?

50. Fritz came home from school and said he had 12 problems to do. He wanted to do them before supper. At supper time his mother asked if he had finished. He answered, "I still have 3 to do." Then his mother knew how many he had done. How many had he done?

51. William paid 16 dollars for a bicycle and 3 dollars for a new tire. How many dollars did he pay in all?

52. Anna went away for a few days of her spring vacation. When she returned she found 19 beautiful pansy blossoms in her garden. This made her very happy; but the next morning she was happier still, for there were 6 more blossoms peeping out. How many pansies now bloomed in Anna's garden?

53. Harry is 19 years old. His dog Jack is 7 years old. How old was Harry when his dog Jack was born?

54. Jack picked 14 quarts of berries and sold 8 quarts. How many quarts did he have left?

55. There were 26 children invited to a party, but only 22 were present. How many of the children were absent?

56. Mother gave Nina 53 cents and told her to buy a pound of butter at the grocery store. Mother said Nina might spend the rest for chocolate cookies. The butter cost 45 cents. How much did Nina have to spend for chocolate cookies?

57. Mrs. Thompson told the storekeeper she wanted a dozen lemons. "I'm afraid," said he, "I haven't a dozen left." But when he counted them, he found 15. How many more than a dozen did he have?

58. Charles and Ben played a game of dominoes. Charles made 8 for his first score, 4 for the next, and 5 for the next. What was his score at the end of the game?

59. Ben's scores were 9, 7, and 2. What was his whole score?

60. There are 43 pupils in the first grade and 34 in the second. How many pupils are there in the first and second grades?

61. Elizabeth was going on a journey. She bought a suitcase for 9 dollars and a hatbox for 4 dollars. How many dollars did she have to pay for both?

62. In our town there is a school for boys. In this school there are 76 boys. There is also a school for girls. In this school there are 87 girls. How many more pupils are there in the girls' school than in the boys'?

63. Edward planted 18 hills of corn in two rows. How many hills of corn were in each row?

64. Mrs. Post paid 7 dollars for a porch chair and 6 dollars for a lawn seat. How many dollars did she pay for both?

65. Henry received 16 dollars for his week's work. He used 5 dollars to buy a pair of shoes and saved the rest. How many dollars did he save that week?

66. A racing boat has 8 oars in all, the same number on each side. How many oars are there on each side?

67. The teacher's ruler is 18 inches long. Elizabeth's ruler is only 1 foot long. How many inches shorter is Elizabeth's ruler than the teacher's?

68. Some boy scouts took a long walk one day. They walked 7 miles in the morning and 5 miles in the afternoon. How many miles did they walk that day?

69. Susie had a piece of ribbon 36 inches long. She cut off 6 inches for a bookmark. How many inches long was the piece that was left?

70. Frank helped care for the garden. One day he made up 24 bunches of rhubarb and went out to sell it. When he returned he still had 5 bunches. How many bunches had he sold?

71. Bessie was in the first grade. There were 48 children in her class. Julia was in the second grade. There were 39 in her class. How many more children were there in Bessie's class than in Julia's?

72. The grocer had 21 oranges left in a box. Mrs. Sammis bought a dozen. How many did the grocer then have?

73. Philip has 16 problems to do. Half of them are addition problems. How many are addition problems?

74. Little Billy Brown came to school crying one morning. Miss Smith said, "What is the matter, Billy?" Billy said, "I had 25 cents in this pocket. Now I have only so much." Billy held out 15 cents. Miss Smith helped Billy search and there in his other pocket they found the rest of his money. How much was it?

75. Alice picked 24 daisies and 15 buttercups. How many flowers did she pick?

76. There were 5 girls and 6 boys at the blackboard. How many children were at the blackboard?

77. Raymond paid 55 cents for a dozen oranges and 42 cents for a dozen eggs. How many cents did he pay for both?

78. Annie's father measured her height and found it was 61 inches. "Why, Annie," said he, "you are just one foot shorter than I am." How many inches tall was Annie's father?

79. Tom bought a new spotlight for his car. It cost 5 dollars. He then bought a heater that cost 8 dollars. How many dollars did both cost?

80. Mrs. Stone needs $28 to pay for her new rug. She has $16 that she earned by sewing. How many dollars does she still need?

81. There were 6 buttons on a card. How many buttons were there on 2 cards?

82. Mr. White can spend 39 days of his vacation at the seashore. He has been there 7 days. How many more days can he stay?

83. Pour what you think is 3 pints of water into a basin. Then measure the water and see how near you were able to guess.

84. There are 9 houses on each side of our street. How many houses are there on our street?

85. Mr. Martin said he would help carry the children to the picnic. He could carry 7 at one trip and would make two trips. How many children could ride to the picnic with Mr. Martin?

86. Mrs. Gates bought a new dress for $25, and then paid $2 to have it fitted. How many dollars did her dress cost?

87. How many cents are a quarter and a nickel?

88. How many oranges are there in half a dozen?

89. Robert sold 7 watermelons in the morning and 12 in the afternoon. How many melons did he sell that day?

90. How many 5-cent tops can you buy for 10 cents?

91. Fred had charge of the classroom library. Friday afternoon he counted the books and found only 86. There should have been 93. How many books were missing?

92. Edith bought a loaf of bread for 8 cents. She gave the storekeeper a quarter. How much change should she get?

93. Frank worked in Mr. Poe's store. Mr. Poe thought he would fill one table with things to be sold at half price. Frank was to change the price cards. The first thing Frank took was an egg beater marked 18 cents. He crossed out the "18 cents." What did he write in its place?

94. Frank next found a needlecase marked 14 cents. What was the new price mark for it?

95. A cup and saucer were marked 16 cents. What was the sale price of these articles?

96. Harry earned 25 cents by raking the yard and 3 cents by going to the store. How many cents did Harry earn?

97. Robert had 32 fine watermelons in his garden and he meant at first to sell them all. Then he decided to send 4 of them as a present to his cousins in the city. How many melons did Robert still have to sell?

98. There were 3 blackbirds walking about on the field where Mr. Lane had just planted oats. Then 19 more blackbirds came flying over and went down to join them. Now how many blackbirds were there on Mr. Lane's oat field?

99. At the school lunch counter Anna paid 5 cents for a cake of chocolate, 8 cents for sandwiches, and 3 cents for an orange. How many cents did she pay for all?

100. If a can of peas costs a quarter, how many cans can you buy for a dollar?

101. Margaret did 8 examples on each side of her paper. How many examples did she do all together?

102. Mr. Rose got 967 bushels of apples from his trees this year and 645 bushels last year. How many more bushels did he get this year than last?

103. Mr. Cole and Raymond drove 67 miles to the town where Aunt Kate lived. They stopped at a drug store to ask where her house was and were told to turn south and drive 2 miles. How many more miles did their speedometer read when they got to Aunt Kate's than it did when they left home?

104. Everett has 23 chickens; 16 of them are pure white. He is going to keep only the white ones and sell the others. How many chickens will he sell?

105. Without using your ruler draw a line which you think is 2 feet long. Now measure it with your ruler. How many inches is your line too long or too short?

106. Elsie bought some bulbs. There were 6 hyacinth, 9 tulip, and 6 lily bulbs. How many bulbs did she buy all together?

107. There are 31 days in July and 31 days in August. How many days are there in July and August together?

108. There were 8 woolly white lambs alone in a field. Then the farmer's boy opened the gate and in came a flock of sheep, 15 of them. How many animals were in the field then?

109. James picked 33 ears of green corn in his garden. His mother used 9 of them for dinner, and he sold the rest. How many ears did he sell?

110. Mrs. Parker sold 14 pounds of butter one week and 15 pounds the next. How many pounds of butter did she sell in the two weeks?

111. There were 14 turtles on a log at the edge of the pond when Frank peeped through the bushes. Then Frank moved to get a little nearer. Splash! There were only 4. How many turtles had jumped off?

112. Mr. Moore has 670 apple trees and 320 peach trees. How many more apple trees than peach trees has he?

113. A trolley car had 7 seats on each side. How many seats were there in the trolley car?

114. Fred had 4 white rabbits, 3 black ones, and 5 gray ones. How many rabbits had he in all?

115. One morning Lillian counted the boys and Harriet counted the girls in the school bus. They told Miss Gray there were 20 boys and 23 girls. Miss Gray then asked how many children there were in all. They both answered at once and they were both right. What was their answer?

116. Florence sent a birthday present to her cousin in the country. The present cost 75 cents, and she paid 4 cents for postage. How many cents did she pay in all?

117. On Saturday Doris had to read 6 pages of geography and 14 pages of "Alice in Wonderland" in order to catch up with her class. How many pages did she have to read in all?

118. There were 28 chickens in Joe's chicken yard when Joe went to school in the morning. At noon he went to give them water, and there were only 6. The rest had gone out through a little hole in the wire fence. How many had gone out?

119. Alfred and Tom went fishing. Alfred caught 21 fish and Tom caught 9. How many more fish did Alfred catch than Tom?

120. Mr. Harris planted 18 cabbage plants one evening. Next morning he went out to look at them and there were only 13. A rabbit had eaten the rest. How many had the rabbit eaten?

121. Two tickets, one for the trip across the lake and the other for the trip back, cost 68 cents. A round-trip ticket costs 62 cents. How many cents does one save by buying the round-trip ticket?

122. Raymond's new white blouse cost 79 cents and his gingham one 75 cents. How many more cents did the white blouse cost than the gingham?

123. Gene and Dorothy played a game of dominoes. Gene made 57 and Dorothy 50. By how many points did Gene win?

124. Henry bought 43 cents worth of meat. He gave the butcher 50 cents. How many cents in change did the butcher give him?

125. Mildred picked 12 pint boxes and 1 quart box of berries. How many pints was that all together?

126. Ralph wanted to send a box of eggs to his aunt in the city. The box had places for a dozen eggs, but Henry had only 8. How many more eggs did he need to fill the box?

127. Jenny bought a drawing tablet for 25 cents at a little store near the school. Hazel bought one like it for 19 cents at a big store down town. How many cents did Hazel save by buying her tablet down town?

128. Ada weighed 45 pounds on her seventh birthday. Just one year later she weighed 53 pounds. How many pounds had she gained?

129. Jack saw a book in the store window. It was marked 25 cents. He counted his money. He had 18 cents. How many more cents did he need before he could buy the book?

130. Ruth and Mabel were jumping rope. Ruth jumped 63 times and Mabel jumped 71 times. How much did Ruth lack of jumping as many times as Mabel?

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